Semi –Coercive Function

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Abstract

In this paper ,we introduce the definition of semi-coercive function and introduce several properties of semi-coercive function.

الخلاصة

في هذا البحث , سوف نقدم تعريف الدالة شبه الاضطرارية وقدمنا بعض المبرهنات حول الدالة شبه الاضطرارية .

Introduction

The notion of semi – open set was introduced by Levine [1] . Semi – compact space and semi – compact function were introduced by Dorsettt [5] and Mustafa[6] respectively**.**We introduce a new definition of semi – coercive function .
Also we have the following results :

(1) Let X , Y and Z be spaces and $f: X \to Y$, $g: Y \to Z$ be function Then :

(i)If f and g are semi-coercive functions ,then $gof: X \to Z$ is semi-coercive function .
(ii) If gof is semi-coercive function , g is semi – irresolute and bijective , then f is semi-coercive function .
(iii)) If gof is semi-coercive function , f is semi – irresolute and onto , then g is semi-coercive function .
(2) Every semi – compact function is semi-coercive function
(3) Let X and Y be spaces , and $f: X \to Y$ be semi- coercive function such that A is clopen set in X , then $f_{/A}: A \to Y$ is semi- coercive function .

1- Basic Concepts

Definition 1.1,[1]

A set B in aspace X is called semi –open (s.o) if there exists an open subset O of X such that $O \subseteq B \subseteq \overline{O}$.
The complement of a semi-open set is defined to be semi-closed (s.o) .

Definition 1.2,[2]

A subset B of aspace X is called pre-open if $B \subseteq \overline{B}^{0}$.The complement of a pre-open set is defined to be per- closed.

Proposition 1.3,[3]

Let $A \subseteq B \subseteq X$,where X is aspace and B is pre- open in X .Then A is semi –open (res. Semi –closed) in B if and only if $A = S \cap B$,where S is semi –open (res. Semi –closed) in X.

Definition 1.4,[1]

Let X and Y be space and $f : X \to Y$ be a function . Then f is called semi-continuous function if $f^{-1}(B)$ is semi –open set in X , for every open set B in Y . **Definition 1.5 ,[3]**

Let X and Y be space and $f : X \to Y$ be a function . Then f is called semi- irresolute function if $f^{-1}(B)$ is semi –open set in X , for every semi- open set B in Y .

Definition 1.6, [4]

A space X is called a compact space if every open cover of X has a finite subcover

Definition 1.7, [5]

Let (X , T) be aspace .Then (X ,T) is s semi-compact iff every semi-open cover of X has a finite sub cover .

Proposition 1.8, [3]

Let B be a pre –open subset of a space X and $A \subseteq B$. Then A is semi-compact set in X if and only if A is semi-compact se in B .

Proposition 1.9, [3]

Let A be a semi –compact set in a space X and B be a semi-closed subset of X . Then $A \bigcap B$ is semi –compact set in X .

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Proposition 1.10 , [3]

Let $f : X \to Y$ be semi – irresolute function ,then if A is semi –compact set in X , then $f(A)$ semi –compact set in Y.

Definition 1.11,[6]

Let X and Y be a space ,the function $f : X \to Y$ is called semi – compact if the inverse image of each semi – compact set in Y is semi – compact set in X .

2- The main results

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Definition (2.1) :

Let X and Y be spaces . A function $f : X \to Y$ is called semi-coercive if for every semi – compact set $J \subseteq Y$ there exists semi – compact set $K \subseteq X$, such that :

$$f(X \setminus K) \subseteq Y \setminus J$$

Example (2.2) :

If X is semi – compact space ,then the function $f : X \to Y$ is semi-coercive .

Proposition (2.3) :

Let X , Y and Z be spaces and $f : X \to Y$, $g : Y \to Z$ be function Then :

(i)If f and g are semi-coercive functions ,then $gof : X \to Z$ is semi- coercive function .

(ii) If gof is semi-coercive function , g is semi – irresolute and bijective , then f is semi-coercive function .

(iii)) If gof is semi-coercive function , f is semi – irresolute and onto , then g is semi-coercive function .

Proof :

(i) Let J be semi – compact set in Z , then there exists semi – compact set K in Y such that :

$$g(Y \setminus K) \subseteq Z \setminus J$$

Since $f : X \to Y$ is semi-coercive function , then there exists semi – compact set D ,such that :

$$f(X \setminus D) \subseteq Y \setminus K \quad \text{, then}$$

$$g(f(X \setminus D)) \subseteq g(Y \setminus K) \subseteq Z \setminus J$$

Hence $gof : X \to Z$ is semi-coercive function .

(ii) Let J be semi – compact set in Y, since g is semi – irresolute function , then by proposition(1.10) , $g(J)$ semi – compact set in Z .since gof is semi – compact function , then there exists semi – compact set in K in X such that :

$$gof(X \setminus K) \subseteq Z \setminus g(J) \text{, then } g^{-1}(gof(X \setminus K)) \subseteq g^{-1}(Z \setminus g(J))$$

Since g is bijective , then $g^{-1}(gof(X \setminus K)) = f(X \setminus K)$ and

$$g^{-1}(Z \setminus g(J)) = g^{-1}(Z \cap (g(J))^c)$$

$$= g^{-1}(Z) \cap g^{-1}(g(J))^c$$

$$= Y \setminus J$$

Thus $f(X \setminus K) \subseteq Y \setminus J$. Hence $f : X \to Y$ is semi-coercive function .

(iii) Let J be semi – compact set in Z , since gof is semi – compact function , then there exists semi – compact set in K in X such that $gof(X \setminus K) \subseteq Z \setminus J$, then $g(f(K^c)) \subseteq Z \setminus J$, since f is on to , then $g((f(K))^c) \subseteq g(f(K^c))$, thus $g((f(K))^c) \subseteq Z \setminus J$. since f is semi – irresolute function , then by proposition(1.10) , $f(K)$ semi – compact set in Y .
Therefore $g : Y \to Z$ semi-coercive function .

Proposition (2.4):

Every semi – compact function is semi-coercive function .

Proof :

Let J be any semi – compact set in Y , since $f : X \to Y$ is semi – compact function , then $f^{-1}(J)$ is semi – compact set in X . Thus $f(X \setminus f^{-1}(J)) \subseteq Y \setminus J$ Therefore $f : X \to Y$ is an semi-coercive function .

Proposition (2.5) :

For any clopen subset F of a space X , the inclusion function $i_F F \to X$ is semi-coercive function.

Proof :

Let K be semi – compact set in X , (To prove the function $i_F : F \to X$ is semi – compact function) .

Since F is clopen set in X , then by proposition (1.9) , $F \cap K$ is semi – compact set in X .

But $F \cap K \subseteq F$, thus by proposition (1.8) , $F \cap K$ is semi – compact set F .
But $i_F^{-1}(K) = F \cap K$, then $i_F^{-1}(K)$ is semi – compact set in F , therefore the inclusion $i_F : F \to X$ is semi – compact function , then by proposition (2.4) , the inclusion function $i_F : F \to X$ is semi- coercive .

Proposition (2.6) :

Let X and Y be spaces , and $f : X \to Y$ be semi- coercive function such that A is clopen set in X , then $f_{/A} : A \to Y$ is semi- coercive function

Proof :

Since A is clopen set in X , then by proposition (2.5) , the inclusion function $i_A : A \to X$ is semi- coercive function .

Since $f : X \to Y$ is semi- coercive function , then by proposition (2.3,i) , foi_A is semi- coercive function
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But $foi_A = f_{/A}$, then $f_{/A} : A \to Y$ is semi- coercive function

Proposition (2.7) :

Let X and Y be a space and $f : X \to Y$ be semi – coercive , semi - continuous , function . If T be clopen subset of Y , then $f_T : f^{-1}(T) \to T$ is semi – coercive function .

Proof :

Let J be semi – compact set in T , since T is clopen subset of Y , then by proposition (1.8) , T is semi – compact set in Y . since f is semi – coercive function , then there exists semi – compact set K in X such that :

$$f(X \setminus K) \subseteq Y \setminus J$$

Since T is clopen set in Y and f is semi - continuous function , then $f^{-1}(T)$ is semi – closed set in X , then by proposition (1.9) , $f^{-1}(T) \cap K$ is semi – compact set in X .

Since $f^{-1}(T)$ is pre – open set in X , then by proposition (1.8) , $f^{-1}(T) \cap K$ is semi – compact set in $f^{-1}(T)$. Notice that :

$$f_{/T}(f^{-1}(T) \setminus f^{-1}(T) \cap K) = f_{/T}(f^{-1}(T) \cap K^C) = f_{/T}(f^{-1}(T) \setminus K)$$

Since $f^{-1}(T)\setminus K) \subseteq X \setminus K$, then $f_T(f^{-1}(T)\setminus K) \subseteq f_T(X \setminus K)$

But $f_T(X \setminus K) = T \cap f(X \setminus K)$. Since $T \cap f(X \setminus K) \subseteq T \cap (Y \setminus J) = T \setminus J$, then $f_T(f^{-1}(T) \setminus f^{-1}(T) \cap K) \subseteq T \setminus J$.

Therefore $f_T : f^{-1}(T) \to T$ is semi – coercive function .

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