Numerical Study of Two-Dimensional Transient Natural Convection in an Inclined Shallow Porous Cavity Exposed to a Constant Heat Flux

Dr. Ayad K. Hassan
Materials Engineering Department, University of Technology/Baghdad
Email: ayad_kadhm@yahoo.com
Dr. Jasim M. A. Al-lateef
Engineering College, University of Diyala/ Diyala
Email: jasimmohE7777@yahoo.com

 ABSTRACT
   Numerical method is used to solve the two-dimensional transient natural convection heat transfer problem in an inclined shallow porous cavity. A constant heat flux is applied for heating and cooling all opposing walls. Solutions for laminar case are obtained within Rayleigh number varied from 20 to 500 and aspect ratio for porous cavity varied from 2 to 4. A finite difference method is used to obtain numerical solutions of full governing equations. Energy equation is solved using alternating direct implicit (ADI) method and stream function equation by successive over relaxation (SOR) method. The results are presented for the flow filed, temperature distributions, and average Nusselt number in terms of the Rayleigh number, aspect ratio, and the inclination angle of cavity. the convection becomes more and more vigorous as the orientation angle of the cavity is increased and for high Rayligh number no steady unicellular flow could be maintained inside the cavity. The effect of inclination angle on Nasselt number is more pronounced as the Rayleigh number is increased. When the inclination angle increased the Nusselt number increased and sudden transition appears and flow becomes unicellular and Nusselt number increased clearly. The value of mean Nusselt number strong function with the value of Rayleigh number, aspect ratio and the orientation of porous cavity.

دراسة عددية للعمل الطبيعي العابر ذي البعدين في فجوة رقيقة مائلة لوسط مسامي معرضة الافيفي حراري ثابت

الخلاصة

    الطـرق العدديـة استخدمت لحـل مسـالة انتقـال الحـرارة ثنـائي الابعـاد بالحمـل الحـر للحالـة غير المسـتقرة مـن خـلال تجويف مسـامي سـطحي مائـل. تم تسـليط فيض حـراري ثابت لتسـخين وتبريـد الجدران المتقابلة. الحل العددي كان ضمن حـالة الجريان الطباقي ضمن رقم رالي يتراوح من 20 الى

Eng. & Tech. Journal, Vol.30, No.15, 2012   Numerical Study of Two-Dimensional Transient
Natural Convection in an Inclined Shallow
Porous Cavity Exposed to a Constant Heat Flux

50 ونسبة باعية تتراوح من 2 الى 4. تم استخدام طريقة الفروقات المحددة للحصول على الحل
العددي للمعادلات الحاكمة. معادلة الطاقة تم حلها باستخدام طريقة الاتجاه الضمني المتناوب و معادلة
دالة الانسياب تم حلها باستخدام طريقة التراخي فوق التعاقب. تم تمثيل نتائج الجريان وتوزيع درجات
الحرارة ومعدلات رقم نسلت بدلالة رقم رالي ونسبة الباعة وزاوية ميلان التجويف.ان انتقال الحرارة
بالحمل يصبح كبير واكثر فعالية عند زيادة زاوية الميلان للتجويف. عند قيم رقم رالي العالية لايمكن
الحفاظ على جريان مستقر واحادي الخلية داخل التجويف . ان تاثير زاوية الميلان على رقم نسلت
يتضح اكثر عند زيادة رقم رالي. ان زيادة زاوية الميلان يؤدي الى زيادة رقم نسلت ويضهر تحول
مفاجأ والجريان يصبح احادي الخلية ورقم نسلت يزداد بوضوح. ان قيم رقم معدل رقم نسلت تعتمد
بشكل كبير على رقم رالي ونسبة الباعة وزاوية ميلان التجويف المسامي.

INTRODUCTION

Over the past years, natural convection heat transfer in cavities filled with a fluid-saturated, porous medium has several important geophysical and engineering applications. These include regenerative heat exchangers containing porous materials, high performance insulation for building and cold storage, solar power collection, underground spread of pollutants, and convection in the earth's curt Buchberg et al. (1976), Seki et al. (1978). Another important area of application is heat transfer from the storage of agriculture products which generate heat transfer as a result of metabolism. Natural convection effects on heat transfer in a differentially heated rectangular porous cavity, with top and bottom walls insulated, is of fundamental interest in each of these areas. Several investigators Seki et al.(1978), Chan et al. (1970) , Burns et al. (1976), Walker and Homsy (1978) , Bejan (1979) , Simpkins and Blythe (1980) and Prasad and Kulacki, (1984) have presented analytical and experimental results for the case when both the vertical walls are at constant temperature. Analytical work includes numerical solutions, boundary layer solutions, integral analyses, and series solutions. Based on the past studies, various correlations, covering a wide range of Rayleigh number and cavity height- to-width (aspect) ratios, have been presented for heat transfer coefficients Seki et al. (1978), Chan et al. (1970), Walker and. Homsy (1978), Weber(1975) and Bories and Combarnous (1973).

The previous theoretical publications deal with vertical Burns et al.(1976) and Weber (1975) or horizontal Eldr (1974) case. For situations involving inclined layers, available studies are relatively limited. The problem of a sloped porous layer, heated isothermal from below has been considered theoretically and experimentally by Bories and Combarnous (1973). Depending on the value of slope of the layer and Rayleigh number, different shapes of free convection movements have been observed.

Holst and Aziz (1972) considered temperature-dependent physical properties, investigated the heated transfer of a tilted square of porous material. More recently, the existence of multiple solutions, in a slightly inclined porous cavity heated from the below, has been studied numerically by Moya et al. (1981) and analytically by Caltagirone and Bories (1985) who determined their stability. It was demonstrated that for small angles of inclinations, three different real solutions may exist for a given Rayleigh number and aspect ratio. Vasseur et al. (1986), studied the effect of

Eng. & Tech. Journal, Vol.30, No.15, 2012   **Numerical Study of Two-Dimensional Transient**
Natural Convection in an Inclined Shallow
Porous Cavity Exposed to a Constant Heat Flux

natural convection in an inclined, rectangular, porous layer when a constant heat flux is applied on two opposing walls, while the other two walls are maintained adiabatic.

Double–diffusion occurs in a wide range of scientific fields, such as oceanography, astrophysics, geology, biology and chemical processes; so, the author's interest more and more for the heat and mass transfer developed in enclosures or cavities. About these case of fluid flows generated by combined temperature and concentration gradients, the studies of double-diffusive natural convection have centered chiefly their analyses on the limit cases of dominating thermal buoyancy force or concentration buoyancy force. The considered spaces are enclosures comprising a fluid completely occupied by porous medium Alavyoon (1993), Chamkha and Al-Naser (2001) and Bennacer et al (2001).

The problem of double-diffusive flow inside an inclined square cavity which is divided by a porous medium was studied numerically by Rahli and Bouhadef(2004).The numerical finite volume method was employed to resolve the governing equations which describe the problem. Graphical results for various parametric conditions were presented and discussed. It was found that the heat and mass transfer mechanisms and the flow evolution inside the enclosure depend strongly on the dimensionless characteristic parameters (Lewis number Le, Darcy number Da, enclosure inclination angle ϕ and buoyancy ration).

Thus, the above studies have considered cavities with isothermal walls, natural convection in porous enclosures, focused on the case of rectangular cavities heated and cooled only through two opposing sides while the other two sides are kept adiabatic, however in practice, all the faces of the enclosure may be thermally active. Despite the fact that in many engineering applications the temperature of a wall is not uniform but, rather, is a result of imposition of a constant heat flux Vasseur et al. (1986). Results available for situations where a constant heat flux is applied on one wall Prasad et al. (1984) or two walls Bejan (1983) have been reported only for the case of a vertical cavity.

The objectives of the present work is to analyze the behavior of natural convection flows in a shallow inclined porous layer, when all four faces of the rectangle enclosure are exposed to constant heat fluxes, opposite boundaries being heated and cooled, respectively, which are new to the author's knowledge.

When the porous layer is slightly inclined with respect to the horizontal line, several types of flow configurations appear Caltagirone and Bories (1985). During the last years, several authors have been studied the criterion for transition between the different configurations of such flows. Weber (1975) demonstrated that a three-dimensional perturbation is steadier than a two-dimensional one if the inclination angle is close to zero. The existence of different flow configurations and the transition between them was also investigated by means of two- and three-dimensional numerical simulations by Chan et al. (1970).

On the basis of the parallel flow approximation, a closed-form solution is obtained for the temperature and velocity distribution in the limit of a shallow enclosure (A>>1). In the following section, the differential equations, which described the physical model considered here, are formulated in a standard manner assuming the validity of Darcy's law and the Boussinesq approximation method. The full

2579

Eng. & Tech. Journal, Vol.30, No.15, 2012   **Numerical Study of Two-Dimensional Transient**
Natural Convection in an Inclined Shallow
Porous Cavity Exposed to a Constant Heat Flux

governing equations are solved numerically, using finite difference method. Effects of various parameter such as Ra, (the Rayleigh number), ϕ, (angle of inclination), and A, (aspect ratio), are analyzed.

STATEMENT OF THE PROBLEM

Consider the natural convection motion of a fluid filling a homogenous, isotropic, porous medium on all sides by an impermeable rectangular box. The enclosure, shown in **Figure (1)**, is of height H, width L and is tilted at an angle ϕ, with respect to horizontal plane. A constant heat flux (q) is applied along the top and bottom the y-axis boundaries which heat and cool respectively at the same rate. A constant heat flux (aq), where (a) is a constant, is also applied in the x-axis on the two boundaries, where assume (a=1) equal unity in the present study to ensure uniform heat flux for all sides of porous layer.

Assuming the validity of Darcy's law and Boussinesq approximation, the equations describing conservation of mass, momentum and energy in the medium are as follows Vasseur et al. (1986) and Torrance (1985).

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad \qquad \text{....(1)}$$

$$u = -\frac{K}{m}\left(\frac{\partial p}{\partial x} + rg\cos f\right) \qquad \qquad \text{...(2a)}$$

$$v = -\frac{K}{m}\left(\frac{\partial p}{\partial y} - rg\sin f\right) \qquad \qquad \text{....(2b)}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = a\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) \qquad \qquad \text{.... (3)}$$

where *u, v, p, g, K, m,* and *a* stand for the velocity components in *x* and *y* directions, pressure, gravitational acceleration, medium permeability, viscosity of fluid and thermal diffusivity of porous medium, respectively. Here, it has been assumed that the fluid properties are constant, except for the density variation in producing the bouncy force. Viscous drag and inertia terms are neglected because their magnitudes are small order compared to other terms. Also, heat transfer by radiation is assumed to be small compared to conduction and convection and hence is neglected in the formulation of the problem Vasseur et al. (1986) and Prasad and Kulacki (1984).
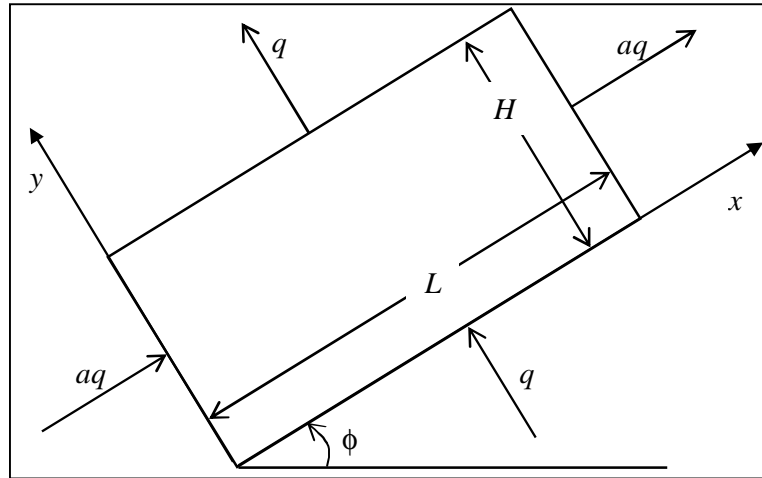
Figure (1) The physical model and coordinate system.

As usual, the governing equations are simplified if u and v is replaced by approaching define a stream function Y' which is satisfies the continuity eq.(1) identically

$$u = \frac{\partial Y'}{\partial y} \, , \quad v = -\frac{\partial Y'}{\partial x} \qquad \qquad …(4)$$

Further, the pressure terms appearing in eq.(2) are eliminated through cross-differential. The momentum and energy equations become:

$$\nabla^2 Y' = \frac{-Kgb}{u}(\frac{\partial T}{\partial x}cosf - \frac{\partial T}{\partial y}sinf) \qquad ….(5)$$

$$\nabla^2 T = \frac{1}{a}(\frac{\partial Y}{\partial y}\frac{\partial T}{\partial x} - \frac{\partial Y}{\partial x}\frac{\partial T}{\partial y}) + \frac{1}{a}\frac{\partial T}{\partial t} \qquad … (6)$$

where υ- is the kinematics viscosity of fluid μ/ρ.

Finally, eqs.(5 and 6) are transferred in non-dimensional form by defining a new set of variables

$$t = \frac{ta}{L^2} \, , \quad X = \frac{x}{L} \, , \quad Y = \frac{y}{L} \, , \quad Y = \frac{Y'}{a} \, , \quad Q = \frac{(T-T_o)}{DT} \qquad … (7)$$

Where

T_o, is the temperature at the geometric center of the cavity and $\Delta T = (qL)/k$, a characteristic temperature difference.

The resulting equation for the stream function Ψ and temperature Θ are:

$$\nabla^2 Y = -Ra\left(\frac{\partial Q}{\partial X}\cos f - \frac{\partial Q}{\partial Y}\sin f\right) \qquad \dots (8)$$

$$\nabla^2 Q = \left(\frac{\partial Y}{\partial Y}\frac{\partial Q}{\partial X} - \frac{\partial Y}{\partial X}\frac{\partial Q}{\partial Y}\right) + \frac{\partial Q}{\partial t} \qquad \dots (9)$$

Where

Ra, is a Rayleigh number based on the constant heat flux (q) and the permeability *K* of the medium

$$Ra = \frac{gbKL^2 q}{auk} \qquad \dots(10)$$

The boundary conditions on Ψ and Θ are:

$$Y = 0 \quad , \quad \frac{\partial Q}{\partial X} = \mathbf{m}a \quad \text{on} \quad X = 0, A \qquad \dots (11a)$$

$$Y = 0 \quad , \quad \frac{\partial Q}{\partial Y} = \mathbf{m}1 \quad \text{on} \quad Y = 0,1 \qquad \dots(11b)$$

And initial condition

$\Theta = 0$ and $\Psi = 0$ everywhere

where

A=H/L, is the cavity aspect ratio.

a, is a constant controlling the fraction of the heat flux imposed on the y-axis walls with respect to that imposed on the x-axis walls, assume equal unity in the present study.

The overall heat transfer cross the enclosure is expressed by average Nusselt number, defined by **Vasseur et al. (1986).**

$$\overline{Nu} = \left(\frac{q}{\overline{DT}}\right)\frac{L}{k} = \frac{1}{DQ} \qquad \dots (12)$$

where : $DQ = Q_{(A/2,1)} - Q_{(A/2,0)}$, is the side to side temperature difference at the center of the cavity.

$\overline{DT}$, is the actual wall-to-wall temperature difference.

Eng. & Tech. Journal, Vol.30, No.15, 2012   **Numerical Study of Two-Dimensional Transient**
Natural Convection in an Inclined Shallow
Porous Cavity Exposed to a Constant Heat Flux

The parameters governing the present problem are the thermal Rayleigh number, Ra, the cavity aspect ratio, A, and the angle of inclination ϕ.

The problem is to find the functions Ψ and Θ which satisfy the governing eqs. (8) and (9) and boundary conditions (11a) and (11b) for the case of long shallow cavity, i.e. for the condition A>>1 with fixed values of Ra.

NUMERICAL SOLUTION

To obtain the numerical solutions of the complete governing eqs. (8) and (9), finite difference method was used. The solution consists of stream function and temperature fields as well as the velocity distribution in x and y directions.

The energy equation was solved using the alternating direction implicit (ADI) method AZIZ and Hallums (1967). The stream function field was obtained from eq.(8) using successive over-relaxation method (SOR) and a known temperature distribution. Forward time and central space differences were used and the advective term in the energy equation was written in conservative form to preserve the transported property.

To test the present method of formulation and the finite difference scheme, various combinations of mesh sizes were used to select one which give better accuracy and requires less computational time. The number of grid points in the x and y directions were varied, depending upon the aspect ratio, A, of the cavity. As expected it was found that the necessary number of grid lines depends on the Rayleigh number, Ra, and the aspect ratio, A, of the cavity. The following are the grid fields used for the several aspect ratios considered in the present work;

Grid field	A
41x31	2
41x31	3
51x41	4

In order to gain confidence in our results, it is tried to compare ours with available previous published results. Thus, we compare our numerical solution but exposed to the same conditions (a=1) by Vasseur et al. (1986). After obtaining confidence in our results, see **Table.(1)**, we processed to compute the transient mean Nusselt number.

Table (1) Mean Nusselt Number for Ra=250, $f = 90^o$, and A=4

Grid field	$\overline{Nu}$ [Present Study]	Grid field	$\overline{Nu}$ [Vasseur et al. (1986)]
51x41	4.327	51x51	4.587
81x81	4.411	81x81	4.546

The iterative procedure for the stream function was repeated until the following condition was satisfied:

Eng. & Tech. Journal, Vol.30, No.15, 2012　　**Numerical Study of Two-Dimensional Transient**
Natural Convection in an Inclined Shallow
Porous Cavity Exposed to a Constant Heat Flux

$$\frac{\sum_i \sum_j \left| Y_{i,j}^{n+1} - Y_{i,j}^n \right|}{\sum_i \sum_j \left| Y_{i,j}^{n+1} \right|} \leq 10^{-4}$$

….(13)

Where the superscripts (n) and (n+1) indicate the value of the (n)th and (n+1)th iterations respectively and i and j indices denote grid location in the (x,y) plane. Further decrease of the convergence criteria (10^{-4}) did not cause any significant change in the final results.

　　The steady state was defined based on the following criteria:

$$\left| \frac{\overline{Nu}^{n+1} - \overline{Nu}^n}{\overline{Nu}^n} \right| \leq 10^{-4}$$

… (14)

where $\overline{Nu}$ is the average Nusselt number. The iterative procedure was carried out until above criteria was satisfied.

RESULTS AND DISCUSSION

Computations were conducted for a range of Rayleigh number Ra, 20,100, and 500 with aspect ratio varied from 2, 3, and 4. The inclination angle of enclosure from horizontal plane also varied from 0° to 180°. Flow patterns and temperature fields for some typical values of Rayleigh number and aspect ratio are presented in **Figs (2-5)**. Compared to the case of the constant temperature at both vertical walls **Prasad and Kulacki (1984)** and constant heat flux from two side and other is insulated **Vasseur et al. (1986),** constant heat flux on one vertical wall **Prasad and Kulacki (1984),** temperature fields in the present case are some different.

When an inclined porous layer saturated by a fluid satisfying the Boussinesq approximation is differentially heated, a wide range of two-three dimensional, stationary or non-stationary flow configuration appear **Caltagirone and Bories (1985).** These configuration depends on the geometric dimensions of the porous media (aspect ratio), angle of inclinationϕ, and the Rayleigh number Ra.

The basic flow which develops in differentially heated inclined porous layer is of an unicellular two-dimensional type. Thus, when the porous layer slightly inclined with respect to the horizontal line, the flow which takes place spontaneously is of unicellular type. At lower value of Rayleigh number, the flow is setting up of longitudinal rolls and remains steady for wide range of inclination angle **Caltagirone and Bories (1985).**

Here isotherms for any size of cavity start either from the heated wall. Similar isotherms patterns have been reported for free convection in non porous vertical cavity by **Said and Trupp (1982) and Balvanz and Kuehn (1983)**, though no adverse temperature gradients or "S" –shaped isotherms **Said and Trupp (1982)** are

2584

Eng. & Tech. Journal, Vol.30, No.15, 2012   **Numerical Study of Two-Dimensional Transient
Natural Convection in an Inclined Shallow
Porous Cavity Exposed to a Constant Heat Flux**

observed for the present case.

The thermal boundary thickness increases on the heated wall, for the velocity boundary layer thickness, the growth is different and is largely due to the change in bouncy effect. We can note that the convection becomes more and more vigorous as the orientation angle of the cavity is increased.

In the numerical results, the flow inside the cavity was steady and unicellular flow in the case of tilted porous rectangular cavity. When the angle of inclination angle approaching to the horizontal position, the flow might be multicelullar pattern, see **Figs. (5)** and (**6**). In fact, when the angle of inclination angle is approaching 180°, the flow might not be two-dimensional as assumed in the theoretical and numerical solutions **Vasseur et al. (1986).** Experimental observations and three-dimensional numerical simulations have show that, in the case of tilted, porous, rectangular cavity, the flow remains two-dimensional for $0^o < \phi < 173^o$ but for $\phi > 173^o$, oblique rolls were obtained Bories and Combarnous (1973). Also, its should be denoted that for very high values of the Rayleigh number, no steady unicellular flow could be maintained inside the cavity.

Streamlines close to the heated surfaces are observed to run parallel to the wall over significantly large portion of its extent. This behavior becomes more prominent as the aspect ratio is increased and increasing in the inclination angle ϕ. As excepted, the temperature field are strong function of Rayleigh number and aspect ratio. As Ra is increased (**Figs.2 - 4**), isotherms shifts toward the constant flux wall and corners. This result in an asymmetric core region flow. As increase in aspect ratio further pushes the isotherms towards the corner of heated surface indicating high velocity.

The value of Nusselt number is important for design proposed because its directly gives the value of a range temperature $\Delta\Theta$, for any applied heat flux which , in turn gives the order of the temperatures to be encountered any particular values of Rayleigh number and aspect ratio. The orientation angle ϕ is seen to have a dominate effect on the Nusselt number for a given Rayleigh number. As the angle of inclination ϕ approach to the horizontal position, the Nusselt number at low Rayleigh number tends towards unity, indicating that the heat transfer is mainly due to conduction. Most of the change in heat transfer occurs in the range $0^o < \phi < 90^o$ and $90^o < \phi < 180^o$. Also, it's noticed that the Nusselt number is strong function of Rayleigh number.

For Ra>500, there is no numerical results are presented since they did not provide sufficient additional insight into the problem and also the computing time necessary to obtained an accurate steady-state solution become rapidly prohibitive.

Generally, the average Nusselt number show fairly large dependence on inclination angle ϕ. Also, we can note that, the effect of heating the cavity from $0^o < \phi < 90^o$ on the Nusselt number is seen to be large in comparison with that heating from $90^o < \phi < 180^o$. It's also noticed that the effect of inclination angle on Nusselt number is some more pronounced as the Rayleigh number is increased. A similar tends, rather fore, has been reported in the case of inclined fluid cavities contain two opposite of thermal surfaces maintained at different temperature **Ozoe et al. (1977)** and for case **Vasseur et al. (1986).**

The results of numerical calculations for mean transition Nusselt number vs. dimensionless time τ, are plotted in **Fig.(7)**. Generally, we can note that the

dimensionless time increases with increasing aspect ratio and Rayleigh number.

Figure (8), presents the results of average Nusselt number as a function of inclination angle ϕ, at Rayleigh number Ra of 20, 100, 500 respectively and an aspect ratio A=3. For small inclination angles from horizontal position, the calculations lead to a stationary state consisting of rotating cells. When the angle ϕ increases, the Nusselt number increases and sudden transition appears and flow becomes unicellular and the Nusselt numbers increase clearly. If the angle ϕ continuous increase, the flow remains unicellular and the Nusselt number decrease when close to the vertical position. So, the value of mean Nusselt number strong function with the value of Rayleigh number, aspect ratio, and the orientation of the porous cavity.

CONCLUSIONS

The numerical results of shallow porous cavity show that when the porous layer slightly inclined with respect to the horizontal line, the flow which takes place spontaneously is of unicellular type.

The thermal boundary thickness increases on the heated wall, for the velocity boundary layer thickness, the growth is different and is largely due to the change in bouncy effect.

The convection becomes more and more vigorous as the orientation angle of the cavity is increased. For very high values of the Rayleigh number, no steady unicellular flow could be maintained inside the cavity.

Streamlines close to the heated surfaces are observed to run parallel to the wall over significantly large portion of its extent. This behavior becomes more prominent as the aspect ratio is increased and increasing in the inclination angle.

As increase in aspect ratio further pushes the isotherms towards the corner of heated surface indicating high velocity.

The Nusselt number at low Rayleigh number tends towards unity, indicating that the heat transfer is mainly due to conduction.

When the angle ϕ increases, the Nusselt number increases and sudden transition appears and flow becomes unicellular and the Nusselt numbers increase clearly. If the angle ϕ continuous increase, the flow remains unicellular and the Nusselt number decrease when close to the vertical position. So, the value of mean Nusselt number strong function with the value of Rayleigh number, aspect ratio, and the orientation of the porous cavity.

REFERENCES

[1]. Alavyoon, F., (1993)." On natural convection in vertical porous enclosure due to prescribed fluxes of heat and mass at vertical boundaries", Int. J. Heat Mass Transfer 36, 2479-2498.

[2]. AZIZ,K., and J. D. Hallums,(1967)." Numerical solutions of the three dimensional equations of motion for laminar natural convection", the physics of fluids 10,2,314-325.

[3].Balvanz, J. L. and Kuehn, T. H. (1983)." Effect of wall conduction and radiation on natural convection in a vertical slot with uniform heat generation on the heated wall", cited in [Caltagirone and Bories (1985)].

Eng. & Tech. Journal, Vol.30, No.15, 2012   Numerical Study of Two-Dimensional Transient
                                                                                    Natural Convection in an Inclined Shallow
                                                                                    Porous Cavity Exposed to a Constant Heat Flux

[5]. Bejan, A., (1979). "On the boundary layer regime in a vertical enclosure filled with a porous medium", Letters in Heat and Mass Transfer 6, 93-102.

[6]. Bejan, A., (1983). "The boundary layer regime in a porous layer with uniform heat flux from the side", Int. J. Heat Mass Transfer 26, 1339-1346

[7] .Bennacer, R., Tobbal, A., Beji, H., and Vasseur, P. (2001)." Double diffusive convection in a vertical enclosure filled with an isotropic porous media", cited in [Rahli and Bouhadef (2004)].

[8]. Bories, S. A., and Combarnous, M. A., (1973). "Natural convection in a sloping porous layer", J. Fluid Mech. 57, 63-79.

[9]. Buchberg, H., Catton, I., and Edwards, D. K. (1976)."Natural convection in enclosed spaces-a review of application to solar energy collection", ASME J. Heat Transfer98,182-188.

[10]. Burns, P. J., Chow L.C. and Tien, C. L. , (1976) "Convection in a vertical slot filled with   porous insulation", Int. J. Heat Mass Transfer 20, 919-926.

[11]. Caltagirone, J. P., and Bories, S., (1985). "Solutions and stability criteria of natural convective flow in an inclined porous layer", J. Fluid Mech.155, 267-287.

[12] .Chamkha, J.,and Al-Naser, H. (2001). "Double-diffusive convection in an inclined porous enclosure with opposing temperature and concentration gradients", , cited in [Rahli and Bouhadef (2004)].

[13]. Chan, B. K. C., Ivey, C. M., and Barry, J. M. (1970). " Natural convection in enclosed porous media with rectangular boundaries", ASME J. Heat Transfer 2, 21-27.

[14]. Chen, C.J. ,and Taie, V.,(1982)." Finite analytic numerical solution of laminar natural convection in two-dimensional inclined rectangular enclosure", cited in [Vasseur et al. (1986)].

[15].Eldr, J. W., (1974). "Convection in a porous media with horizontal and vertical temperature gradients", Int. J. Heat, Mass17, 241-248.

[16] Holst, P. H. and Aziz, K., (1972)."Transient natural convection in confined porous media", Int. J. Heat Mass15, 73-90.

[17] .Inaba, H. ,and Kanayama, K.,(1983). "Natural convection heat transfer in an inclined rectangular cavity", cited in [Vasseur et al. (1986)].].

[18]. Moya S. L., Ramos, E., and Sen, M (1980). "Numerical study of natural convection in a tilted rectangular porous material", cited in [Bejan (1983)].

[19].Ozoe, H., Sayama, H., and Churchill, S. W., (1977) " Natural convection in a long inclined rectangular box heated from below", Int. J. Heat Mass Transfer 20, 123-129.

[20] .Prasad, V. and Kulacki, F. A., (1984). "Natural convection in a rectangular porous cavity with constant heat flux on one vertical wall", J. Heat Transfer106, 152-157.

[21] ,.Prasad V. and F. A. Kulacki, (1984). "Convection heat transfer in a rectangular porous cavity –effect of aspect ratio on flow structure and heat transfer", J. Heat Transfer 106, 158-165.

[22]. Rahli, O., and Bouhadef, K., (2004). "Double-diffusive natural convection in a partially porous square enclosure; effect of the inclination", e-mail: khedbouh@yahoo .fr.

Eng. & Tech. Journal, Vol.30, No.15, 2012   **Numerical Study of Two-Dimensional Transient**
Natural Convection in an Inclined Shallow
Porous Cavity Exposed to a Constant Heat Flux

[23] Said, M. N. A., and Trupp (1982)." Laminar free convection in vertical air-filled cavities with mixed boundary conditions", cited in [Caltagirone and Bories (1985)].

[24] Seki, N., Fukusako, S., and Inaba, H., (1978). " Heat transfer in a confined rectangular cavity packed with porous media", Int. J. Heat Mass Transfer 21,985-989.

[25] Simpkins, P. G. and Blythe, P. A. (1980). "Convection in a porous layer", Int. J. Heat Mass Transfer 23, 881-887.

[26] Torrance K. E, (1985)."Numerical method in heat transfer "Hand Book of Heat transfer Fundamentals,McGraw-Hill,2nd edition.

[27] Vasseur, M. G. Satish and L. Robillard, (1986). " Natural convection in a thin inclined porous layer exposed to a constant heat flux", Int. J. Heat Mass Transfer 30, 537-549.

[28] Walker, K. L., and Homsy, G. M., (1978). "Convection in a porous cavity", J. Fluid Mech. 97,449-474.

[29] Weber, J. E., (1975) " The boundary layer regime for convection in a vertical porous layer", Int. J. Heat Mass Transer18, 569-573.

[30] Weber, J. E., (1975)" Thermal convection in a tilted porous layer", Int. J. Heat Mass Transfer 18, 474-475.

NOMENCLATURE

A aspect ratio of the cavity, H/L.

c specific heat of fluid, J/kg.K.

g gravitational acceleration, m/s^2.

H thickness of the porous cavity, m.

k thermal conductivity of fluid saturated porous medium, W/m.K.

K permeability of porous medium, m^2.

L length of the porous cavity, m.

Nu Nusselt number.

P pressure, kPa.

q constant heat flux, W/m^2.

Ra Rayleigh number, $g\beta KL^2 q/k a u$.

T temperature, K.

T_o reference temperature at x=y=0, K.

ΔT characteristic temperature differenc, qL/k.

ΔQ wall-to-wall temperature difference at x=0, eq. (12).

u fluid velocity in x-direction.

v fluid velocity in y-direction.

U dimensionless velocity in x-direction, uL/a.

V dimensionless velocity in y-direction, vL/a.

x, y   *cartesian coordinate, m.*

X *dimensionless distance on x-axis, x/L.*

Y *dimensionless distance on y-axis, y/L.*

Eng. & Tech. Journal, Vol.30, No.15, 2012 **Numerical Study of Two-Dimensional Transient**
Natural Convection in an Inclined Shallow
Porous Cavity Exposed to a Constant Heat Flux

Greek symbols.

a   thermal diffusivity of porous medium, k/ρc.
b   coefficient of thermal expansion, K^{-1}.
Q   dimensionless temperature.
τ   dimensionless time.
m   dynamic viscosity of fluid, kg/m.s.
u   kinematic viscosity of fluid, m^2/s.
r   density of fluid, kg/m^3.
Y   stream function.
f   angle of inclination of the enclosure.

Superscript.
-       average.

n   iteration.

Subscript.
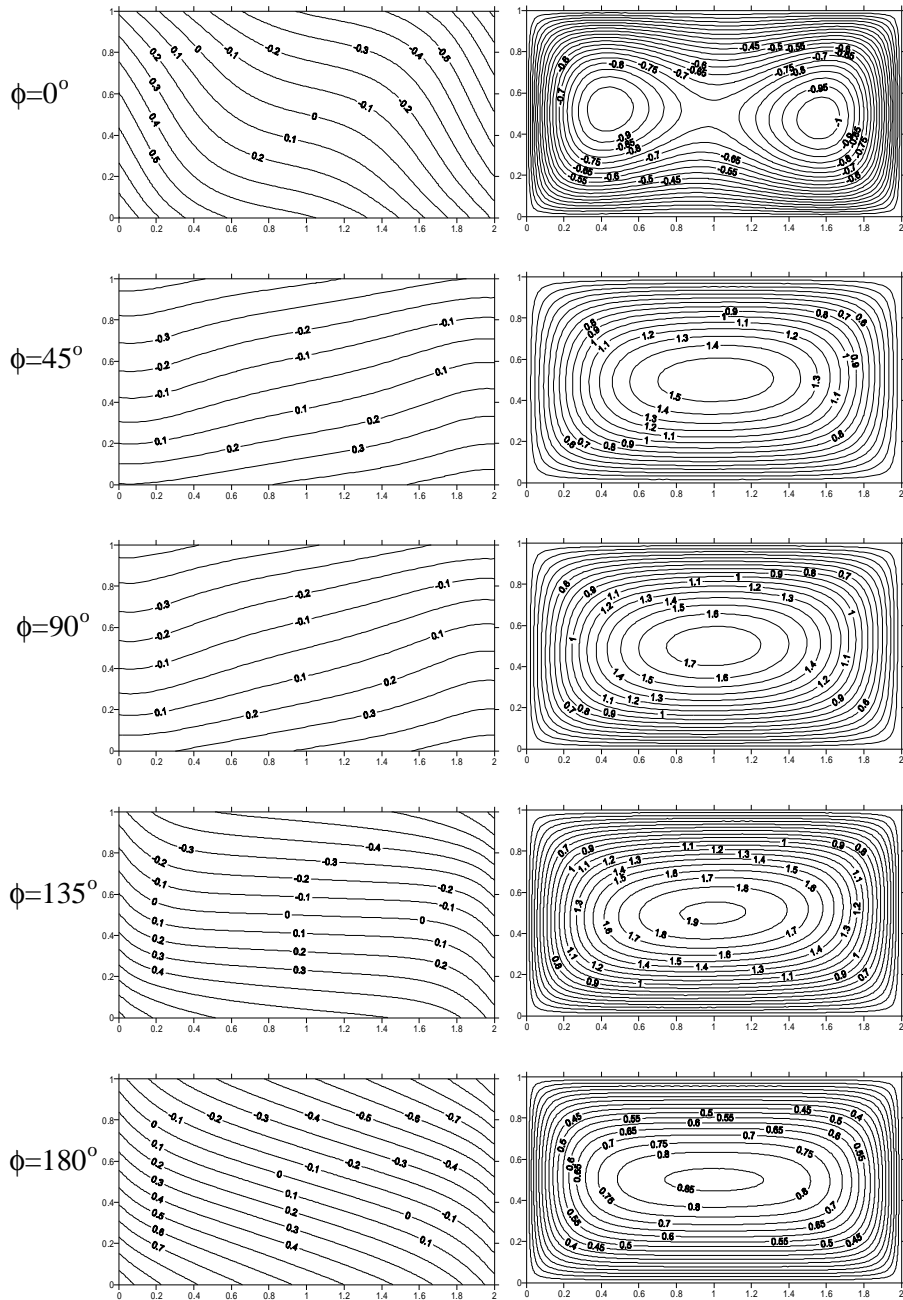i, j   indices denote grid location.
- •   reference temperature.

Eng. & Tech. Journal, Vol.30, No.15, 2012   **Numerical Study of Two-Dimensional Transient
Natural Convection in an Inclined Shallow
Porous Cavity Exposed to a Constant Heat Flux**

**Figure (2) Isotherms and streamlines for
porous cavity for Ra=20. A=2.**

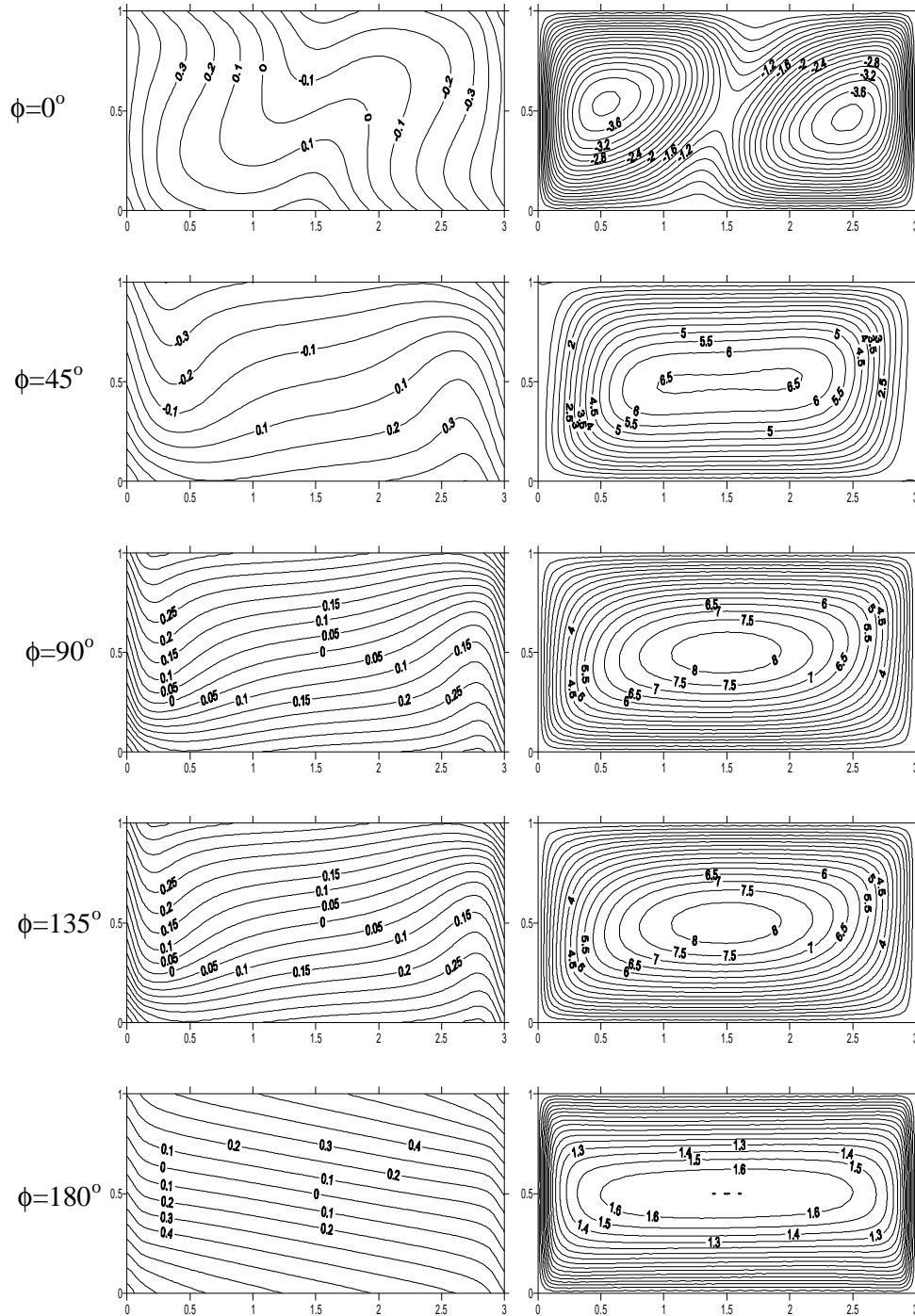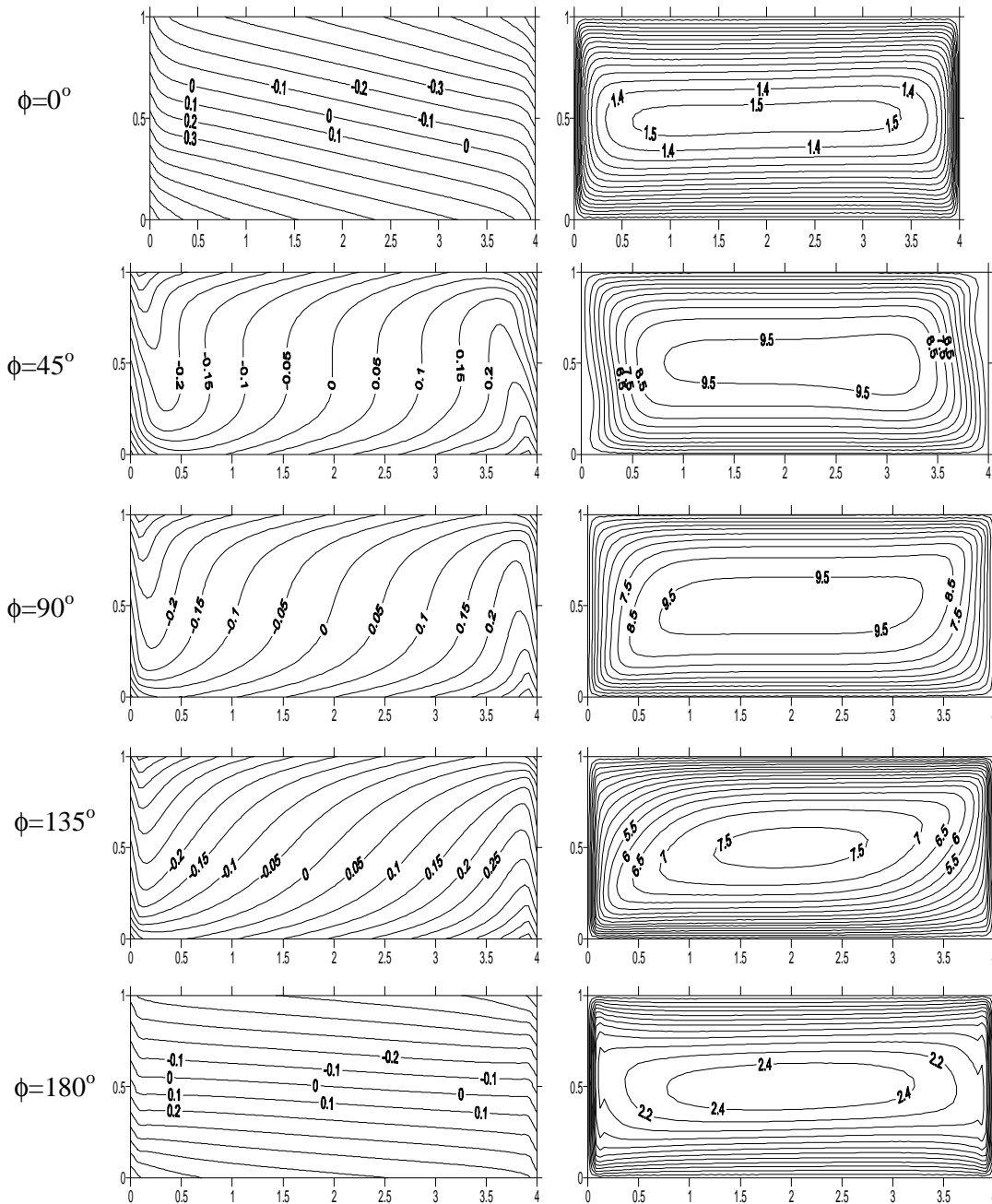**Figure (3) Isotherms and streamlines for porous cavity for Ra=100, A=3.**

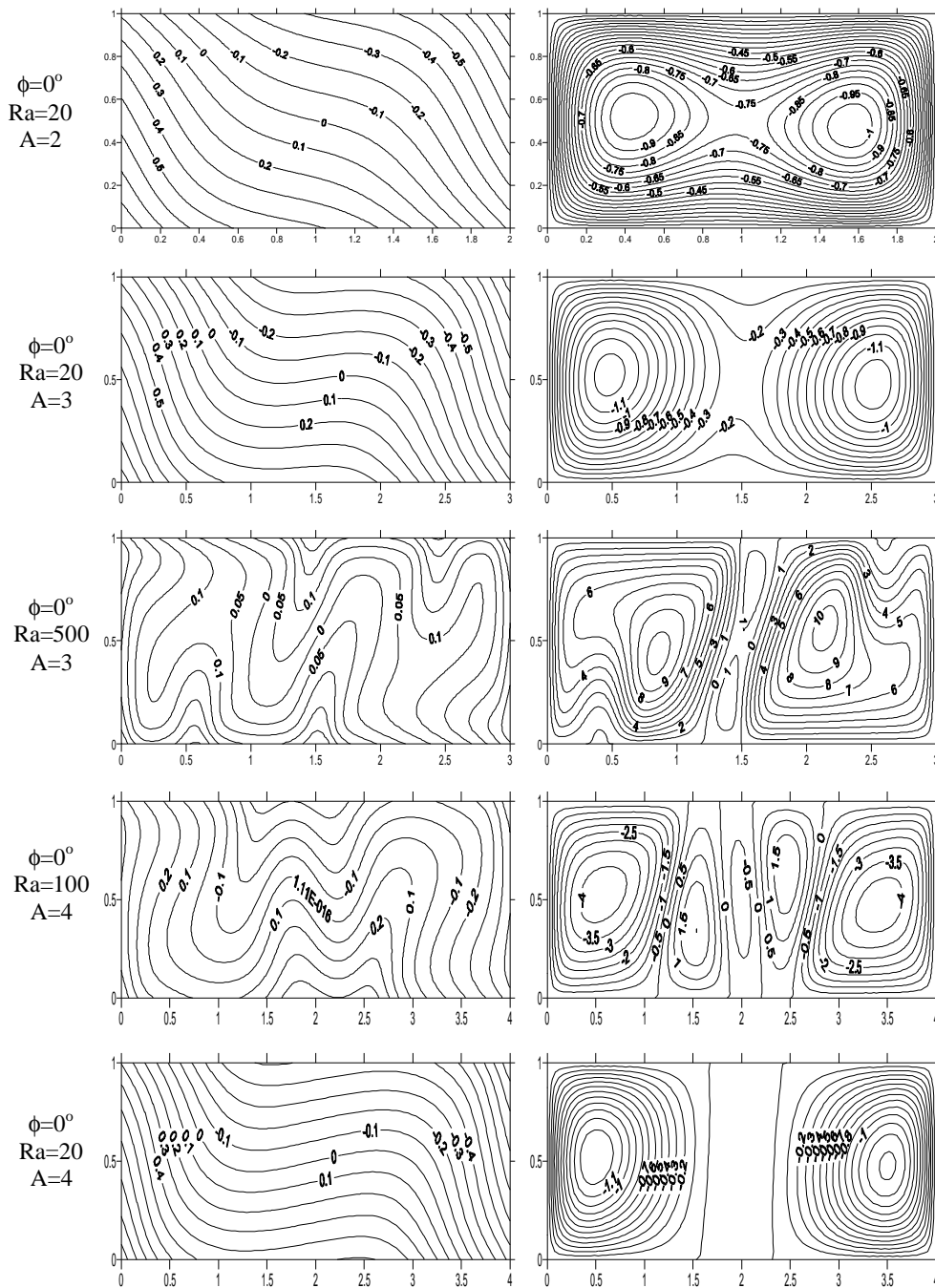**Figure (4) Isotherms and streamlines for porous cavity for Ra=500, A=4.**

Eng. & Tech. Journal, Vol.30, No.15, 2012   **Numerical Study of Two-Dimensional Transient
Natural Convection in an Inclined Shallow
Porous Cavity Exposed to a Constant Heat Flux**

φ=0°
Ra=20
A=2

φ=0°
Ra=20
A=3

φ=0°
Ra=500
A=3

φ=0°
Ra=100
A=4

φ=0°
Ra=20
A=4

**Figure (5) Isotherms and streamlines for porous cavity at horizontal position
at different aspect ratios and Rayleigh number.**

2593

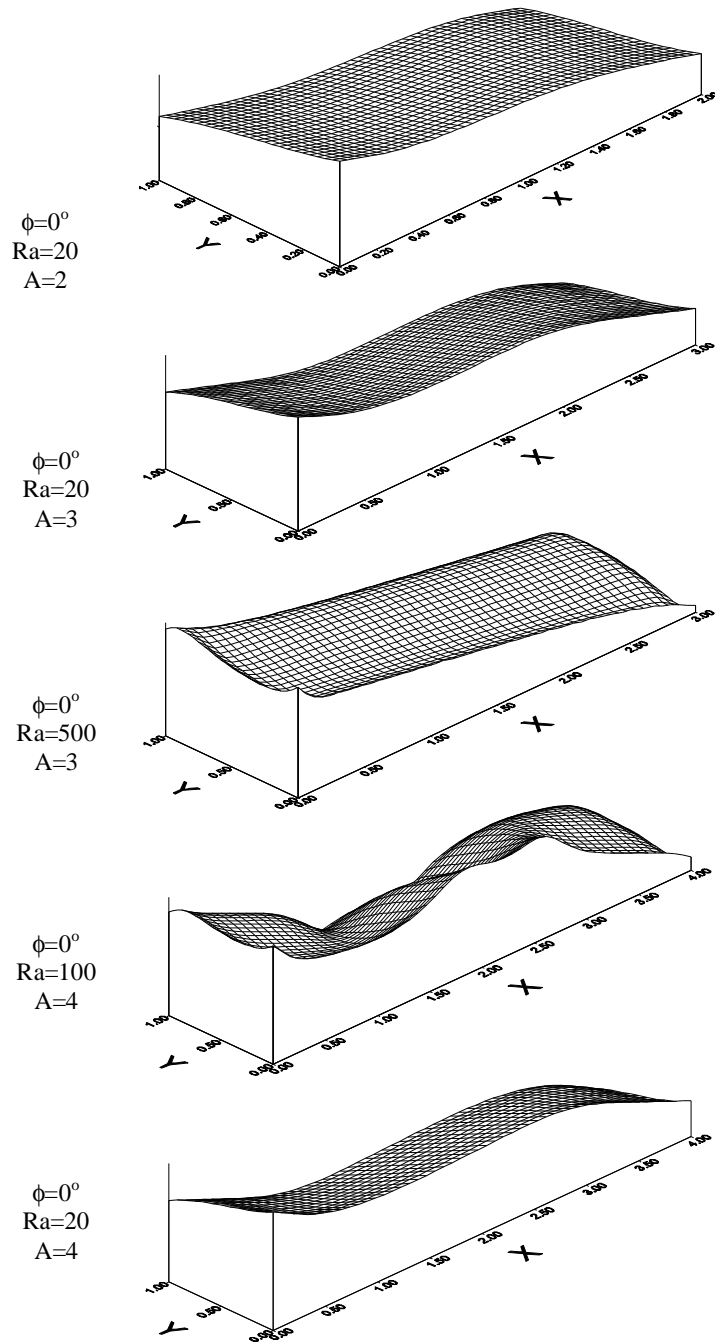*Eng. & Tech. Journal, Vol.30, No.15, 2012*   **Numerical Study of Two-Dimensional Transient
Natural Convection in an Inclined Shallow
Porous Cavity Exposed to a Constant Heat Flux**

$\phi=0^{o}$
Ra=20
A=2

$\phi=0^{o}$
Ra=20
A=3

$\phi=0^{o}$
Ra=500
A=3

$\phi=0^{o}$
Ra=100
A=4

$\phi=0^{o}$
Ra=20
A=4

**Figure (6) Temperature distribution for porous cavity
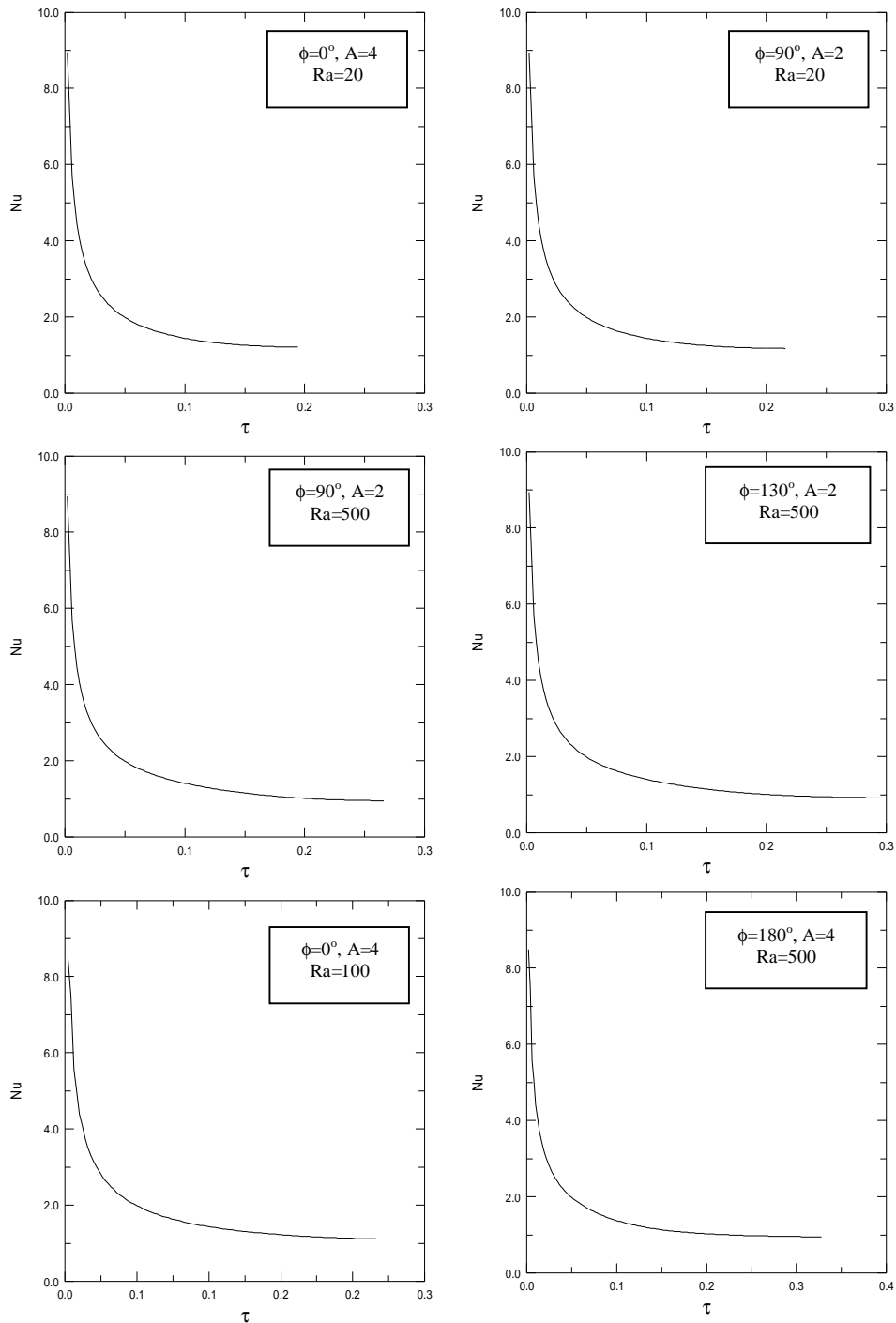at horizontal position with different aspect ratios and Rayleigh number**

2594

Eng. & Tech. Journal, Vol.30, No.15, 2012   **Numerical Study of Two-Dimensional Transient**
Natural Convection in an Inclined Shallow
Porous Cavity Exposed to a Constant Heat Flux

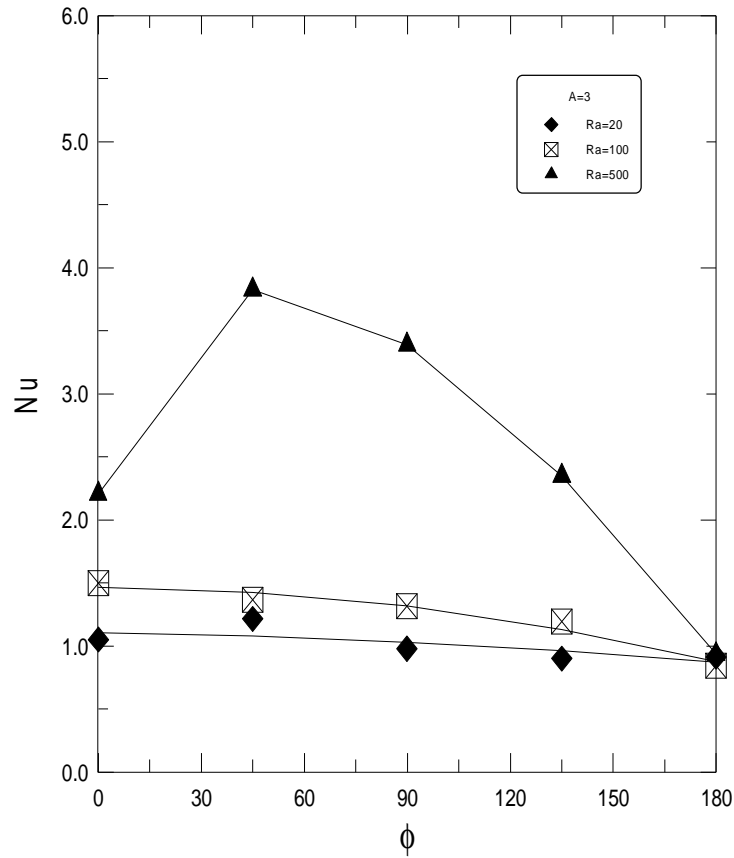**Figure (7) Mean Nusselt number vs. non dimensional time.**

2595

Figure (8) Effect of inclination angle ϕ**, on the mean Nusselt number, A=3.**