Derivation and analyzation of steady state error (s.s.e) performance of rejected disturbances in the variable structure of control (VSC) systems

اشتقاق وتحليل نسبة حالة الخطأ المستقرة لأداء الاضطرابات المرفوضة للمنظومات أشتقاق وتحليل نسبة حالة الخطأ المستقرة لأداء الاضطرابات المرفوضة للمنظومات

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Abstract:

This research presents an analytical study in the variable structure of control (VSC), where many equations had been derived in the (VSC) and designed to meet the recorrects for the fast response, low overshoot and small (S.S.E) for the 2nd order control system. Through using (VSC) efficiency feature, it is found that, it is possible to obtain high quality control which can't be obtained through using conventional control. It is noticed that, these controllers are easy and also, the switching cct's is simply implemented, and the system works even if the first structure is unstable, however it is necessary to have stable over damped structure. The most switching function algorithms which proposed here for (VSC) systems in different application fields (lift), claim a 2nd order and high order of robustness with respect to parameters variation and external disturbances. The analysis presented here in treating the question of changing the structure when a deterministic disturbance acts anywhere in the control system using Matlab programs.

An adaptive model reference algorithm is established to make the control system respond satisfactory or not there is a disturbance.

الخلاصة:

يمثل هذا البحث در اسة تحليلية للسيطرة على المنظومات ذات التركيبة المتغيرة حيث تم اشتقاق المعادلات الأساسية لها والتي تكون ذات استجابة سريعة، أقل سعة للموجه الأولى وأقل نسبة لحالة الخطأ المستقرة في المنظومات من الدرجة الثانية و بكفاءة عالية مقارنة مع منظومات السيطرة التقليدية والتي تمتاز بسهولة السيطرة عليها وسهولة التصميم. لقد لوحظ في هذا العمل انه لو كانت المنظومة مكونة من تركيبين (S1 و S2) فأذا كان التركيب الأولى اS1 غير مستقر فمن المحروري ان يكون التركيب الثاني S2 مستقر. كما يمكن ملاحظة انه في التطبيقات المختلفة للمنظومات ذات التركيبة المتركيبة الأول التركيبة التركيب الثانية منظومات متينة من الدرجة الثانية المصنعين من تركيبين (S1 و S1) فأذا كان التركيب الأول S1 غير مستقر فمن الضروري ان يكون التركيب الثاني S2 مستقر. كما يمكن ملاحظة انه في التطبيقات المختلفة للمنظومات ذات التركيبة المتغيرة مثل المصعد تتطلب منظومات متينة من الدرجة الثانية او أعلى حيث تظهر قدرة وفعالية خوارزمية ودوال التبديل المقترحة في إمكانية التعامل

ومن خلال البر امج الذي كتبت بلغة المصفوفات المختبرية (Matlab) فان التحليل المقترح سوف يعالج المشاكل الذي تظهر من خلال تغيير التركيب عندما يدخل اضطراب منتظم في مواقع مختلفة من منظومة السيطرة. كما شمل الاختبار صنف محدد من الاضطرابات العشوائية والذي من خلالها تم الحصول على أفضل النتائج والذي تستجيب لها المنظومة باستخدام التعشيق بين وظائف المنظومة ذات التركيبة المتغيرة ووظائف نظام السيطرة المكيفة باستخدام نموذج مرجعي.

1- Introduction

The search and application in the field of variable structure systems (VSS) has been maintained at a high level and dedicated to sliding mode control which matter reflects the interest of control theorists and practicing engineers. Sliding mode control enables efficient control of second-order and high order nonlinear plants operating under uncertainty conditions, which is common for a wide range of modern technological processes [1].

The sliding mode is principle operation mode in variable structure systems consisting of a set of continuous subsystems, referred to a structures and supplied with an appropriate switching logic [2]. The second-order system is used as a base for such analysis, for variable structure control (VSC) is a variable high-speed switching feedback control system, this (VSC) is advance in electronic technology and high-speed switching circuit of (on-off switching) control by the two values of the different structures to achieved the stability system and minimum error steady state (S.S.E).

The main effect of disturbance leads to a steady state error in the control system, however this error can be made small by increasing controller gain providing that the stability conditions remain valid [3].

Variable structure systems offer to the control designer new possibilities for improving the quality of control in comparison with range of control action, simples a large range of admissible transient processes in the system. Therefore such systems possess attractive advantage for example fast response, good transient performance and in sensitivity to variation in plant parameters or external disturbance [2, 4].

2- Theory

In the field of applied control technology it is well known that all realistic control systems operate in environments that produce system disturbances of one kind or another [5].

Here the term (disturbances) refers to that special category of system inputs which are not accurately known and which cannot be manipulated by the control designer, e.g. uncontrollable inputs. If the presence of disturbance is an inevitable feature, it follows that a properly designed control system must be capable of effectively coping with the range of disturbances that might conceivably act on the system, precisely a high performance control system should be designed so as to maintain the given control designed specification in the face of all disturbances that might act on the system under actual operating field conductions [6]. The classical control technology coping with disturbances, are known as (integral action, feed forward action and notch filters). To alter the steady-state error c/cs of the system closed loop transfer function. (modern control technology) cope effectively with realistic disturbances in complex multivariable control problems [7].

Research on the development of a modern state variable approach to the disturbance problems in feed back control design, present an approach called the method of disturbance accommodation controller (DAC) [8].

The basic idea of variable structure systems can be illustrated by considering the (2-nd order) with state feedback K_x .

 $\ddot{X} + a\dot{X} + (b+k)x = 0....(1)$

Where a and b are the plant parameters and K is the feedback gain. The structure of the dynamic system by eq. (1) in the phase plane takes different shapes depending on the values of K and the trajectories, can be sources, sinks spirals saddles etc as shown in Fig.(1). The values of the root of the (c/cs) equation decide the type of the trajectory the system takes. Therefore if the control law (Kx) is chosen so that K switches between two different values which charge the structure of the closed loop system from one of these forms to another the trajectories of qualitatively different form can be obtained. A simplified block diagram for (VSC) as shown in Fig. (2) [9].



Spiral sink

Fig.(1) Different types of the phase plane depend on the value of gain and trajectories



Fig.(2) Simplified block diagram for (VSC)

There are many ways to analyses the (VSC) systems performance depending on the criteria of determining the switching function S(t). In this work the error signal performance will be used to determine the switching function.

A second-order type one control system can be described by the following differential equation [10] :-

 $\frac{d^2}{dt^2}c(t) + 2\Im \frac{d}{dt}c(t) = k_s u(t)....(2)$ Where C(t) is the system output.

u(t) is the system input.

≤ is the damping coefficient (ratio).

 K_s is the system gain

A closed-loop with negative feedback (-ve F.B) and proportional feed forward controllers as shown in Fig.(3) will be new considered, in which the error differential equation for constant reference, input $r(t)=r_0$ is derived as follow:-

Since u(t) = $k_c e(t)$ and e(t) = r(t) - c(t). The equation (1) becomes, $\frac{d^2e}{dt^2} + 2\Im \frac{de}{dt} + k_s k_c e(t) = \overline{r}(t) + 2\Im \overline{r}(t)$



Fig.(3) Closed loop negative F.B.

And when $r(t) = \text{constant} = r_o(r'(t) = r''(t) = 0)$ the error differential equation is obtained in the form: $\frac{d^2e}{dt^2} + 2\Im \frac{de}{dt} + k e(t) = 0$ where the constant k=k_s k_c. The characteristic (c/c_s) equation of the system shown in Fig.(3) is $s^2 + 2\Im s + k = 0$ which has the following two root.

$$S_1 = -\mathfrak{z} - \sqrt{\mathfrak{z}^2 - k} \qquad (4)$$
$$S_2 = -\mathfrak{z} + \sqrt{\mathfrak{z}^2 + k}$$

Clearly the value of these roots and hence the system performance depends on the system damping coefficient and the product of the system and controller gains.

Therefore at the instant of changing the control input or when the error signal is large than the system has to respond fastly. This can be achieved by either decreasing the damping coefficient or increasing the gain of the controlled system. The root in Eq. (4) will have in this case complex conjugate values which yield harmonic oscillation in the system output around the steady state. Such output is not preferable specially if the magnitude the first peak (over shoot) is of percentage greater then 5%.

If however the values of damping coefficient and gain are switched to largest and smaller values respectively at certain moment of time (tsw) the one may obtain better performance, see Fig.(4). As shown in the Fig.(4) the system output in (Vsc) is composed two different response:

$$c(t)_{vsc} = \begin{cases} c_a(t); is \quad under \quad damped; 0 \le t \le t_{sw} \\ c_b(t); is \quad over \quad damped; t \ge t_{sw} \end{cases}$$
....(5)



Fig.(4) The system output (S.S.E) with different response

The evaluation of the moment of switching (tsw) is in general difficult, instead one way determine a switching condition when it satisfies the switching performed. [11]. Note that the evaluation of t_{sw} in (2nd-order) system is not difficult even thought it is used for practical implemented. Consider the variable damping coefficient system shown in the Fig.(5).

The under damping response $e_a(t)$ is obtained when the $(+\alpha 1)$ is connected and in this case the system (i/p) u1(t) will be given by [12]:



Fig.(5) The variable damping coefficient system with VSC

And hence error differential equation is given by: $\bar{e}_a(t) + (2\mathbf{z} - k_s \alpha_1)\bar{e}_a(t) + k e_a(t) = 0$

for the sake of simplicity the system is normalized by the following definitions (dimension less).

 $\tau = t\sqrt{k}$ Then $\frac{d^2e}{d\tau^2} + 2\mathfrak{Z}_1 \frac{de}{d\tau} + e(\tau) = 0....(8)$ In similar way for over damped response $e_b(t)$ the error differential equation is obtained $u_2(t) = k_c e(t) - \alpha_2 \bar{c}(t)$(9) where $2\mathbf{g}_2 = \frac{2\mathbf{J} + \alpha_2 \ k_s}{\sqrt{k}}$(11) Eq. (10) is the differential equation derived the error signal after switching. It solution is where λ_1 and λ_2 are the characteristics equation root given by $\lambda_1 = -\underline{3}_2 - \sqrt{\underline{3}_2^2 - 1}$ $\lambda_2 = -\underline{z}_2 + \sqrt{\underline{z}_2^2 - 1}$ The constant of integration c1 and c2 are given in terms of the initial conditions $e_b(\tau = 0)$ and $\bar{e}_b(\tau = 0)$ denote $E_{0} = e_{b}(\tau = 0) = e_{a}(\tau = \tau_{sw})$ $E_{0} = \bar{e}_{b}(\tau = 0) = \bar{e}_{a}(\tau = \tau_{sw})$(14)

Substituting in eq. (12) and its derivative for $t \approx 0$, then $c_1 + c_2 = E_0$

sub eq. (15) and (16) in (12) yields

Eq. (17) gives the possibilities of determining the condition of switching according to different criteria. Now if is assumed that the switching will take place only one time then the response of the variable structure control system will given by Eq. (17) and it will be denoted next by:

the possible criteria to be used for performing the switching of the requirement to obtain a periodic response after switching.

We assume that the switching will taken place such that the system response cross the zero value in Fig.(6). Then it is required to determine first the moment (τ_m) . Making the differential on Eq.(17).



Fig.(6) system response time before and after switching VSC

in order that the first derivative equally to zero, should have at (τ_m) ; $\lambda_1(E_1 - \lambda_2 E_0) e^{\lambda_1 \tau_m} = \lambda_2 (E_1 - \lambda_1 E_0) e^{\lambda_1 \tau_m}$ OR

$$e^{(\lambda_1 - \lambda_2)\tau_m} = \frac{\lambda_2 (E_1 - \lambda_1 E_o)}{\lambda_1 (E_1 - \lambda_2 E_o)}....(20.a)$$

$$\therefore \tau_m = \frac{1}{\lambda_1 - \lambda_2} \ln \left[\frac{\lambda_2 (E_1 - \lambda_1 E_o)}{\lambda_1 (E_1 - \lambda_2 E_o)} \right]....(20.b)$$

and in order that we will have zero (em) then from eq. (17), we get: (E₁- λ_2 E₀) $e^{\lambda_1 \times \tau_m} - (E_1 - \lambda_1 E_0) e^{\lambda_2 \times \tau_m} = 0$ (21) OR

$$e^{(\lambda_{1}-\lambda_{2})\tau_{m}} - \frac{E_{1}-\lambda_{1}E_{o}}{E_{o}-\lambda_{2}E_{o}} = 0$$
(22)

substitute for $e^{(\lambda_1 - \lambda_2) \tau_m}$ $\left[\frac{\lambda_2(E_1 - \lambda_1 E_o)}{\lambda_1(E_1 - \lambda_2 E_o)}\right] - \left[\frac{(E_1 - \lambda_1 E_o)}{(E_1 - \lambda_2 E_o)}\right] = 0$(23) After simplification we get: $\frac{E_1 - \lambda_1 E_o}{E_1 - \lambda_2 E_o} \left(\frac{\lambda_2}{\lambda_1} - 1\right) = 0$(24) Eq. (24) represents a condition for dead beat response. It is satisfied in two cases $either\left(\frac{\lambda_2}{\lambda_1} - 1\right) = 0$(25)

Which means $\lambda_2 = \lambda_1$ in which nothing now, or $(E_1 - \lambda_1 E_o) = 0$(26)

Which relates the values of the error signal and its derivative of the pervious response before switching.

In order word or can define the switching condition to be:

The under-damped structure to over damped structure and from Eq.(17) the error response after switching become according to $e(\tau) = E_o \times e^{\lambda_1 \tau}$(28)

from Fig.(7) illustrate the variable structure response [13].



Fig.(7) The response system of variable system structure

Now its required to determine analytically the moment of switching τ_1 The solution of eq.(8) is ea(τ) = $m_1 e^{r_1 \cdot \tau} + m_2 e^{r_2 \cdot \tau}$ (29) Where r_1 and r_2 the (c/cs) root given by:-

 $\mathbf{r_{1,2}} = -\mathbf{3_1} \neq \mathbf{3_1^2} - \mathbf{1}$ and constants m₁ and m₂ can be computed from the initial condition. $\mathbf{e}_a(0) = 1$ and $\mathbf{e}_a(0) = 0$; i-e from $1 = \mathbf{m}_1 + \mathbf{m}_2$ $0 = \mathbf{m}_1 \mathbf{r}_1 + \mathbf{m}_2 \mathbf{r}_2$ Which yield; $\mathbf{m}_1 = \frac{-r^2}{r_1 - r_2}$; $\mathbf{m}_2 = \frac{r_1}{r_1 - r_2}$ (31) Then

$$e_{a}(\tau) = \frac{1}{r_{1} - r_{2}} \left[-r_{2} \times e^{r_{1}\tau} + r_{1} \times e^{r_{2}\tau} \right].$$
 (32)

At the switching moment τ_1 ; s(τ_1) = 0 OR

$$\frac{r_{1} \times r_{2}}{r_{1} - r_{2}} \left[-e^{r_{1}\tau_{1}} + e^{r_{2}\tau_{1}} \right] - \frac{\lambda_{1}}{r_{1} - r_{2}} \left[-r_{2}e^{r_{1}\tau_{1}} + r_{1}e^{r_{2}\tau_{1}} \right] = 0 \dots (33)$$

$$\left[-\frac{r_{1} \times r_{2}}{r_{1} - r_{2}} + \frac{\lambda_{1} \times r_{2}}{r_{1} - r_{2}} \right] e^{r_{1}\tau_{1}} + \left[\frac{r_{1} \times r_{2}}{r_{1} - r_{2}} - \frac{\lambda_{1} \times r_{1}}{r_{1} - r_{2}} \right] e^{r_{2}\tau_{1}} = 0 \dots (34)$$

$$e^{(r_1 - r_2)\tau_1} = \frac{r_1(\lambda_1 - r_2)}{r_2(\lambda_1 - r_1)}....(35)$$

$$\therefore \tau_1 = \frac{1}{r_1 - r_2} \ln \left[\frac{r_1(\lambda_1 - r_2)}{r_2(\lambda_1 - r_1)} \right]....(26)$$

3- Illustrative results and calculations:

The purpose of (vsc) is to investigate the (S.S.E) in error response of a second order control system in the following without disturbance with disturbance and S.S.E operates to get the maximum and settling values of error signal are the objectives to be noticed.

To find the transfer function (T.F) of the closed loop system by;

 $\frac{c(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)} \tag{37}$

From Fig(8) to give that:



Fig.(8)Closed loop (-ve F.B)with controller

To get wn = 6.5; 3 = 0.59

And gives the error response of the system by eq.(36) that: $\mathbf{e}(\mathbf{t}) = \mathbf{e}^{-\mathbf{z}\mathbf{w}\mathbf{n}\mathbf{t}} \left[\cos w_n \sqrt{1 - \mathbf{z}^2} t + \frac{\mathbf{z}}{\sqrt{1 - \mathbf{z}^2}} \sin w_n \sqrt{1 - \mathbf{z}^2} t\right] \dots (39)$ $\mathbf{t}_m \approx 0.6 \text{ ; over shoot \% in this system; settling time (t_s)}$ $\mathbf{e}_m = over shoot \% = \bar{\mathbf{e}} \frac{\mathbf{z}}{\sqrt{1 - \mathbf{z}^2}} \pi = 10\% \dots (40)$ $\mathbf{t}_s = \frac{4}{w_n \sqrt{1 - \mathbf{z}^2}} = 0.763 \dots (41)$

From Fig.(9-a,b) to plot e(t) ; c(t) respectively and from Fig.(5) to determine that 2- structure in the system from table (1).



output In the Vsc

system	Properties of structure	Controller parameters (gain)	Damping coefficient	Feed back gain coefficient	k gain eient Roots equation for the system System		Time required to the switch in the system	Moment of switching	Error response for the system
Second order system 42.3 $\overline{s(s+7.7)}$	s ₁	k ₁ = 1	3 1 ₌ 0.1	$\alpha_1 = 0.15$	r ₁ =-0.1-j0.999	$\lambda_1 = -2.618$	t ₁ =0.2 msec	$\tau_1=1.3$ msec	e(t)=0.325
		$k_1 = 42.3$	3 1 ₌ -0.25	$\alpha_1=0.258$	r ₁ =0.25-j0.937	λ1=-2	t ₁ =0.15 msec	τ ₁ =0.981 msec	e(t)=0.0703
		$k_1 = 21.5$	3 ₁ =0.25	$\alpha_1 = 0.01$	r ₁ =-0.25-j0.937	$\lambda_1 = -3.732$	t ₁ =0.114 msec	τ_1 =1.67 msec	e(t)=0.0937
		$k_1 = 423$	3 1 ₌ 0.1	$\alpha_1 = 0.084$	r ₁ =-0.1-j0.990	$\lambda_1 = -2$	t ₁ =0.05 msec	$\tau_1=1.2$ msec	e(t)=-0.054

Table(1) simulation the (vsc) and properties

system	Properties	Controller k ₂	Damping 32	F. B. α ₂	Roots r_2 , λ_2	Time t ₂	Moment of switching τ_2	Error e(t)
$\frac{42.3}{s(s+7.7)}$	S ₂	k ₁ = 1	3₂ =1.5	0.279	r_2 = -0.1+j0.99; λ_2 = -0.381	0.432msec	2.303msec	0.523
		$k_1 = 42.3$	1.25	0.202	$r_2 = 0.25 + j0.937; \lambda_2 = -0.5$	0.353 msec	3.056 msec	0.435
		$k_1 = 21.5$	0.25	0.687	$r_2 = -0.25 + j0.37; \lambda_2 = -0.267$	0.236 msec	3.624 msec	0.366
		$k_1 = 423$	1.25	0.361	$r_2 = -0.1 + j0.99;$ $\lambda_2 = -0.5$	0.203 msec	3.836 msec	0.2

From table(1) shows the error transfer function of the structure s_1 and s_2 are similar approximately because the same type of disturbance the only difference is the switching sequence because the system starts with over damped structure then switches to the under damped structure. Fig.(10) shows the effectiveness of the error response performances in Fig.(11) shows the switching occurrence for 2-nd order (S.S.E) with disturbance the main effect of external disturbances is to impose a (S.S.E) theoretically (S.S.E) represents one of the powerful control tools to eliminate disturbance effects on the system. But in fact the simulation show that a complete rejection of disturbance effects can not to achieved through the purposed (S.S.E) design in Fig.(11).



Fig.(11) Switching occurrence for second order in the VSC with disturbance

Accordingly the simulation results summarize that there are steady errors according to the following cases:

1- disturbance at the input i.e, G(s) = 1.

2- disturbance at the output state feedback.

3- selection of switching conditions and sequence mode matched.

In order to evaluate the efficiency of different types of switching conditions (on-off) state or types of variable structure conditions on the (S.E.E) behavior, its interesting to make performance comparison.

Calculated the switching timing of (S.E.E) performance under effect of different input types as shows in table (2).

Input type		Dauformanaa				
input type	t ₁	t ₂	t ₃ t ₄ t ₅		renoimance	
Reference input	1	1.33	2.1	2.3	3.4	Acceptable ess = 0
disturbance	2.7	3.55	No other switching			Oscillatory unacceptable
Simultaneously reference i/p and distribution	1.3	1.7	1.98	2.31		Acceptable performance with $ess = 0.13$

Table (2) S.E.E performance under effect of different input types

The vsc needs least amount of information about the uncertainties and exhibit the best performance. This represents a superiority in addition to the more commonly known advantages of variable structure techniques like insensitivity to model uncertainties [11].

Conclusions

The following points summarize the resulted conclusions :

1- Using (VSC) it is possible to obtain high quality control which cant be obtained by conventional control.

2- The realization of these controllers is easy and also the switching circuit is simply implemented.

3- The system works even if the first structure is unstable however it is necessary to have stable over damped second structure.

4- Analysis and mathematical formulation improve output state feedback a logarithm response against disturbance.

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