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Kinematic Analysis of Semi-Flexible Robot

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ABSTRACT

In this work, a kinematic model of semi- flexible robot with two degrees of freedom based on the joint angles and arm's deflections is presented with taking in consideration the small deflection parameter (exact model), and used experimental deformation results to make a comparison between the approximate model and the exact model. Due to the difficulties of using flexible robot in the real live, a two degrees semi-flexible robot was built; this robot will be used to get the experimental results for comparison. The comparison shows a small difference between the approximate model and the exact model, so with increasing the flexibility this difference will increase and in some applications of robot this difference will be significant and worth to take in consideration.

Keywords: semi-flexible robot, kinematic equation of flexible and semi- flexible robot.

التحليل الكينماتيكي لروبوت شبه مرن

الخلاصة

في هذا العمل، تم تقديم نموذج كينماتيكي لروبوت شبه مرن بدرجتين من الحرية، اخـذا بنظـر الاعتبار وجود انحناء الاذرع مع الاخذ بنظر الاعتبار القيم الصغيرة لمتغيرات التشوه. وباسـتخدام نتائج عملية لقيم التشوه تم عمل مقارنة مع هذا النموذج بدون تقريب مع نموذج مع التقريب. نتيجـة لصعوبة استخدام روبوت مرن في الحياة العملية، تم بناء روبوت شبه مرن لاستخدامه في الحصـول على النتائج العملية اللازمة لعملية المقارنة. ان المقارنة بينت وجود فرق صغير مـابين النمـوذجين، وانه مع زيادة المرونة فأن الفرق سيزيد وفي بعض التطبيقات فأن هذا الفرق يكون ذو تـأثير كبيـر يستحق ان يؤخذ بالحسبان.

INTRODUCTION

The kinematic analysis of a mechanical system means the deformation of the position, velocity and acceleration of the various mechanical elements forming the mechanism under consideration. The combination of position velocity and

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acceleration of an element at a certain time is referred to henceforth as the state of this element [1]. In the case of open kinematic chain, a direct kinematic analysis determines the state of the end effector as function of the known state of the various joints. The direct kinematic analysis also includes the determination of the link's position and orientation and their time derivatives, such information is essential for any subsequent dynamic analysis or when the position and orientation of a sensor mounted on a link are required for the data processing.

The references [2], [3] and [4] have supposed that the generalized deflection parameters are small, so a first order approximation can be applied to their trigonometric functions and product, and higher order equals zero.

In this work, the kinematic of robot will be analyzed using a homogeneous transformation without and with deflection and elongation.

POSITION AND ORIENTATION OF A RIGID BODY

The arm linkage of a manipulator can be modeled as a system of rigid bodies. The location of each single rigid body is completely described by its position and orientation [5]. The position can be represented by the coordinates of an arbitrary point fixed with respect to the rigid body. Figure (1) shows the coordinate frame O-xyz fixed to the ground and the point O^b fixed to the rigid body.

The position of the coordinate frame (O^b) of the rigid body is represented with reference to the coordinate frame of the base O_o-xyz by:

$$X_o = \begin{matrix} x \\ y \\ z \end{matrix} \qquad (1)$$

The subscript ($_o$) shows that the vector is defined with reference to the coordinates frame of the base.

Also, the position of point P on the rigid body can be represented with reference to the coordinate frame O^b fixe to the rigid body by:

$$X^b = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \qquad (2)$$

The subscript (^{b}) indicates that the vector is defined with reference to the body coordinate's frame [6].

The three angles ψ, ϕ and θ determine the orientation of the coordinate frame uniquely about the axes x, y and z, respectively and referred to as Euler angles. The Euler angles are independent in that each of them can vary arbitrarily [6].

The matrices R (ψ), R (ϕ) and R (θ) represent the rotation about the axis x, y, z, respectively, also called yaw, pitch and roll, respectively, as shown in figure (2).

$$R(\psi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\psi & -\sin\psi \\ 0 & \sin\psi & \cos\psi \end{bmatrix} \tag{3}$$

$$R(\phi) = \begin{bmatrix} \cos\phi & 0 & -\sin\phi \\ 0 & 1 & 0 \\ \sin\phi & 0 & \cos\phi \end{bmatrix} \tag{4}$$

$$R(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{5}$$

From figure (1), the relationship, that defines the coordinate transformation between the fixed frame and the coordinate of any point on the rigid body, will be:

$$X = X_o + RX^b \tag{6}$$

Equation (6) provides the desired coordinate transformation from the body coordinate Xb to the fixed coordinates X, Xo and R represent the position and orientation of the rigid body or of the body's coordinate frame relative to the fixed frame, respectively [6].

HOMOGENOUS TRANSFORMATION

Equation (6) can resemble the position and orientation in a conceptual form identified henceforth by the matrix [A]:

$$A = \begin{bmatrix} R & \vdots & X_o \\ \cdots & \vdots & \cdots \\ 0 & \vdots & 1 \end{bmatrix} \tag{7}$$

The original vector X and X^b are augmented by adding 1 as the fourth element, so that the result is a 4x1 vector . Also, the rotation matrix R is extended to 4x4 matrix by combining it with 3x1 position vector X_o with three 0's and 1 in the fourth raw. Equation (6) can then be written as:

$$X = AX^b \tag{8}$$

Thus

$$\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} R & \vdots & X_o \\ \cdots & \vdots & \cdots \\ 0 & \vdots & 1 \end{bmatrix} \begin{bmatrix} x^b \\ y^b \\ z^b \\ 1 \end{bmatrix} \tag{9}$$

The coordinate transformation given by equation (9) is referred to as the homogenous transformation equation [1].

DENAVIT-HARTENBERG (DH) MATRIX

Any robot can be kinematically described by giving the values of four quantities for each link. Two describe the link itself and two describe the link's connection to a neighboring link. In usual case of a revolute joint, θ_i is called the joint variable, and the other three quantities would be fixed link parameters. For prismatic joints, d_i is the joint variable and the other three quantities are fixe link parameters. The definition of mechanisms by means of these quantities is a convention, usually called the Denavit-Hartenber (DH) notation [7].

The DH displacement matrix for a rigid link is:

$$A_i^{i-1} = \begin{bmatrix} \cos\theta_i & -\sin\theta_i\cos\alpha_i & \sin\theta_i\sin\alpha_i & a_i\cos\theta_i \\ \sin\theta_i & \cos\theta_i\cos\alpha_i & -\cos\theta_i\sin\alpha_i & a_i\sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (10)$$

The four geometric quantities θ_i , α_i , a_i and di represent the joint parameters defined as follow, as shown in figure (3):

θi is the angle between the x_{i-1} and the xi axis, obtained by screwing xi-1 into xi around the z_{i-1} axis. For a revolute joint, θ_i is a variable parameter.

d_i is the coordinate of the origin of O_i frame on the z_{i-1} axis, the distance between the origin of O_{i-1} frame to the intersection of the z_{i-1} axis with the x_i axis, for a revolute joint, d_i is a constant parameter.

a_i is the common normal distance between z_{i-1} and z_i axis, measured along the negative direction of x_i from its origin, to where it intersects the z_{i-1} axis (a constant parameter).

α_i is the angle between the z_{i-1} axis and the z_i axis, obtained by screwing z_{i-1} into z_i around the x_i axis (a constant parameter) [1].

KINEMATIC EQUATION OF SEMI-FLEXIBLE ROBOT (WITH ERROR)

The link flexibility can cause elastic deformations of the structural members of the manipulator, resulting in large end-effector errors, especially in long reach manipulator systems. Hence as a result, the frames defined at the manipulator joints are displaced from their expected locations. So that, the using of kinematic equation of the rigid robot to position the manipulator end–effector, will place the manipulator in a different position than the desired one.

In figure (4) [8], the frame O_{i-1} is the base frame , O_i^I is the ideal location and O_i^a is the actual location due to the deflection.

The transformation equation between the coordinates O_i^a and O_i^I consists of two parts, rotation and translation. According to the Euler angle principle, the rotation part is the result of three sets of rotations which are roll, pitch and yaw about the axes z, y, x, respectively. The sequence of rotation is [8]:

$$E^{I}_{Ra} = \mathrm{Rot}\,(z, d\phi)\,\mathrm{Rot}(y, d\beta)\,\mathrm{Rot}(x, d\psi) \tag{11}$$

That is a rotation of $d\psi$ about the x axis , followed by a rotation $d\beta$ about the y axis and finally a rotation of $d\phi$ about the z axis.

$$\mathrm{Rot}(x, d\psi) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos d\psi & -\sin d\psi & 0 \\ 0 & \sin d\psi & \cos d\psi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{12}$$

$$\mathrm{Rot}(y, d\beta) = \begin{bmatrix} \cos d\beta & 0 & -\sin d\beta & 0 \\ 0 & 1 & 0 & 0 \\ \sin d\beta & 0 & \cos d\beta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{13}$$

$$\mathrm{Rot}(z, d\phi) = \begin{bmatrix} \cos d\phi & -\sin d\phi & 0 & 0 \\ \sin d\phi & \cos d\phi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{14}$$

$$E^{I}_{Ra} = \begin{bmatrix} a_{11} & b_{12} & c_{13} & 0 \\ a_{21} & b_{22} & c_{23} & 0 \\ a_{31} & b_{32} & c_{33} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{15}$$

$a_{11}=\cos d\phi \cos d\beta$
$a_{21}=\sin d\phi \cos d\beta$
$a_{31}=-\sin d\beta$
$b_{12}=\cos d\phi \sin d\beta \sin d\psi - \sin d\phi \cos d\psi$
$b_{22}=\sin d\phi \sin d\beta \sin d\psi + \cos d\phi \cos d\beta$
$b_{32}=\cos d\beta \sin d\psi$
$c_{13}=\cos d\phi \sin d\beta \cos d\psi + \sin d\phi \sin d\psi$
$c_{23}=\sin d\phi \sin d\beta \cos d\psi - \cos d\phi \sin d\psi$
$c_{33}=\cos d\beta \cos d\psi$

The equation (15) represents the rotation part of the transformation matrix. The translation matrix is:

$$E^{I}_{Ta} = \begin{bmatrix} 1 & 0 & 0 & \delta x \\ 0 & 1 & 0 & \delta y \\ 0 & 0 & 1 & \delta z \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{16}$$

The transformation between coordinates O_i^a and O_i^I is become:

$$E_a^I = E_{Ta}^I\, E_{Ra}^I \tag{17}$$

$$E_a^I = \begin{bmatrix} A_{11} & B_{12} & C_{13} & \delta x \\ A_{21} & B_{22} & C_{33} & \delta y \\ A_{31} & B_{32} & C_{33} & \delta z \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{18}$$

$A_{11}=\cos d\phi \cos d\beta$
$A_{21}=\sin d\phi \cos d\beta$
$A_{31}=-\sin d\beta$
$B_{12}=\cos d\phi \sin d\beta \sin d\psi - \sin d\phi \cos d\psi$
$B_{22}=\sin d\phi \sin d\beta \sin d\psi + \cos d\phi \cos d\beta$
$B_{32}=\cos d\beta \sin d\psi$
$C_{13}=\cos d\phi \sin d\beta \cos d\psi + \sin d\phi \sin d\psi$
$C_{23}=\sin d\phi \sin d\beta \cos d\psi - \cos d\phi \sin d\psi$
$C_{33}=\cos d\beta \cos d\psi$

Here, the δx, δy, δz, $d\psi$, $d\beta$ and $d\phi$ are called generalized error parameters. Usually these generalized parameters are introduced on the basis of different approximations such as assumed modes, finite elements, or Ritz-Kantorovich expansions, with different implications on the model complexity and accuracy [9].

The references [2], [3], and [4] have supposed that the generalized deflection parameters are small, so a first order approximation can be applied to their trigonometric functions and product, and higher order equals zero.

$\sin d\psi = d\psi$, $\sin d\beta = d\beta$,
$\sin d\phi = d\phi$
$\cos d\psi = \cos d\beta = \cos d\phi = 1$

Based on this assumptions the matrix E_a^I will be equal to:

$$E_{defl} = \begin{bmatrix} 1 & -d\phi & d\beta & \delta x \\ d\phi & 1 & -d\psi & \delta y \\ -d\beta & d\psi & 1 & \delta z \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{19}$$

In this work, the parameters deflection's values are taken in consideration.
For a serial of links the kinematic equation covering the positions of this links with presence of flexible coordinates is:

$$T_{defl}=A_1^o(q_1)E_{a\,1}^{I\,1}\; A_2^1(q_2)E_{a\,2}^{I2}\; \cdots\cdots A_n^{n-1}(q_n)E_{a\,n}^{I\,n} \tag{20}$$

A_n^{n-1} is the homogenous transformation matrix in equation (10).
By using equation (20), the transformation relationship between the two frames O_i^a and O_{i-1} could be found.

T_{defl} may be expressed in simple form as a nonlinear function of q and ϵ vectors:

$$p_{defl} = f(q, \epsilon) \tag{21}$$

Where:

p_{defl} is the set of Cartesian coordinates describing the position and orientation of the manipulator's end- effector with respect to the inertial frame.

$$q = [q_1 \quad q_2 \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad q_n]^T \tag{22}$$

$$\epsilon = [\epsilon_1 \quad \epsilon_2 \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \epsilon_g]^T \tag{23}$$

$$\epsilon = [\delta x_m \quad \delta y_m \quad \delta z_m \quad d\psi_m \quad d\beta_m \quad d\phi_m]^T \tag{24}$$

Where:

ϵ is a dimensional link deflection vector space, spanned to be the link deflections of the manipulator arm.

n is the number of joints.

g generally equals to 6 x m.

m is the number of flexible links.

The elements of the vector ϵ cannot specify their values, since these elastic coordinates are dependent on the rigid coordinate's q and on the applied forces F.

For more generally:

$$\epsilon = E(F, q) \tag{25}$$

By substituting equation (24) into equation (23), p_{defl} will be:

$$p_{defl} = f(q, E(F, q)) \tag{26}$$

As mentioned in the previous section, the two degrees of freedom of the arm robot manipulator used in this study is confined to move within the vertical plane, and so is the deformation of each link. Hence, there is no rotation about the axes x and y, also there is no transformation in the z direction (the parameters **δ**z, dψ and dβ are equal zero). Figure (5) shows a single link coordinate system , the O_{i-1} represents the base local coordinate system, and the frame O_i^I is the local coordinate system assigned to link i in its undeformed position, while the O_i^a is the actual local coordinate system of link i , when it is under deformation.

The homogenous transformation matrix for this single link was derived as follow:

- According to the motion of the links within the vertical plane, the homogenous transformation matrix E_{ia}^{I} between O_i^a and O_i^I becomes:

$$E_{ia}^{il} = \begin{bmatrix} \cos d\phi & -\sin d\phi & 0 & \delta x \\ \sin d\phi & \cos d\phi & 0 & \delta y \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (27)$$

- The homogenous transformation matrix between the O_i^I and O_{i-1} is presented by the matrix A_i^{i-1} in equation (10).
- The homogeneous transformation matrix for this a single link is found from the relations (10) and (27) :

$$T_{ia}^{i-1a} = A_i^{i-1a} \, E_{ia}^{il} \qquad (28)$$

$$T_{ia}^{i-1} = \begin{bmatrix} T11 & T12 & T13 & T14 \\ T21 & T22 & T23 & T24 \\ T31 & T32 & T33 & T34 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (29)$$

$T_{11} = c_i cd_i - c\alpha_i s_i sd_i$
$T_{21} = s_i cd_i + sd_i c\alpha_i c_i$
$T_{31} = s\alpha_i sd_i$
$T_{12} = -c_i sd_i - c\alpha_i s_i cd_i$
$T_{22} = -s_i sd_i + c_i cd_i c\alpha_i$
$T_{32} = s\alpha_i cd_i$
$T_{13} = s\alpha_i s_i$
$T_{23} = -s\alpha_i c_i$
$T_{33} = c\alpha_i$
$T_{14} = c_i[\delta x_i + a_i] - c\alpha_i s_i \delta y_i$
$T_{24} = s_i[\delta x_i + a_i] + \delta y_i c\alpha_i c_i$
$T_{34} = s\alpha_i \delta y_i + d_i$

As shown in figure (6), the total kinematic equation of semi-flexible robot of two degrees of freedom is:

$$T_{defl} = T_{1a}^{0a} \, T_{2a}^{1a} \qquad (30)$$

The transformation matrix of robot in general form is [10]:

$$T_2^0 = \begin{bmatrix} n_x & o_x & t_x & p_x \\ n_y & o_y & t_y & p_y \\ n_z & o_z & t_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (31)$$

T_{defl} can be written in the general form (equation (31)) as follow:

$$n_x = c_1c_2[cd_1cd_2 - sd_1sd_2c\alpha_2] - c_1s_2[cd_1c\alpha_2sd_2 + sd_1cd_2] + s_1s_2[c\alpha_1c\alpha_2sd_1sd_2 - c\alpha_1cd_1cd_2] - c_2s_1[cd_2sd_1c\alpha_1 + c\alpha_1cd_1c\alpha_2sd_2] + s_1s\alpha_1s\alpha_2sd_2 \quad (32)$$

$$n_y = c_1c_2[cd_2sd_1c\alpha_1 + c\alpha_1cd_1c\alpha_2sd_2] - s_1s_2[sd_1cd_2 + cd_1c\alpha_2sd_2] + c_2s_1[cd_1cd_2 - sd_1sd_2c\alpha_2] + c_1s_2[cd_2cd_1c\alpha_1 - sd_1c\alpha_1sd_2c\alpha_2] - c_1[s\alpha_1sd_2s\alpha_2] \quad (33)$$

$$n_z = c_2[s\alpha_1sd_1cd_2 + s\alpha_1cd_1c\alpha_2sd_2] + s_2[s\alpha_1cd_1cd_2 - s\alpha_1sd_1sd_2c\alpha_2] + c\alpha_1sd_2s\alpha_2 \quad (34)$$

$$o_x = s_1s_2[c\alpha_1sd_1c\alpha_2cd_2 + c\alpha_1cd_1sd_2] - c_1c_2[cd_1sd_2 + sd_1cd_2c\alpha_2] + c_1s_2[sd_1sd_2 - cd_1c\alpha_2cd_2] + s_1c_2[c\alpha_1cd_1cd_2 - c\alpha_1sd_1sd_2c\alpha_2] + s_1s\alpha_1s\alpha_2cd_2 \quad (35)$$

$$o_y = c_1c_2[cd_1c\alpha_1cd_2c\alpha_2 - sd_1c\alpha_1sd_2] - s_1c_2[sd_1cd_2c\alpha_2 + cd_1sd_2] - c_1s_2[cd_1c\alpha_1sd_2 + sd_1c\alpha_1c\alpha_2cd_2] + s_1s_2[sd_1sd_2 - cd_1 c\alpha_2cd_2] - c_1[s\alpha_1s\alpha_2cd_2] \quad (36)$$

$$o_z = c_2[s\alpha_1cd_1cd_2c\alpha_2 - s\alpha_1sd_1sd_2] + s_2[s\alpha_1cd_1sd_2 - s\alpha_1sd_1c\alpha_2cd_2] + c\alpha_1s\alpha_2cd_2 \quad (37)$$

$$t_x = c_1s_2cd_1s\alpha_2 - s_1s_2c\alpha_1sd_1s\alpha_2 + c_1c_2sd_1s\alpha_2 + s_1c_2c\alpha_1cd_1s\alpha_2 + s_1s\alpha_1c\alpha_2 \quad (38)$$

$$t_y = s_1s_2cd_1s\alpha_2 + c_1s_2sd_1c\alpha_1s\alpha_2 + s_1c_2sd_1s\alpha_2 - c_1c_2cd_1c\alpha_1s\alpha_2 - c_1s\alpha_1c\alpha_2 \quad (39)$$

$$t_z = s_2s\alpha_1sd_1s\alpha_2 - c_2s\alpha_1cd_1s\alpha_2 + c\alpha_1c\alpha_2 \quad (40)$$

$$p_x = c_1c_2[cd_1(\delta x_2 + a_2) - sd_1\delta y_2c\alpha_2] - c_1s_2[cd_1c\alpha_2\delta y_2 + sd_1(\delta x_2 + a_2)] - s_1c_2[c\alpha_1sd_1(\delta x_2 + a_2) + c\alpha_1cd_1\delta y_2c\alpha_2] + s_1s_2[c\alpha_1sd_1c\alpha_2\delta y_2 - c\alpha_1cd_1(\delta x_2 + a_2)] + s\alpha_1s_1s\alpha_2\delta y_2 + s\alpha_1s_1d_2 + c_1(\delta x_1 + a_1) - c\alpha_1s_1\delta y_1 \quad (41)$$

$$p_y = s_1c_2[cd_1(\delta x_2 + a_2) - sd_1\delta y_2c\alpha_2] - s_1s_2[sd_1(\delta x_2 + a_2) + cd_1c\alpha_2\delta y_2] + c_1c_2[sd_1c\alpha_1(\delta x_2 + a_2) + cd_1c\alpha_1\delta y_2c\alpha_2] + c_1s_2[cd_1c\alpha_1(\delta x_2 + a_2) - sd_1c\alpha_1c\alpha_2\delta y_2] + c_1[c\alpha_1\delta y_1 - s\alpha_1s\alpha_2\delta y_2 - s\alpha_1d_2] + s_1[\delta x_1 + a_1] \quad (42)$$

$$p_z = c_2[s\alpha_1sd_1(\delta x_2 + a_2) + s\alpha_1cd_1\delta y_2c\alpha_2] + s_2[s\alpha_1cd_1(\delta x_2 + a_2) - s\alpha_1sd_1c\alpha_2\delta y_2] + c\alpha_1s\alpha_2\delta y_2 + c\alpha_1d_2 + s\alpha_1\delta y_1 + d_1 \quad (43)$$

Where:
$c_{\mathrm{i}} = cos\,\theta_i \quad cd_i = \cos d\phi_i$
$c\alpha_{\mathrm{i}} = \cos\alpha_i$
$s_i = \sin\theta_i \qquad sd_i = \sin d\phi_i \qquad s\alpha_i = \sin\alpha_i$

The degree of freedom of the robot arm manipulator which is used in this work, is two degrees of freedom, and confined to move within the vertical plane with the kinematic parameters listed in the Table (1).
By substituting the values of parameters of the manipulator from the Table (1) in the equations (32) to (43) and with triangular operations, these equations will be as follow:

$$n_x = [\mathrm{c}_{12}][\mathrm{cd}_{12}] - [\mathrm{s}_{12}][\mathrm{sd}_{12}] \quad (44)$$

$$n_y = \mathrm{c}_1\mathrm{c}_2[\mathrm{sd}_{12}] + \mathrm{s}_1\mathrm{s}_2[\mathrm{sd}_{1-2}] + \mathrm{c}_2\mathrm{s}_1[\mathrm{cd}_{1-2}] + \mathrm{c}_1\mathrm{s}_2[\mathrm{cd}_{12}] \quad (45)$$

$$n_z = 0 \quad (46)$$

$$o_x = \mathrm{s}_1\mathrm{s}_2[\mathrm{sd}_{1-2}] - \mathrm{c}_1\mathrm{c}_2[\mathrm{sd}_{12}] - \mathrm{c}_1\mathrm{s}_2[\mathrm{cd}_{1-2}] - \mathrm{s}_1\mathrm{c}_2[\mathrm{cd}_{12}] \quad (47)$$

$$o_y = \mathrm{c}_{12}[\mathrm{cd}_{12}] - \mathrm{s}_{12}[\mathrm{sd}_{1-2}]] \quad (48)$$

$$o_z = 0 \quad (49)$$

$$t_x = 0 \quad (50)$$

$$t_y = 0 \quad (51)$$

$$t_z = 1 \quad (52)$$

$$p_x = c_1c_2[cd_1(\delta x_2 + a_2) - sd_1\delta y_2] - c_1s_2[cd_1\delta y_2 + sd_1(\delta x_2 + a_2)] - s_1c_2[sd_1(\delta x_2 + a_2) + cd_1\delta y_2] + s_1s_2[sd_1\delta y_2 - cd_1(\delta x_2 + a_2)] + c_1(\delta x_1 + a_1) - s_1\delta y_1 \quad (53)$$

$$p_y = s_1c_2[cd_1(\delta x_2 + a_2) - sd_1\delta y_2] - s_1s_2[sd_1(\delta x_2 + a_2) + cd_1\delta y_2] + c_1c_2[sd_1(\delta x_2 + a_2) + cd_1\delta y_2] + c_1s_2[cd_1(\delta x_2 + a_2) - sd_1\delta y_2] + c_1\delta y_1 + s_1[\delta x_1 + a_1] \quad (54)$$

$$p_z = 0 \quad (55)$$

Where:

$$\left.\begin{aligned} \mathrm{c}_{12} &= \cos(\theta_1 + \theta_2) \\ \mathrm{s}_{12} &= \sin(\theta_1 + \theta_2) \end{aligned}\right\}$$

$$\mathrm{cd}_{12} = \cos(d\phi_1 + d\phi_2) \qquad \mathrm{sd}_{12} = \sin(d\phi_1 + d\phi_2) \quad (56)$$

$$\mathrm{cd}_{1-2} = \cos(d\phi_1 - d\phi_2)$$

$sd_{1-2} = \sin(d\phi_1 - d\phi_2)$

Equations (53) to (54) represent the end-effector position with presence of flexibility of robot.

With using the equation (19) instead of equation (18) to represent E_a^I in equation (20), the p_x and p_y will be as follow:

$$p_x = c_1c_2[(\delta x_2 + a_2) - d\phi_1\delta y_2] - c_1s_2[\delta y_2 + d\phi_1(\delta x_2 + a_2)] - s_1c_2[d\phi_1(\delta x_2 + a_2) + \delta y_2] + s_1s_2[d\phi_1\delta y_2 - (\delta x_2 + a_2)] + c_1(\delta x_1 + a_1) - s_1\delta y_1 \quad (57)$$

$$p_y = s_1c_2[(\delta x_2 + a_2) - d\phi_1\delta y_2] - s_1s_2[d\phi_1(\delta x_2 + a_2) + \delta y_2] + c_1c_2[d\phi_1(\delta x_2 + a_2) + \delta y_2] + c_1s_2[(\delta x_2 + a_2) - d\phi_1\delta y_2] + c_1\delta y_1 + s_1[\delta x_1 + a_1]$$

The set of equations (53) and (54) and the set of equations (57) and (58) will be called, the flexible kinematic equation and the approximate flexible kinematic equation, respectively.

Experimental Work

In this work, the test rig was represented by two degrees manipulator (two semi-flexible links) confined to move in a vertical plane; each link has its own control system for movement. These control systems (drivers for the actuators) were connected to the computer to control them by the main program which was built by the Visual Basic. net software. Also, the rig has measurement systems and data acquisition system (DAQ). Figure (7) shows the semi-flexible robot for the experimental data.

The experimental relations between the loads and deflection δy were found by applying different values of load and measuring the deflection at the tip for each load, and then by using the curve fitting, the relations were found.

To find the rotation of the tip of the beam $d\phi$ experimentally, the measuring of the deflections at two points along the beam was done, as shown in figure (8), at the points where the strain gauge No.3 and at the tip, and by applying these values of the deflections into the following equation [11]:

$$\tan d\phi = \frac{\Delta\delta y}{\Delta x} \quad (59)$$

Where:

$\Delta\delta y$ represents the difference between the two deflections.

Δx represents the distance between the two points where the deflections measured.

δx was experimentally found by extracting the strain from two sensors at the upper and lower surface at the same position of the strain gauge No.1 and No. 2 as shown in the figure (8). The difference value represents the extension of the beam in the longitudinal direction, as represented by the relation:

$$\delta x = \epsilon . L \quad (60)$$

These values of δx, δy and $d\phi$ will be compensated in the kinematic equations (53), (54), (57) and (58) to be the deflection parameters.

The procedure that is worked on it is presented as follows:

1- Assembly the two links to work as a manipulator with their servo motor and DAQ system.

2- In this test, the sequence mode of the main program was activated. The range of angles for the first link was (4^o to 29^o step 5^o) , the second link was (-35^o to 60^o step 5^o) . So, the first link is still at angle 4^o, and the second link is still awhile at the start angle then moves 5^o to the next step, after completion the other steps, it returns back to the start angle, and the first joint moves 5^o to next step and so on. This procedure was repeated for four values of load (0, 1, 2 and 3) kg hanging at the end of the link two. Figure (9) shows samples of positions during this test.

The samples of results of the parameters of error which were read in this test are shown in the figures from (10) to (13) which represent the change of values for δy, δx and $d\phi$ at the tip with angles of the two links. The angles for the first link represent the angle of joint with addition 4o, while to the second link; the angles were taken relative to the dimension a1. From the figures (10) to (12) the deformation variables values of the first link decreased with increasing θ_2 for each value of θ_1, while figure (13) shows that; with increasing θ_2 the values of the variables δy_2 and $d\phi_2$ decrease and δx_2 increases.

These results were used for the following:

To compare between the two sets of equations (53 and 54) and (57 and 58).

The comparison between the two sets of flexible kinematic equations (53 to 54) and (57to 58)

The results, that were obtained in the part two, were used to calculate the px and py for the two sets of flexible kinematic equations (53) to (54) and (57) to (58) to compare between these two sets to explain why in this research the kinematic equation without approximations were used.

The figures (14) and (15) show the differences between the values of px and py of the two sets of equations. The difference were taken by subtracting the px and py of equations (53) and (54) from the px and py of equations (57) and (58) respectively.

The maximum value of differences in the x-direction (px) is (6.3E-8 m) as shown in the figure (14). While from figure (15) It can be easily to see that, the differences of the py (in the y- direction), where the maximum value equals to (2.9E-8 m).

CONCLUSIONS

The difference between the two sets of equations is small, and it could be ignore in some applications while in the applications that need to high accuracy like micro applications it is not worth to ignore this difference. Also with increasing the flexibility of links this difference will be increase may be in significant values.

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Symbols:

Sym.	Definition	Unit
a_i	Perpendicular distance between axis of shaft for two parallel joints	m
c_{12}	$\cos(\theta_1 + \theta_2)$	
cd_{12}	$\cos(d\phi_1 + d\phi_2)$	
cd_{1-2}	$\cos(d\phi_1 - d\phi_2)$	
cd_i	$\cos d\phi_i$	
c_i	$cos\,\theta_i$	
d_i	Prismatic joint displacement	m
$d\phi$	Rotation the tip of arm due to the bending	Deg.
p_{defl}	Transformation matrix with error	
q_i	Configuration variable for robot	m or Deg.
R	Rotation matrix	Deg.
s_{12}	$\sin(\theta_1 + \theta_2)$	
s_{12}	$\sin(\theta_1 + \theta_2)$	
sd_{12}	$\sin(d\phi_1 + d\phi_2)$	
sd_{1-2}	$\sin(d\phi_1 - d\phi_2)$	
sd_i	$\sin d\phi_i$	
s_i	$\sin\theta_i$	
δx	Elongation of arm	m
δy	Deflection of arm	m
ε	A dimensional link deflection vector	
θ_i	Joint variables	Deg.

Table (1) The kinematic parameters of robot

Joints	θ_i	d_i	α_i	a_i
1	θ_1	0	0	a_1
2	θ_2	0	0	a_2

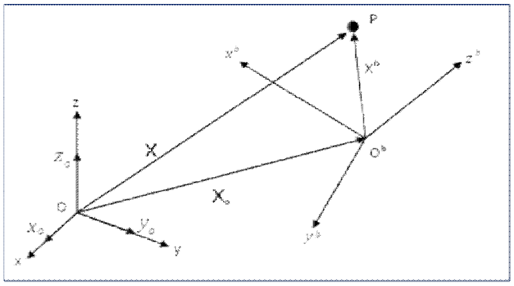


Figure (1) The position of point relative to coordinate frame.

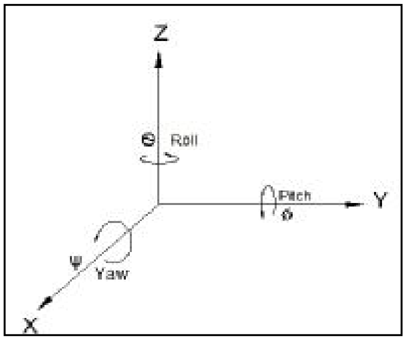


Figure (2) Roll, Pitch and Yaw coordinates for a manipulator.

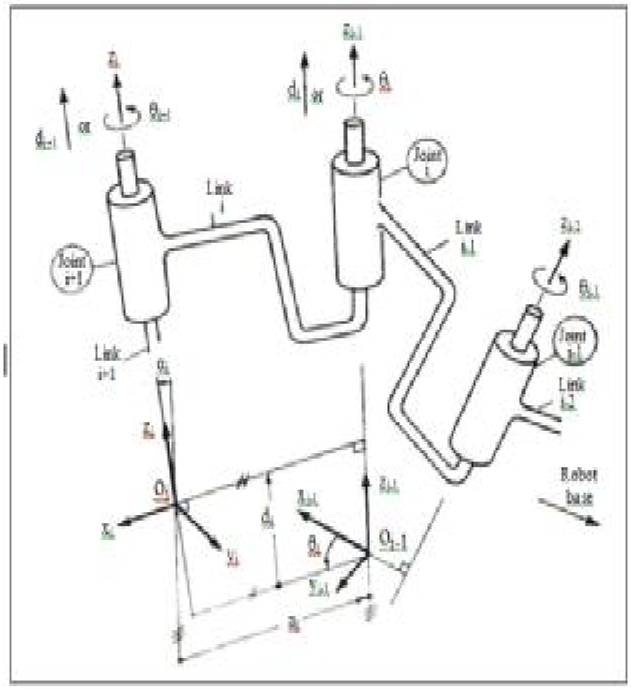
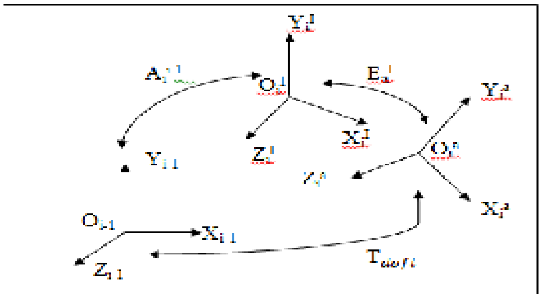


Figure (3) The Denavit-Hartenberg



Figur (4) Frame displacement due to errors.

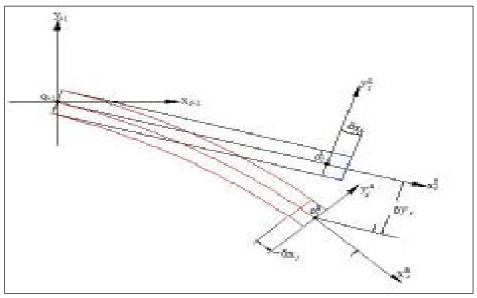


Figure (5) The coordinates system of a single semi-flexible link under deformation.

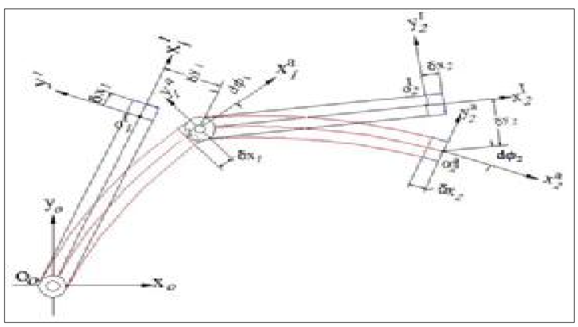


Figure (6) Coordinates systems of two links of semi- flexible robot arm.

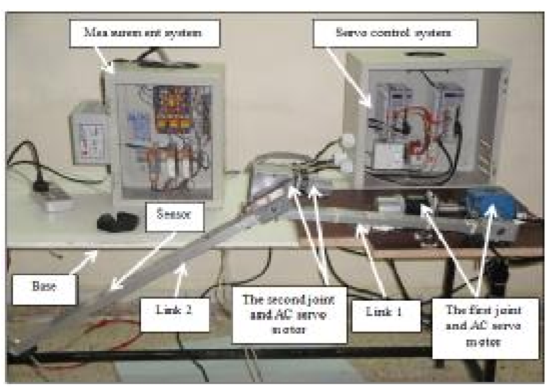


Figure (7) The semi-flexible robot for experimental work.

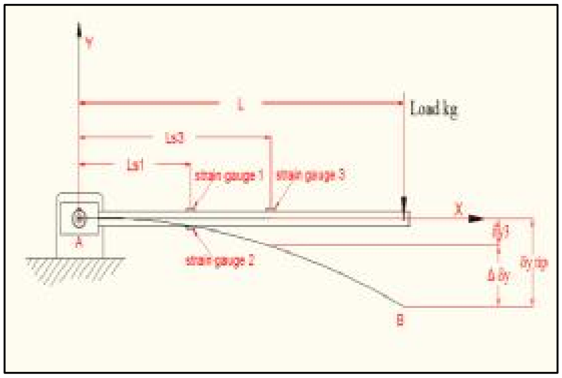


Figure (8) Deflection curve of the beam

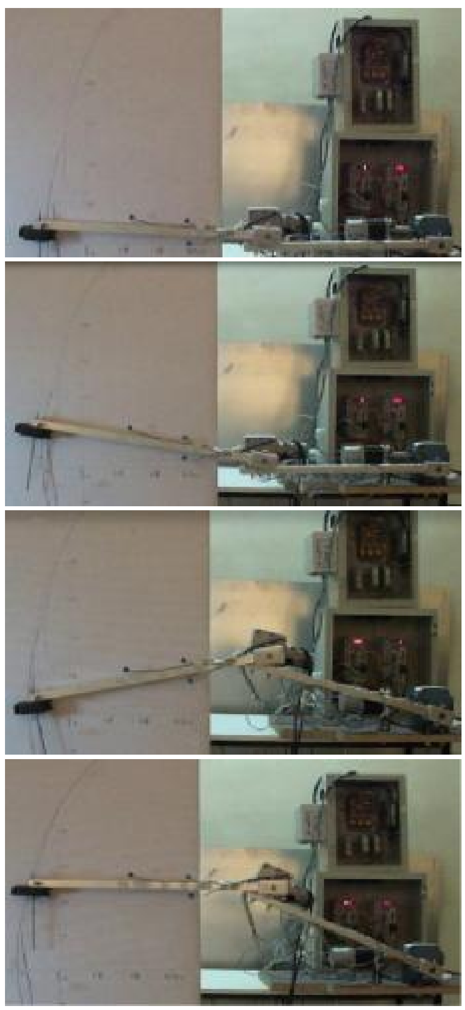


Figure (9) Samples of positions during experimental work.

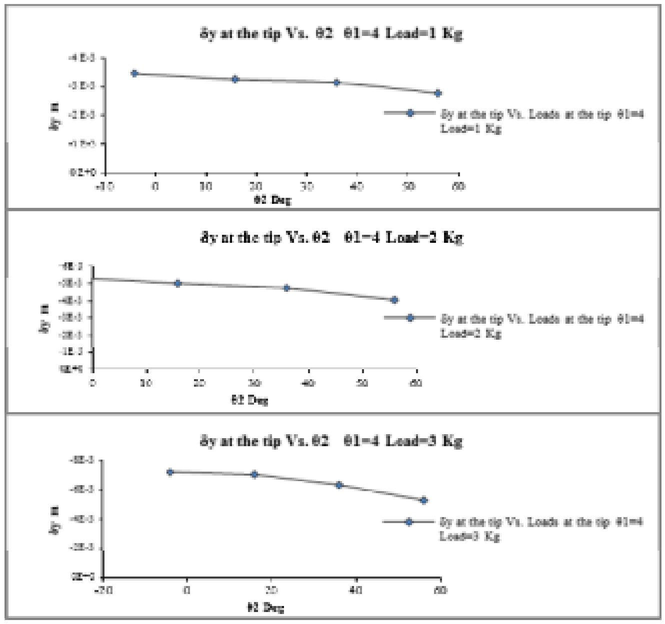


Figure (10) Vs. θ2 at θ1=4 Deg.
Link-1

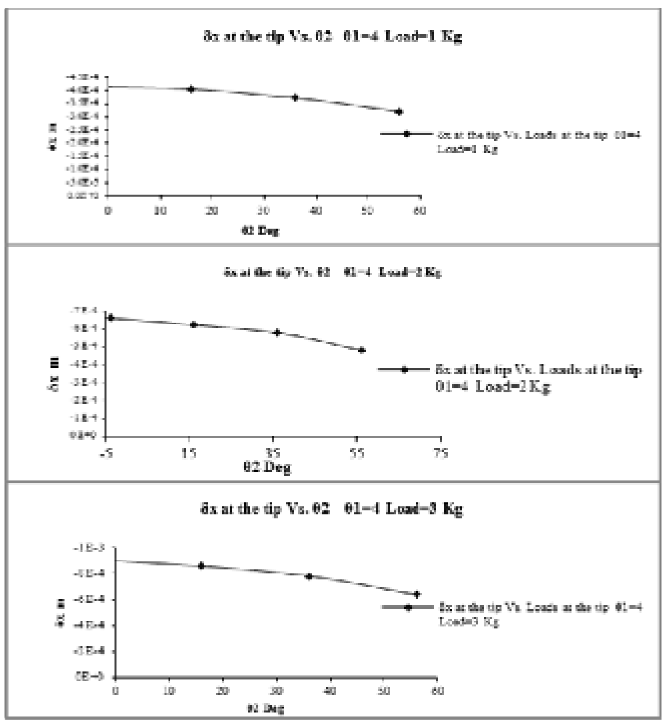


Figure (11) δx vs. θ2 at θ1=4 Deg.
Link-1

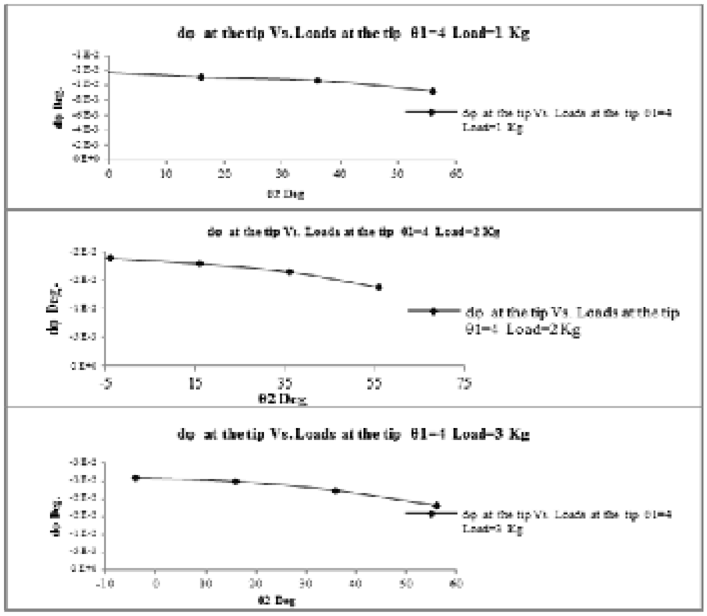


Figure (12) dΦ Vs. θ2 at θ1=4 Deg.
Link-1

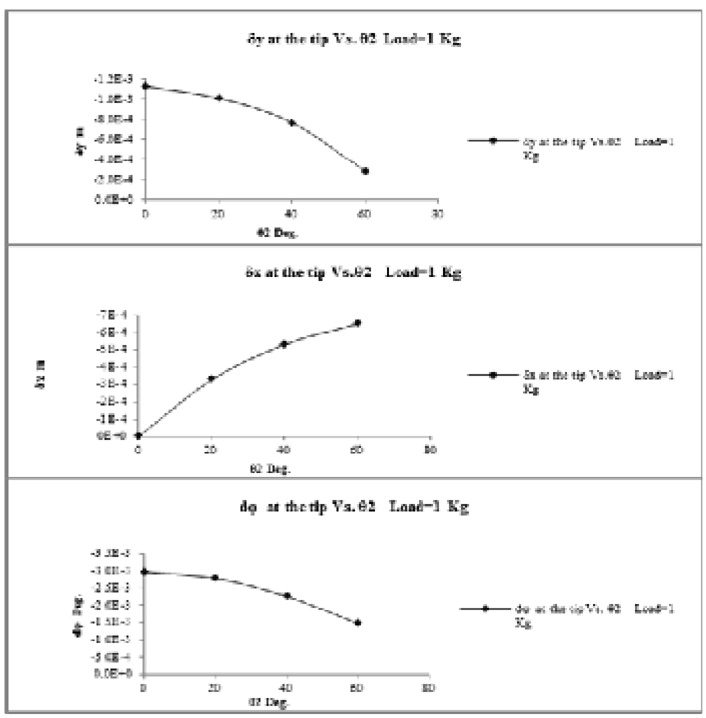


Figure (13) δy, δx and dΦ Vs. θ2 Deg.
Link-2

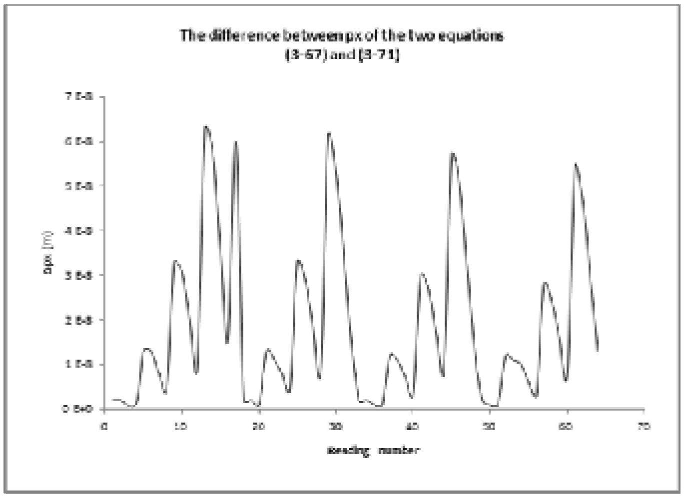


Figure (14) The difference between the px of the two equations (53) and (57)

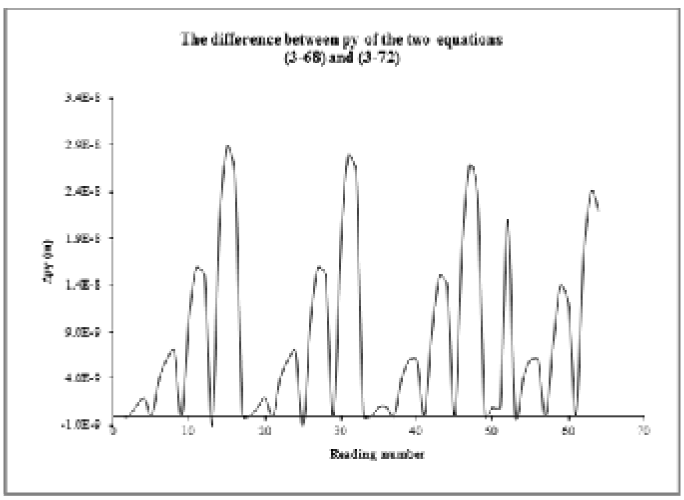


Figure (15) The difference between the px of the two equations (54) and (58)