

()

Employment of the Factor Analysis Approach to Predict the Transfer Function Models

Abstract

The research deals with the employing of the factor analysis approach to structure the transfer function models and then predict depending on the principle component method and then apply that on the climate data (temperature, wind speed, solar radiation and the relative humidity) related to the evaporation - transpiration phenomenon, considering that the climate data is the input for the model and the evaporation - transpiration phenomenon stands for the outputs. The model was estimated twice: in the first estimation the transfer function was estimated using the data resulted from the

/ / / / 1
2

تاريخ التسليم 2011/6/5 تاريخ القبول 2011/10/4

.....

factorial analysis as inputs, while in the second estimation, the transfer function model was estimated using the climate data as inputs and then determining the best approach in prediction by means of prediction accuracy criterions.

: 1-1

: 2-1

:(Factor Analysis) 1-2-1

(Factor Analysis)

. (Hardle and Hlvka,2007)

:(The Factor Analysis Model) 1-1-2-1

m , n , m
 (Common Factors) (Unique Factors)

$$\left. \begin{aligned}
 x_1 &= l_{11}F_1 + l_{12}F_2 + \dots + l_{1p}F_p + e_1 \\
 x_2 &= l_{21}F_1 + l_{22}F_2 + \dots + l_{2p}F_p + e_2 \\
 &\vdots \\
 x_m &= l_{m1}F_1 + l_{m2}F_2 + \dots + l_{mp}F_p + e_m
 \end{aligned} \right\} \dots\dots\dots(1)$$

$$\begin{aligned}
 &\dots\dots\dots \\
 &\dots\dots\dots (m) \dots\dots\dots : F_1, F_2, \dots, F_p \\
 &j = 1, 2, \dots, m \quad x_j \quad F_i \quad : l_{ji} \\
 &\dots\dots\dots x_j \quad i \\
 &\dots\dots\dots : e_j
 \end{aligned}$$

(General

$$\begin{aligned}
 &\dots\dots\dots, h_j^2 \quad \text{Variance)} \\
 &\dots\dots\dots, u_j^2 \\
 &(\dots\dots\dots, (Afifi and Clark, 1984) \quad x_j \\
 &\dots\dots\dots : Anderson, 1984) \\
 &Var(x_j) = h_j^2 + u_j^2 \quad \dots\dots\dots(2)
 \end{aligned}$$

\vdots
 $\dots\dots\dots : h_j^2$

.....

: u_j^2

(Factor Analysis Methods) **2-1-2-1**

 . (Principal Component Method) -1

 .(Diagonal Method) -2

 . (Maximum Likelihood Method) -3

 . (Imaginary Method) -4

(Unweighted Least Square -5

.Method)

 .(Generalized Least Square Method) -6

 .(Alpha Method) -7

 (The Centroid Method) -8

 (Multiple Factors Method) -9

:(Principal Component Method) **3-1-2-1**

.(Afifi and Clark,1984)

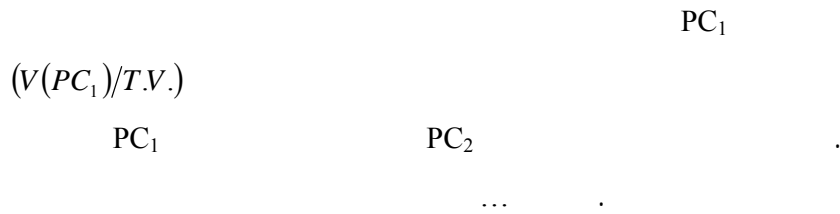
: (Morrison,1976) PC_i

$$\left. \begin{aligned} PC_1 &= a_{11}x_1 + a_{21}x_2 + \dots + a_{m1}x_m \\ PC_2 &= a_{12}x_1 + a_{22}x_2 + \dots + a_{m2}x_m \\ &\vdots \\ PC_p &= a_{1p}x_1 + a_{2p}x_2 + \dots + a_{mp}x_m \end{aligned} \right\} \dots\dots\dots(3)$$

:

. i : PC_i

. i : a_i



4-1-2-1

: (The Component Number Choice Methods)

: (Kaiser) -1

λ_j 1960 Kaiser

:(Cattel's) -2

(Scree Test) 1966 Cattel

$(1, \lambda_1), (2, \lambda_2), \dots, (m, \lambda_m)$

-3

: (Ratio Variance Principal Component Of The Variance Overall)

.(2000 ,) %75

5-1-2-1

(The Factor Analysis For The Principal Components)

(Morrison,1976):

.....

(m) :

.....
 : (PC) X

$$\left. \begin{aligned} x_1 &= a_{11}PC_1 + a_{21}PC_2 + \dots + a_{m1}PC_m \\ x_2 &= a_{12}PC_1 + a_{22}PC_2 + \dots + a_{m2}PC_m \\ &\vdots \\ x_m &= a_{1m}PC_1 + a_{2m}PC_2 + \dots + a_{mm}PC_m \end{aligned} \right\} \dots\dots\dots(5)$$

$$\begin{aligned} F_i &= PC_i / \sqrt{v(PC_i)} \\ &= PC_i / \sqrt{\lambda_i} \end{aligned} \dots\dots\dots(6)$$

x_j

$$v(x_j) = h_j^2 + u_j^2$$

$$h_j^2 = l_{j1}^2 + l_{j2}^2 + \dots + l_{jp}^2$$

$$v(x_j) = 1 = h_j^2 + u_j^2 \dots\dots\dots(7)$$

: (Rotating The Axes) 6-1-2-1

(Gailly and

. Adler,2004)

: (Transfer Function Models)

2-2-1

(System)

Y_t (Outputs)

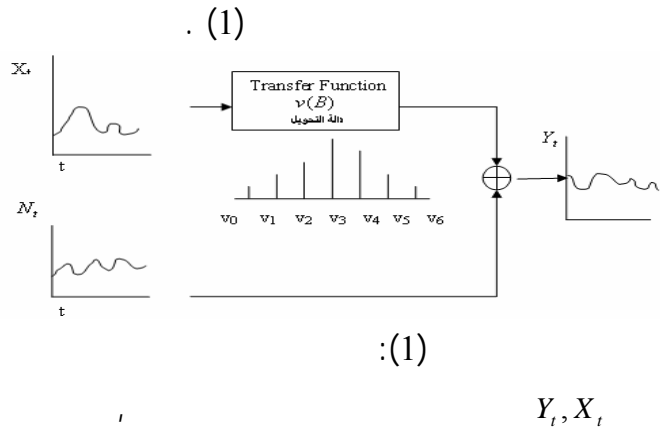
(Inputs)

X_t

N_t (Disturbances)

(Transfer Function)

. (Ljung,1999)



:
 $Y_t = v(B)X_t + N_t$

.....(8)

$$\phi(B)N_t = \theta(B)a_t \Rightarrow N_t = \frac{\theta(B)}{\phi(B)}a_t$$

$$Y_t = v(B)X_t + N_t$$

.....(9)

$$\phi(B)\Phi(B^s)N_t = \theta(B)\Theta(B^s)a_t \Rightarrow N_t = \frac{\theta(B)\Theta(B^s)}{\phi(B)\Phi(B^s)}a_t$$

: $\Theta(B^s), \Phi(B^s)$:

Box and) (Impulse Response Function)

: $v(B)$:

. (Hipel and Mcleod,1994) (Jenkins,1976

:(Impulse Response Function)

1-2-2-1

(Impulse Response Function)

$(v_0, v_1, v_2, v_3, \dots)$

$$v(B) = v_0 + v_1B + v_2B^2 + \dots$$

.....(10)

$\delta(B)$ $\omega(B)$ (Polynomial)

$$v(B) = \frac{\omega_s(B)B^b}{\delta_r(B)}$$

.....(11)

,(Initial Effect)

: $\omega(B)$

(Damping Factor)

: $\delta(B)$

. Y_{t-r}

$\omega(B)$:s

$\delta(B)$:r

(2011 ,) , () :b

- 2-2-2-1

(Box-Jenkins Methodology In Transfer Function Models Analysis)

:

Box-Jenkins

: Y_t, X_t

1-2-2-2-1

: (Identification Transfer Function Model)

Box-Jenkins

:

: (Estimation of transfer function weights) -1

:

(Estimation of transfer function weights depending on the cross-correlation function -a

-

(Cross Correlations Function)

(2011 ,) ,

-b

: (Linear Transfer Function Method)LTF

(1982) LTF

Pankratz (1991)

Hanssens Liu
Hudak Liu (1992)

:

$$Y_t = C + (v_0 + v_1 B + v_2 B^2 + \dots + v_k B^k) X_t + N_t \quad \dots\dots\dots (12)$$

k

Y_t

X_{1t}

:

$$Y_t = C + (v_{10} + v_{11} B + v_{12} B^2 + \dots + v_{1k} B^k) X_{1t} + N_t \quad \dots\dots\dots (13)$$

:

:C

$$Y_t = \sum_{j=0}^k v_{1j} X_{1t}^j + N_t \quad (j = 0, 1, \dots, k)$$

:k

$$m = n - k \quad \dots\dots\dots(14 - a)$$

$$\underline{\beta} = [c \ v_{10} \ v_{12} \ \dots \ v_{1k}]' \quad \dots\dots\dots(14 - b)$$

$$\underline{Y} = [Y_{k+1} \ Y_{k+2} \ \dots \ Y_{k+m}]' \quad \dots\dots\dots(14 - c)$$

$$\underline{X} = [1 \ X_1^0 \ X_1^1 \ \dots \ X_1^k] \quad \dots\dots\dots(14 - d)$$

$$\underline{N} = [N_t] \quad \dots\dots\dots(14 - e)$$

$$X = [X_{1(k+1)} \ X_{1(k+2)} \ \dots \ X_{1(k+m)}]' \quad X_1^j = B^j X_1^0 :$$

(Least Square)

(14-e) (14-d) (14-c) (14-b) (14-a) (13)

$$\underline{Y} = \underline{X} \hat{\underline{\beta}} + \underline{N} \dots\dots\dots(15)$$

$$\hat{\underline{\beta}} = (\underline{X}' \underline{X})^{-1} \underline{X}' \underline{Y} \dots\dots\dots(16)$$

Liu,) (2007,) , (2011 ,) , (1994

(LTF)

(2007,)

N_t ARMA -2

: (Determination Of the ARMA Model for the Disturbance Term)

ARMA

$$N_t = Y_t - v(B)X_t$$

$$Y_t = v(B)X_t + N_t \dots\dots\dots(17)$$

$$\therefore N_t = Y_t - v(B)X_t$$

.....

$$\begin{array}{ccc}
 a_t & & N_t \\
 & & : \\
 \phi(B)N_t = \theta(B) a_t & & \dots\dots\dots(18)
 \end{array}$$

SARMA

$$\begin{array}{ccc}
 \phi(B)\Phi(B)N_t = \theta(B)\Theta(B)a_t & & \dots\dots\dots(19)
 \end{array}$$

2-2-2-2-1

:(Parameters Estimation of Transfer Function Model)

:

,(Parameters)

$$\underline{\omega} = [\omega_0, \omega_1, \dots, \omega_s]', \underline{\delta} = [\delta_1, \delta_2, \dots, \delta_r]', \underline{\phi} = [\phi_1, \phi_2, \dots, \phi_p]', \underline{\theta} = [\theta_1, \theta_2, \dots, \theta_q]', \\
 \underline{\Phi} = [\Phi_1, \Phi_2, \dots, \Phi_p]', \underline{\Theta} = [\Theta_1, \Theta_2, \dots, \Theta_q]'$$

$$\underline{\beta}' = (\underline{\omega}, \underline{\delta}, \underline{\phi}, \underline{\theta}, \underline{\Phi}, \underline{\Theta}) \quad (20)$$

:

$$\delta(B)\phi(B)\Phi(B)Y_t = \phi(B)\omega(B)\Phi(B)X_{t-b} + \delta(B)\theta(B)\Theta(B)a_t \quad \dots\dots\dots(21)$$

: (21)

$$a_t = Y_t + d_1Y_{t-1} + \dots + d_{p+r}Y_{t-p-r} + c_0X_{t-b} + c_1X_{t-b-1} + \dots + c_{p+s}X_{t-b-p-s} + \\
 b_1a_{t-1} + b_2a_{t-2} + \dots + b_{r+q}a_{t-r-q} \quad \dots\dots\dots(22)$$

:

$$\begin{array}{lcl}
 \cdot \Phi & \phi & \delta & : d \\
 \cdot \Phi & \phi & \omega & : c \\
 \cdot \Theta & \theta & \delta & : b
 \end{array}$$

β' . σ_a^2
(Conditional Likelihood Function)

:

$$L(\beta, \sigma_a^2 | X, Y, X_0, Y_0, a_0) \propto \sigma^{-n} \exp\left(-\frac{1}{2\sigma^2} \sum_{t=1}^n a_t^2\right) \dots\dots\dots(23)$$

. a_t (Proper Starting) X_0, Y_0, a_0

(Liu,1992- ,(Box and Jenkins,1976) ,(Abraham and Ledolter,2005)
.1994)

3-2-2-2-1

(Diagnostic checking of the Model)

ARMA

\hat{a}_t .
(Makridakis et al.,1998)

:(Prediction) 4-2-2-2-1

ARIMA

Y_t Y_t
, X_t

.(MMSE) (Minimum Mean Square Error)

:(Prediction Accuracy Criteria) 3-2-2-1

:

.....
 1. (Mean Absolute Errors) MAE :

$$MAE = \frac{\sum_{t=1}^n |Y_t - \hat{Y}_t|}{n} \dots\dots\dots(24)$$

2. (Mean Square Error) MSE :

$$MSE = \frac{\sum_{t=1}^n (Y_t - \hat{Y}_t)^2}{n} \dots\dots\dots(25)$$

3. MAPE معدل

:(Mean Absolute Percentage Error)

$$MAPE = \sum_{t=1}^n \frac{|Y_t - \hat{Y}_t|}{Y_t} / n \dots\dots\dots(26)$$

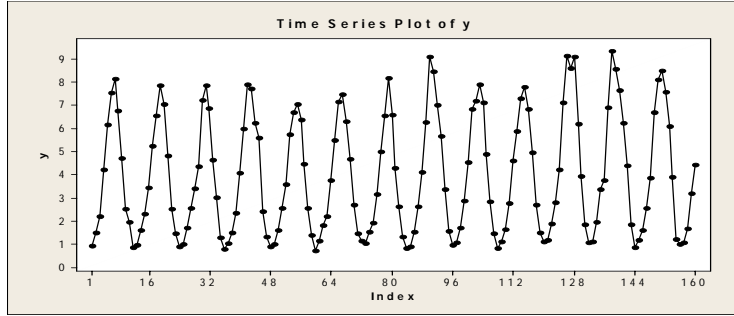
.(Makridakis et al.,1998)

3-1 :

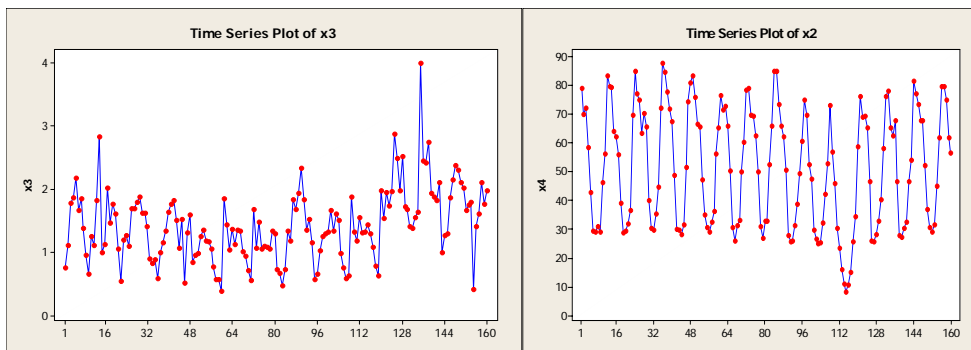
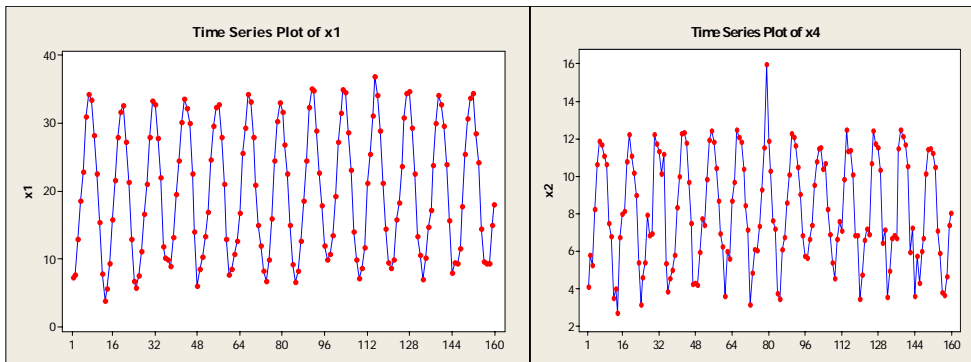
، °)
 (/ ، % ،
 ، 160 2004/04/30 1991/1/1
 . /

(Input)
 - (Output) (Transfer Function Models)
 ، (/) ET_o
 :
 . ET_o - : Y
 . (°) : X_1

. (%) : X_2
. (/) : X_3
. () : X_4



Y -



:(2)

.....

(X_1, X_2, X_3, X_4)

. (varmax)

(%69.5)

(%25.3)

. (%94.8)

(%69.5)

((%)

,(()

)

((°)

)

(%25.3)

. ((/)

)

:

:(1)

كميات الشبوع	تحميلات العامل F ₂	تحميلات العامل F ₁	المتغيرات
0.953673	-0.005897	-0.932896	1
0.926094	-0.086654	0.97239	2
0.999281	0.997953	0.063926	3
0.936069	0.094789	0.979641	4

.(2011 ,)

Liu

,(SCA)

(Statistical System)

Box- Jenkins

156

(Out of

4

Sample)

1-3-1

:

()

(Identification)

(Diagnostic Checking)

(Estimation)

. (Prediction)

SCA

Liu

-

:

-

$$\hat{Y}_t = -0.4294 \times F1_{t-2} + N_t \quad \dots\dots\dots(26)$$

:

$$N_t = (1 - 0.6251B)a_t \quad \dots\dots\dots(26 - a)$$

: a_t

:

-

: (2)

NO.	Y	\hat{Y}
157	-0.233	-0.25857
158	0.195	0.220919
159	0.567	0.550371
160	-0.105	-0.09797

وبما أن a_{t+i} هي القيمة التنبؤية لخطوة مستقبلية مقدارها i وهذه القيمة لا يمكن التنبؤ بها لذلك تكون مساوية إلى الصفر .

:

-

$$X_j = l_{ji} F_i \quad \dots\dots\dots(27)$$

:

-

$$\hat{Y}_t = -0.4294 \times F1_{1t-2} - 0.4294 \times F1_{2t-2} - 0.4294 \times F1_{3t-2} - 0.4294 \times F1_{4t-2} + N_t \quad \dots\dots\dots(28)$$

:

-

-

: (3)

NO.	Y	\hat{Y}
157	-0.233	-0.21619
158	0.195	0.178138
159	0.567	0.581602
160	-0.105	-0.12168

2-3-1

()

:

: SCA

$$Y_t = 0.1274F2_{t-3} - 0.1943F2_{t-4} + N_t \dots\dots\dots(29)$$

:

$$N_t = (1 - 0.6468) a_t \dots\dots\dots(30)$$

: a_t

(29)

:

: (4)

NO.	Y	\hat{Y}
157	-0.233	-0.22911
158	0.195	0.22492
159	0.567	0.607728
160	-0.105	-0.02475

:

$$Y_t = 0.1274X_{1t-3} + 0.1274X_{2t-3} + 0.1274X_{3t-3} + 0.1274X_{4t-3} - 0.1943X_{1t-4} - 0.1943X_{2t-4} - 0.1943X_{3t-4} - 0.1943X_{4t-4} + N_t \dots\dots(31)$$

: (5)

NO.	Y	\hat{Y}
157	-0.233	-0.20362
158	0.195	-0.16934
159	0.567	0.78799
160	-0.105	1.0452

3-3-1

$$\hat{Y}_t = 0.2994 \times X3_{t-3} + N_t \quad \dots\dots\dots(32)$$

$$N_t = (1 - 0.6128B)a_t \quad \dots\dots\dots(33)$$

: a_t

: (6)

NO.	Y	\hat{Y}
157	-0.233	-0.30596
158	0.195	0.229946
159	0.567	0.573642
160	-0.105	-0.10307

.(2011 ,)

4-1

.(7)

: (7)

MAE	MSE	MAPE	النهج المستخدم
0.029019	0.00165	-0.00586	النهج التقليدي
0.018687	0.00041	-0.01425	نهج التحليل العاملي قبل تحويل العامل الأول إلى متغيرات
0.016338***	0.000268***	-0.02937***	نهج التحليل العاملي بعد تحويل العامل الأول إلى متغيرات
0.038447	0.002236	-0.01392	نهج التحليل العاملي قبل تحويل العامل الثاني إلى متغيرات
0.441228	0.376323	-0.01057	نهج التحليل العاملي بعد تحويل العامل الثاني إلى متغيرات

:

: 5-1

:

,"

":(2000)

-1

-
- " : (2007) -2
- " : (2011) -3
- " -
- 4- Abraham, Bovas and Ledolter, Johannes (2005) : " Statistical Methods for Forecasting" , John Wily and Sons, United States of America.
 - 5- Afifi, A.A and Clark, V. (1984): "Computer Aided Multivariate Analysis", Life Time Learning Publications, California, USA.
 - 6- Anderson, T.W., (1984) : "An Introduction to Multivariate Statistical Analysis", 2nd ed., John Wily and Sons, New York-USA.
 - 7- Box, G.E.P and Jenkins, G.M. (1976): "Time series Analysis Forecasting and Control", Holden – Day, San Francisco.
 - 8- Gailly, Jean-loup and Adler Mark (2004) : " SPSS 13.0 Base User's Guide " SPSS Inc, United States of America.
 - 9- Hardle, Wolfgang and Hlvka, Zdenek, (2007): "Multivariate Statistics: Exercises and Solution" . Springer-Verlag, Berlin .
 - 10- Hipel, K.W. and Mcleod, A.I. (1994): " Time Series Modeling Of Water Resources And Environmental Systems", Elsevier Science B.V. All rights reserved, Netherlands.
 - 11- Liu, L.-M. and Hudak, G.B., (1994): "Forecasting and Time Series Analysis Using the SCA Statistical System", Volume 1, Scientific Computing Associates Crop. , Chicago.
 - 12- Ljung, L., (1999): "System Identification Theory for user", 2nd ed. Prentice Hall Upper Saddle River N.J. London UK.
 - 13- Makridaskis, S. Wheelwright, S. and Hyndman, R. (1998): "Forecasting: Methods and Applications", 3rd ed., John Wiley and Sons, New York, USA.
 - 14- Morrison, D.F. (1976): "Multivariate Statistical Methods " 2nd ed., Mcgraw-Hill. Inc. New York, USA.
 - 15- Wei, W. W. S. (1990): "Time Series Analysis Univariate and Multivariate Methods", Addison-Wesley Publishing Company, Inc., The Advanced Book Program, California, USA.