ELASTIC BUCKLING OF PLATES UNDER IN-PLANE PATCH LOADING USING FINITE DIFFERENCE METHOD

الانبعاج المرن للصفائح تحت تأثير رقعة التحميل المستوية باستخدام طريقة الاختلافات المحددة

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Abstract:

The present study investigates the problem of linear buckling of thin steel plates subjected to in-plane patch compression loading. The stability problem treated using finite difference method. The present procedure is general and applicable to the buckling and free vibration of thin rectangular plates with various thickness variations. The influences of thickness variation, plate aspect ratios, and boundary conditions, and length of patch loading on the buckling load are shown graphically. The plate was analyzed with different tapering ratios (t_a/t_o) (1.0, 1.25, 1.5, 1.75, and 2.0), so different lengths of axial patch loading ($\boldsymbol{b_p/b}$)(1.0-0.4) were taken. A comparison with previous works is made. Finally, it is shown that the buckling load factor will increase with decreasing length of axial patch loading where the decreasing the length of axial patch loading to 0.4 will increase the buckling load factor by about 40% for plate with aspect ratio ($\boldsymbol{a/b}$=1) and tapering ratio ($\boldsymbol{t_a/t_o}$=1.0).

Keywords: Linear buckling, Thin plates, Tapered plates, Patch compression, Finite Difference method

المستخلص:

الدراسة الحالية تتحرى مسألة الانبعاج الخطي للصفائح الفولاذية النحيفة المعرضة إلى أحمال ضغط رقعة في المستوي. مسألة الاستقرارية تم معالجتها باستخدام طريقة الاختلافات المحدودة (finite difference). الطريقة المقدمة هي عامة وتطبق للانبعاج والاهتزاز الحر للصفائح المستطيلة النحيفة مع تنوع اختلافات السمك. إن تأثيرات تغير السمك, نسب الطول للعرض, الإسناد وطول رقعة التحميل على حمل الانبعاج قد بينت بالرسم. تم تحليل الصفائح مع مختلف نسب المستدقة (tapering ratios) ($\boldsymbol{t_a/t_o}$) (1.0, 1.25, 1.5, 1.75, and 2.0) وكذلك مختلف أطوال الرقعة للحمل المحوري. وأخيرا ظهر إن معامل حمل الانبعاج سوف يزداد مع تناقص طول رقعة الحمل المحوري حيث التناقص في طول رقعة الحمل المحوري إلى (0.4) سوف يزيد معامل حمل الانبعاج إلى (40%) تقريبا للصفيحة ذات نسبة أبعاد ($\boldsymbol{a/b}$=1) والنسبة المستدقة (tapering ratios) ($\boldsymbol{t_a/t_o}$=1.0).

1. INTRODUCTION

The use of thin panels in many technical fields such as aerospace, mechanical and civil engineering is nowadays quite common. Since modern design process requires the evaluation of appropriate safety levels, many studies have been carried out in the last decades in order to describe the buckling due to uniform compression, tension and shear for such structures[4]. On the other hand, a limited number of studies have been carried out to evaluate the influence of patch loading on the critical buckling load in the compressed plates although designers are always confronted with this issue. Such a problem is encountered in airframe where the action of the air loading on an aircraft wing develops an axial loading that gives a non-uniform compression that can lead to loss of stability. Also, the aerodynamic heating of panels in supersonic aircraft can be approximated by non-uniform thermal stresses as the temperature distribution is not uniform throughout the volume of the restrained plate. In civil engineering structures, engineers are often confronted with designs involving partial edge loading, such as the buckling of the web plate of a crane girder under the action of heavy wheel loads applied to the flanges. It is worth pointing out that since constructional elements are frequently subjected to in-plane patch loading and often prone to buckling, it is important that further design data should be provided to deal with this important stability problem. If such an issue has so far received relatively little attention from researchers, the reason for this is undoubtedly due to the additional theoretical difficulties involved in obtaining rigorous solutions to the buckling of plate when subjected to non-uniform compression. Undeniably, the solution of this stability problem is mathematically difficult to obtain as the stress distribution throughout the plate varies considerably(4). However, using the finite difference method, one can easily deal with these buckling problems. Few researches published their works in buckling behavior of thin steel plate under in-plane patch loading. **Pavlovic** and **Baker**[8] used an analytical method to investigate thin plate buckling, **Rockey**[9] used finite element method to investigate the buckling stiffened plate. **Stephen** and **Steven**[12] worked on the error estimation for plate buckling element. **Ikhenazen**, **et. al.** [4] used the total energy to treat the stability problems where their study showed that the resolution of the plate buckling problem using true stress distribution with the finite element method leads to a good agreement with results previously obtained by means of analytical methods using an exact stress distribution.

Hussein, **et. al.** [3] used simplified computational procedure for the elastic buckling problems of rectangular thin plates with variable cross section thickness. **Kobayashi** and **Sonod** and **Ohga et. al.** [6] used the power series method to solve the differential equation for tapered thin-walled members.

In the present study, the buckling of thin elastic plates non-uniformly compressed in one direction (see Figure (1)) is investigated using the finite difference method. This numerical analysis is performed with the FORTRAN90 program that was written by **Amash**[1]. The aim of this paper is to show some representative elastic buckling coefficient results of a simply supported plate under in-plane patch loading with constant and variable thickness. The influence of edge ratio and load breadth ratio on the critical buckling load is investigated. The obtained numerical results are graphically summarized through a `buckling load factor with varying aspect ratio, varying boundary condition, varying length of patch loading ratio, and varying tapering ratio and some interesting conclusions are drawn.

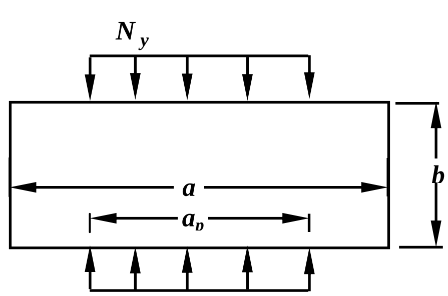


Fig. (1): Thin plate under uniaxial patch loading

2. GOVERNING EQUATION AND SOLUTION

The buckling of isotropic rectangular plates with linearly tapered thickness in the ***x***-direction is considered as shown in Figure(2). The plate is subjected to uniform compressive load in ***y***-direction. The thickness $t(x)$ and moment of inertia $I(x)$ are expressed as: -

$$t(x) = t_o(1 + c_t x) \tag{1}$$

$$I(x) = I_o(1 + c_t x)^3 \tag{2}$$

in which $c_t = (t_a - t_o)/at_o$; t_o and t_a denote the thickness at the sides $x = 0$ and $x = a$, respectively; $I_o = t_o^3/12$ is the second moment of area(per unit width) for the plate cross section at the side $x = 0$.

Within the classical small deflection theory of thin plates, the differential equation for the rectangular plate under consideration can be written in the form [Husain, et al (2002)]: -

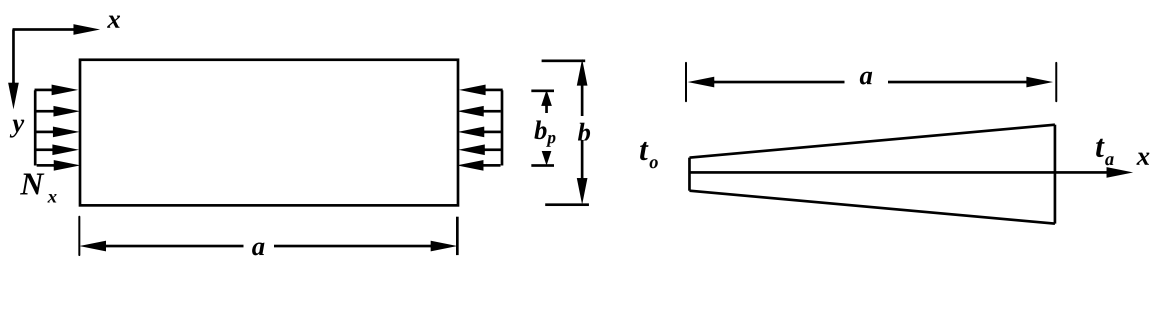
$$\nabla^4 w + \frac{2I'_x}{I_x}\left(\frac{\partial^3 w}{\partial x^3} + \frac{\partial^3 w}{\partial x \partial y^2}\right) + \frac{I''_x}{I_x}\left(\frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 w}{\partial y^2}\right) = \frac{1}{D_{(x)}}\left(N_x \frac{\partial^2 w}{\partial x^2}\right) \tag{3}$$

In which

$D_{(x)} = Et_{(x)}^3/12(1 - \nu^2)$ = is the flexural (or bending) rigidity of the section of the plate (and this is varying with respect to ***x***).

$$\nabla^4 = \frac{\partial^4}{\partial x^4} + 2\frac{\partial^4}{\partial x^2 \partial y^2} + \frac{\partial^4}{\partial y^4}$$

$$I_x = I_o(1 + c_t x)^3$$
$$I'_x = 3c_t I_o(1 + c_t x)^2$$
$$I''_x = 6c_t^2 I_o(1 + c_t x)$$



(a) Plate under Axial Compressive Patch Load (b) Plate Cross Section

Fig. (2): Buckling load of linearly tapered plate under in-plane patch loading

The solution of Equation (3) may be achieved by finite difference method as shown in Figure (3). By applying the finite difference molecules at the interior nodes of the subdivided plate, the following system of simultaneous linear equations in matrices will be obtained: -

$$[K]\{w\}+\lambda[B]\{w\}=0 \qquad (4)$$

where the matrices $[K]$ and $[B]$ may be named as follows:

$[K]$: is the stiffness matrix for the plate

$[B]$: is the geometry matrix for the plate

$\lambda = \dfrac{N_x D\pi^2 h_y^2 h_x^4}{a^2}$: is the Eigen-value

Notice that Equation (4) is an Eigen-value problem. For a given thickness (t_o, t_a) and plate–aspect ratio (a/b), the Eigen-value (λ) can be determined numerically by using any relevant technique. The smallest Eigen-value gives the most (fundamental) buckling load.

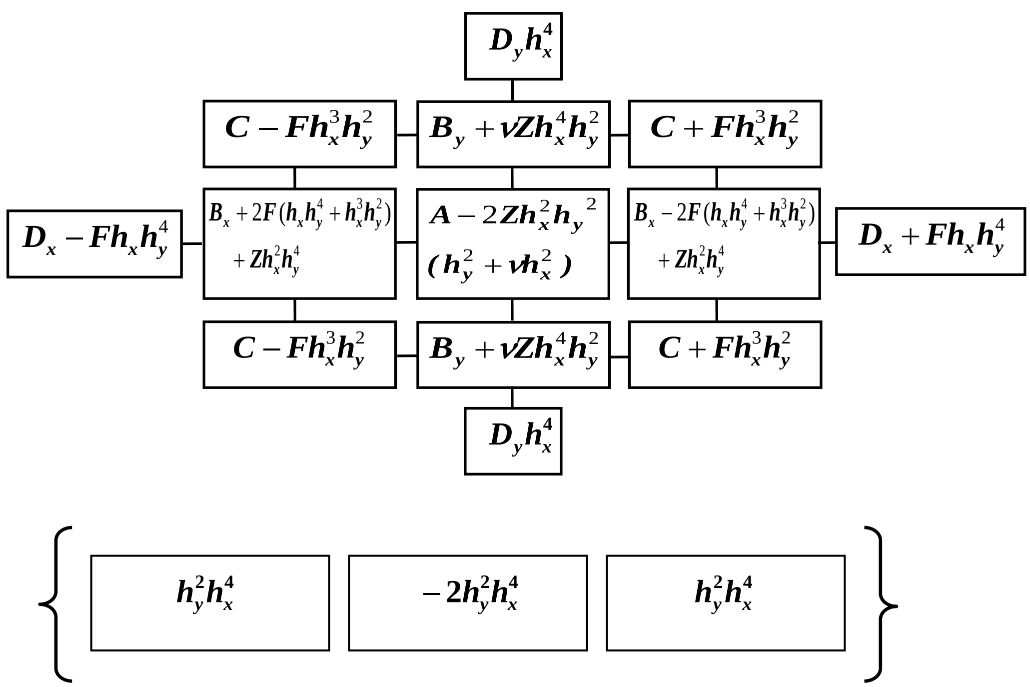


Fig. (3): Plate equation in finite difference molecule form

3. NUMERICAL RESULTS

In order to find the more appropriate mesh size that is to be used in the present stability problem, the case of a simply supported rectangular plate under axial compressive load in the direction of its length is investigated. The plate ratio is ($\boldsymbol{a/b}$=**1**), slenderness ratio ($\boldsymbol{b/t}$=**100**), Young's modulus ($\boldsymbol{E}$=**200 GPa**) and Poisson's ratio ($\boldsymbol{v}$=**0.3**) are considered. Table (1) gives a measure of convergence as a function of mesh size. It can be seen that a (14 × 14) mesh for this problem that gives results to within (0.4%) of the exact Timoshenko's value [13], which in this case corresponds to (4.00).

Table (1): Convergence of buckling coefficient ($\boldsymbol{k}^*$) for a square simply supported plate with constant thickness ($\boldsymbol{a/b}$=1.0)

Mesh size	Buckling coefficient
8 × 8	3.948
10 × 10	3.967
12 × 12	3.977
14 × 14	3.983
16 × 16	3.987
18 × 18	3.989
20 × 20	3.994

$$*\boldsymbol{k} = \boldsymbol{N}_x \boldsymbol{b}^2 / \boldsymbol{D}\boldsymbol{\pi}^2$$

A present study is performed to assess the influence of several important parameters on the linear buckling of a rectangular steel plate subjected to in-plane compressive patch load. Each parameter was studied individually by analyzing a type of rectangular steel plate. In all cases, a finite difference method was used by considering the full plates with (14×14) mesh. The following geometry and material properties of steel plate are used in the analysis: ($\boldsymbol{E}$=200 GPa; $\boldsymbol{v}$=0.30, $\boldsymbol{F_y}$=250 MPa,).

Figure (5) presents the buckling factor-patch length ratio curve of a simply supported rectangular steel plate with variable thickness under in-plane compressive patch loading with slenderness ratios ($\boldsymbol{b/t}$=100), aspect ratio ($\boldsymbol{a/b}$=1.0). The compression patch load is in $\boldsymbol{x}$-direction (the direction parallel to thickness variation). The range of tapering ratio ($\boldsymbol{t_a / t_o}$) was used from (1.0) to (2.0) and patch length ratio($\boldsymbol{b_p/b}$) was used from (0.4) to (1.0). From this figure, it can be noticed that the buckling factor will decrease with increasing patch length ratio for all tapering ratios about (38%) for plate with tapering ratio (2.0) when patch length increase from (0.4) to (1.0).

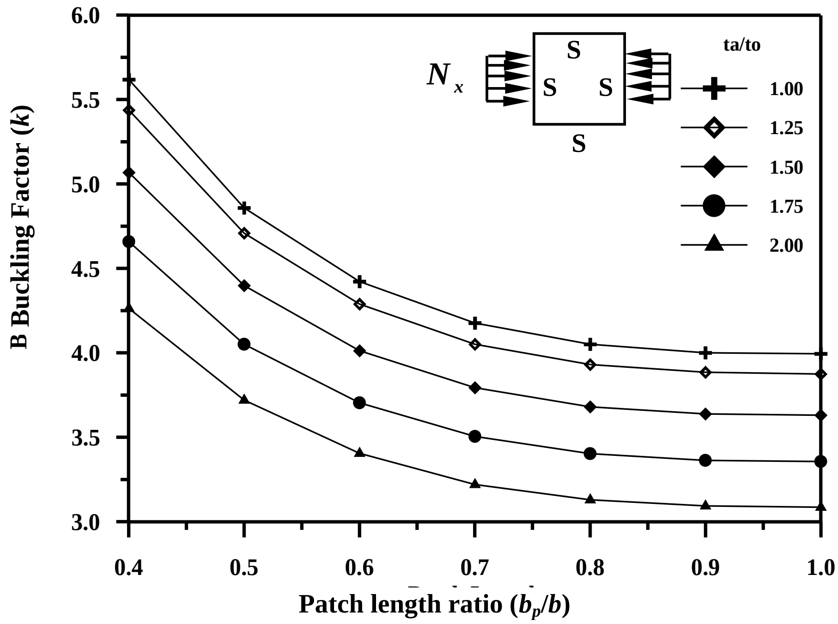


Fig. (5): Buckling Factor-Patch length ratio of simply supported rectangular steel plate with variable thickness under in-plane patch loading along ***x***-direction

Figure (6) presents the buckling factor-patch length ratio curve of simply supported rectangular steel plate with variable thickness under in-plane compressive patch loading with slenderness ratios (***b/t***=100), aspect ratio (***a***/***b***=1.0). The compression patch load is in ***y***-direction (the direction parallel to thickness variation). The range of tapering ratio (t_a / t_o) was used from (1.0) to (2.0) and patch length ratio(a_p/a) was used from (0.4) to (1.0). From this figure, it can be noticed that the buckling factor will decrease with increasing patch length ratio for all tapering ratios about (42%) for plate with tapering ratio (2.0) when patch length increases from (0.4) to (1.0). This can be attributed to the fact that the increase in the thickness ratio amounts to the decrease in the flexural rigidity of the plate.

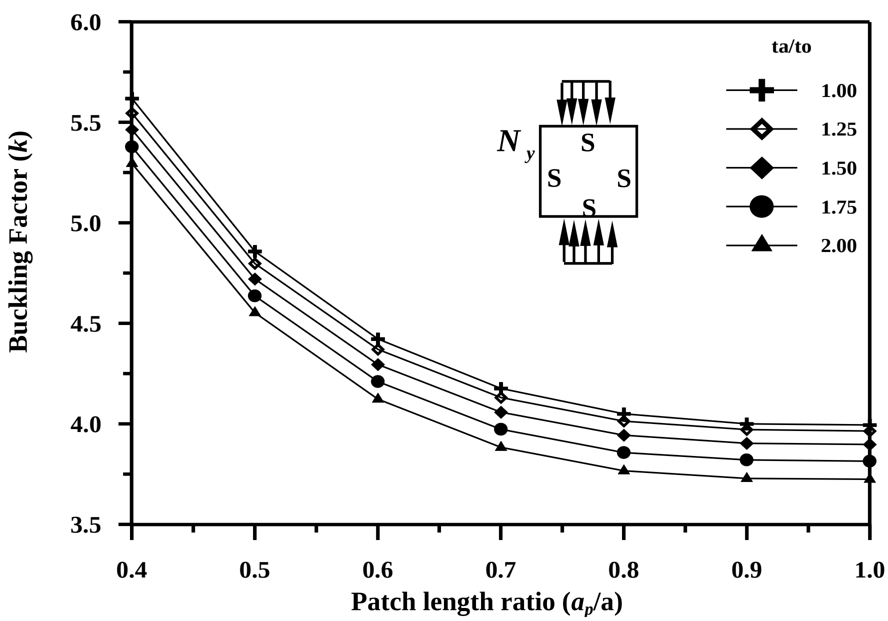


Fig. (6): Buckling Factor-Patch length ratio of simply supported rectangular steel plate with variable thickness under in-plane patch loading along ***y***-direction

Figure (7) presents the buckling factor-patch length ratio curve of clamped rectangular steel plate with variable thickness under in-plane compressive patch loading with slenderness ratios (***b/t***) (100), aspect ratio (***a***/***b***=1.0). The compression patch load is in ***x***-direction (the direction parallel to thickness variation). The range of tapering ratio (t_a / t_o) was used from (1.0) to (2.0) and patch length ratio(b_p/b) was used from (0.4) to (1.0). From this figure, it can be noticed that the buckling factor will decrease with increasing patch length ratio for all tapering ratios about (31%) for plate with tapering ratio (2.0) when patch length increases from (0.4) to (1.0).

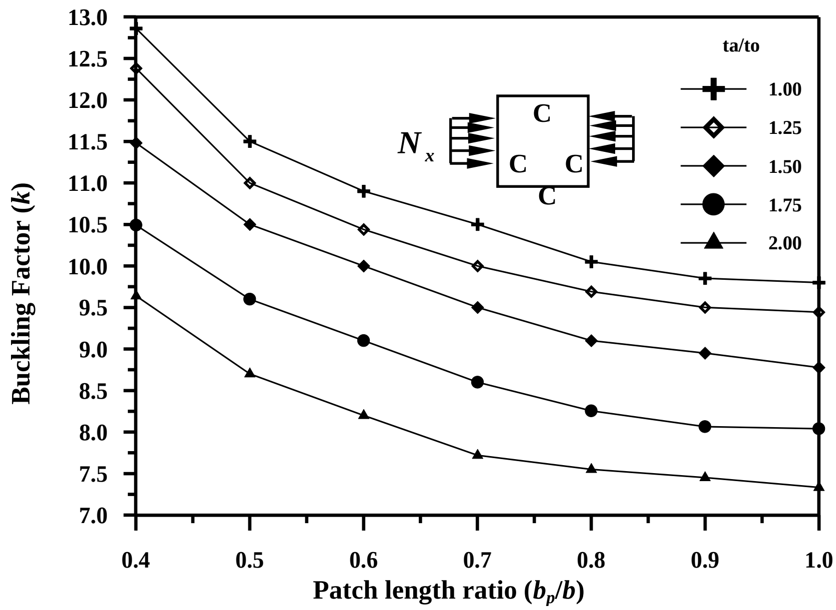


Fig. (7): Buckling Factor-Patch length ratio of rectangular steel plate with variable thickness under in-plane patch loading at ***x***-direction

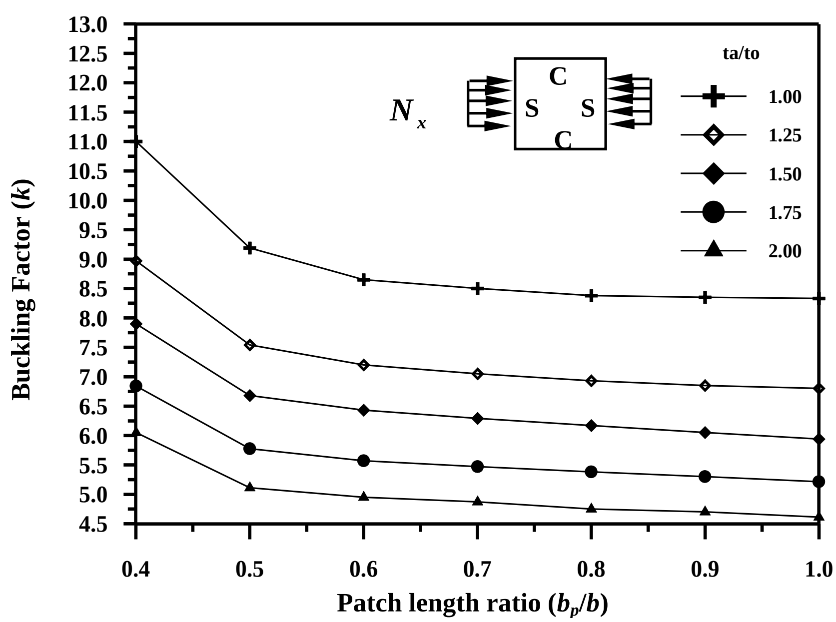


Fig. (8): Buckling Factor-Patch length ratio of simply supported rectangular steel plate with variable thickness under in-plane patch loading at ***x***-direction

Figure (9) presents the curves of buckling coefficient of rectangular steel plates with variable thickness under in-plane compressive patch loading which represent the changes in the buckling mode in the ***x***-direction as the plate aspect ratio changes. The compression patch load in ***x***-direction (the direction parallel to thickness variation). The range of aspect ratios (***a***/***b***) were used from (0.5) to (4.0), tapering ratio (t_a / t_o =1.0) and patch length ratio (b_p/b) was used from (0.4) to (1.0). It can be seen from the figure that the plate aspect ratios at which changes in such buckling mode occur appear to be almost independent of the patch length ratio.

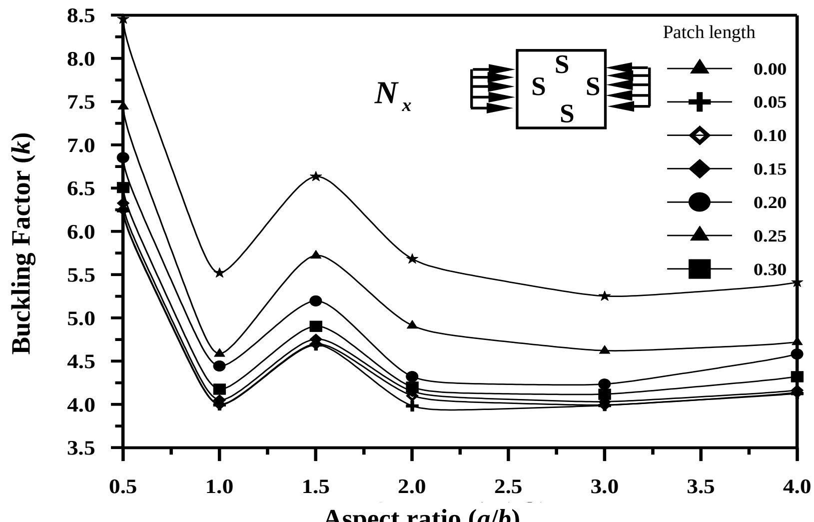


Fig. (9): Buckling Factor-aspect ratio of simply supported rectangular steel plate with tapering ratio (t_a / t_o =1.0) under in-plane patch loading at ***x***-direction

Figure (10) presents the curves of buckling coefficient of rectangular steel plates with variable thickness under in-plane compressive patch loading which represent the changes in the buckling mode in the ***x***-direction as the plate aspect ratio changes. The compression patch load in ***x***-direction (the direction parallel to thickness variation). The range of aspect ratios (***a***/***b***) were used from (0.5) to (4.0), tapering ratio (t_a / t_o =1.5) and patch length ratio (b_p/b) was used from (0.4) to (1.0). It can be seen from the figure that the plate aspect ratios at which changes in such buckling mode occur appear to be almost independent of the patch length ratio.

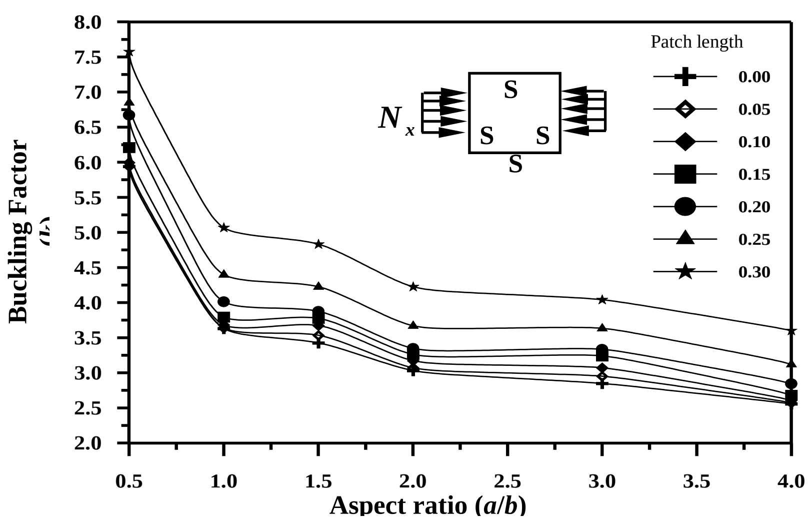


Fig. (10): Buckling Factor-aspect ratio of simply supported rectangular steel plate with tapering ratio (t_a / t_o =1.5) under in-plane patch loading at ***x***-direction

4. CONCLUSIONS

A finite difference method has been employed to solve numerically the buckling problem of rectangular plates with linearly tapered thickness under in-plane patch loading. The effects of aspect ratio, boundary condition, tapering ratio, and patch length ratio on the buckling behavior are considered. The values of buckling coefficients decrease with an increase in the tapering ratio (for the same volume of the plate) and so with increasing patch length ratio. The buckling coefficient of simply supported plate under in-plane patch loading at x-direction will decrease about (40.6%) for tapering ratio (t_a / t_o =1.0) while it will decrease by about (38%) for tapering ratio (t_a / t_o =2.0) but when the plate under in-plane patch loading at y-direction the buckling coefficient will decrease about (28.8%) for tapering ratio (t_a / t_o =1.0) while will decrease about (42%) for tapering ratio (t_a / t_o =2.0).

REFERENCES

[1] Amash,H.K., “Post-buckling and Post-yielding analysis of Imperfect thin plate by finite difference method.” M.Sc. thesis, University of Babylon, February 2003.

[2] Chehil, D.S., and Dua, S.S., “Buckling of Rectangular Plates with General Variation in Thickness “, J. Appl. Mech., Vol.40, 1973, pp.745-751.

[3] Hussein, H.M., Al-Daami, H., and Amash, H.K., “Buckling Behavior of Rectangular Plate with Variable Thickness.” Eng. of Technology Vol.21, No.10, 2002, pp.736-745.

[4] Ikhenazen,G., Saidani,M.m Chelghoum, A. “ Finite element analysis of linear plates buckling under in-plane patch loading ”, Journal of Constructional Steel Research, Vol.66, 2010, pp. 1112-1117.

[5] Jia-rang, F., “Plates of Varying Thickness with Four Simply Supported Edges”, ASCE, J. Eng. Mech. Div., Vol.108, No. EM1, Feb., 1982, pp.167-179.

[6] Kopayashi, H., and Sonoda, K., “Buckling of Rectangular Plates with Tapered Thickness”, ASCE, J. Struct. Eng., Vol.116, No.5, May, 1990, pp.1278-1289.

[7] Ohga, M., Shigematsu, T., and Kawagouchi, K., “Buckling Analysis of Thin-Walled Members with Variable Thickness”, ASCE, J. Struct. Eng., Vol.121, No.6, June, 1995, pp.919-924.

[8] Pavlovic NM, Baker G. Buckling of non-uniformly compressed plates. Theoret Appl Mecha._Transaction of the Yugoslav Society of Mechanics, vol. 9. 1983.pp. 91_103.

[9] Rockey KC, Bagchi DK. Buckling of plate girder webs under partial edgeloadings. Int J Mech Sci 1970;12(1):61_76.

[10] Salvadori, M.G., “Numerical Computation of Buckling Loads by Finite Differences”, ASCE, Trans., Dec., 1949, pp.590-636.

[11] Szilard, R., “Theory and Analysis of Plates: Classical and Numerical Methods”, Prentice- Hall, Inc., Englewood Cliffs, New York, 1974.

[12] Stephen DB, Steven GP. Error estimation for plate buckling elements. Comput & Structures 1996;61(4):747_61.

[13] Timoshenko, S.P., and Woinowshy, S., “Theory of Plates and Shells”, 2nd Ed, McGraw-Hill Book Co., Inc., New York, 1959.

[14] Timoshenko, S.P., and Gere, J.M., “Theory of Elastic Stability”, 2nd Ed, Mc Graw-Hill Book Co., Inc., New York, 1961