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Investigation of Some Even -Even Nuclei with Mass Numbers (A=98,100) by Interacting Boson Model

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Abstract :

In this work, some nuclei with mass numbers (A=98,100) have been investigated by using the both (IBM-1 and IBM-2) versions of interacting boson model (IBM). In calculations, the theoretical energy levels have been obtained by using PHINT and NP-BOS program codes. The presented results are compared with the experimental data in respective tables and figures. At the end, it was seen that the obtained theoretical results are in good agreement with the experimental data. Also the mixing ratios are calculated .

الخلاصة :

تم في هذه العمل دراسة بعض النوى ذات العدد الكتلي (A=98,100) باستعمال أنموذجي IBM-1 و IBM-2 , حيث تم الحصول على قيم مستويات الطاقة النظرية باستعمال برنامجي PHINT و NP-BOS , وقورنت النتائج مع القيم العملية وكانت النتائج في هذه الدراسة ذات توافق جيد مع القيم العملية , وكذلك تم في هذه الدراسة حساب قيم نسب الخلط .

1-Introdution:

A good deal of works has been carried out on studying the structure and properties of complex nuclei, which propose several types of nuclear model. In order to understand the structure of complex nuclei, it must therefore be resorted to approximations. The choice of an appropriate approximations depends on nuclei nature. a nuclear model was proposed by Arima and Iachello, called Interacting Boson Model (IBM) to study the structure and properties of even – even nuclei [1] .

The existence and role of symmetries in the IBM framework represents its most unique and characteristic feature. Their description is simple and analytic. They have clear geometrical relationships and physical interpretations, and their predictions depend on an absolute minimum of parameters [2] .over the last decade the IBM has generated considerable interest, as well as its fair share of controversy, and has prompted a large number of new studies in nuclear structure and spectroscopy [3]. In previous years, some nuclear properties of nuclei are studied by (IBM) such as : Maras et al. [4] calculate the energy levels and the electric quadrupole transition probabilities $B(E2;I_i \rightarrow I_f)$ and gamma-ray E2/M1 mixing ratios for selected transitions of some even-even erbium isotopes, Turkan et al. [5] studied some Pd isotopes of the mass region of A ~ 100 by the IBM-2 .

2- Theoretical Basis :

2-1 Interacting Boson Model : (IBM-1)

The early version of the Interacting Boson Approximation Model (IBA), or (IBM-1), in which there was no distinction made between proton and neutron bosons, and number of bosons taken to be the number of nucleons outside the closed shell divided by two, and the most general Hamiltonian written as: [4]

$$H = \varepsilon n_d + a_0 P \cdot P + a_1 L \cdot L + a_2 Q \cdot Q + a_3 T_3 \cdot T_3 + a_4 T_4 \cdot T_4 \ldots, \qquad (1)$$

where ε, a_o, a_1, a_2, a_3, a_4 are the model parameters, n_d is the d-boson number operator, P and Q represent pairing and quadrupole operators written in the language of second quantization s,s+, d,d+, where s, s+, d,d+ are the annihilation and creation operators of s- and d-bosons, respectively,

$$Q = (s^\dagger \tilde{d} + d^\dagger \tilde{s})^{(2)} + \chi (d^\dagger \tilde{d})^{(2)}, P = \frac{1}{2}(\tilde{d}.\tilde{d} + \tilde{s}.\tilde{s}) \qquad (2)$$

The reduced quadrupole transition probability calculated from the relation:

$$B(E2;\, I_f \to I_i) = \frac{1}{2I_f+1} \left\langle I_f || T^{E2} || I_i \right\rangle^2 , \qquad (3)$$

where

$$T^{(E2)} = \alpha_2 \left(d^+ s + s^+ d\right)^{(2)} + \beta_2 \left(d^+ d\right)^{(2)} , \qquad (4)$$

α_2 and β_2 are two parameters which refer to the effective charge.

2-2 Proton-Neutron Interacting Boson Model : (IBM-2)

the IBM-2 Hamiltonian that has been used to calculate the level energies is:[4,5]

$$H = \varepsilon_\nu n_{d\nu} + \varepsilon_\pi n_{d\pi} + \kappa Q_\pi Q_\nu + V_{\pi\pi} + V_{\nu\nu} + M_{\pi\nu} \qquad (5)$$

where($n_{d\rho}$) is the neutron (proton) d-boson number operator.

$$n_{d\rho} = d^\dagger \tilde{d} \quad , \rho = \pi, \nu$$

$$\tilde{d}_{\rho m} = (-1)^m d_{\rho,-m}$$

Where $s^\dagger_\rho$, $d^\dagger_{\rho m}$and s_ρ , $d_{\rho m}$ represent the s and d -boson creation and annihilation operators. The rest of the operators in the Eq. (5) are defined as: [5]

$$Q_\rho = (s^\dagger_\rho \tilde{d}_\rho + d^\dagger_\rho s_\rho)^{(2)} + \chi_\rho (d^\dagger_\rho \tilde{d}_\rho)^{(2)}$$

$$V_{\rho\rho} = \sum_{L=0,2,4} C_{L\rho} \left((d^\dagger_\rho d^\dagger_\rho)^{(L)} \cdot (d^\dagger_\rho \tilde{d}_\rho)^{(L)} \right)^{(0)} \qquad ; \; \rho = \pi, \nu \qquad (6)$$

and

$$M_{\nu\pi} = \frac{1}{2} \xi_2 [(s^\dagger_\nu d^\dagger_\pi - d^\dagger_\nu s^\dagger_\pi)^{(2)} . (s_\nu \tilde{d}_\pi - \tilde{d}_\nu s_\pi)^{(2)}] - \sum_{L=1,3} \xi_L [(d^\dagger_\nu d^\dagger_\pi)^{(L)} . (\tilde{d}_\nu \; \tilde{d}_\pi)^{(L)}] \qquad (7)$$

In this case $M_{\pi\nu}$ affects only the position of the non-fully symmetric states relative to the symmetric ones. For this reason $M_{\pi\nu}$is often referred to Majorana force.[6,7]

The mixing ratio E2/M1, where T(E2; $Ji \rightarrow Jf$) is the number of E2 transitions per secondand T(M1; $Ji \rightarrow Jf$) is the number of M1 transitions per second, is given by:[8,5]

$$\delta(E2/M1; J_i \to J_f) \; = \; \frac{\sqrt{T(E2; J_i \to J_f)}}{\sqrt{T(M1; J_i \to J_f)}}. \qquad (8)$$

The ratio of δ(E2/M1) can be written in terms of matrix elements as follows :

$$\delta(E2/M1) \; = \; 0.836 \; E\gamma \; (MeV) \frac{\langle J_f || M(E2) || J_i \rangle}{\langle J_f || M(M1) || J_i \rangle}. \qquad (9)$$

3- Result s and Discussions :

In this section, the general features of some nuclei with the mass numbers (A=98,100) are reviewed and it was seen that the presented results have better agreement with the experiment.[9,10] some of the energy results calculated by IBM-1 are compared with the values obtained by IBM-2 .It must be stressed that the choice of parameters is undertaken iteratively by allowing one parameter to vary while keeping the others constant until an overall best fit was achieved . They are treated as free parameters that have been determined so as to reproduce as closely as possible the excitation energy of all positive parity levels for which a clear indication of the spin value exists. Tables (1 and 2) are showing such IBM-1 and IBM-2 parameters mentioned with the fit to the experimental data .

Table 1 . The Hamiltonian parameters set used in the present study for the IBM-1 calculations of some nuclei with mass numbers (A=98,100) (in MeV except CHI & SO6)

Nucleus	EPS	P.P.	L.L.	Q.Q.	$T_3.T_3$	$T_4.T_4$	CHI	SO6
^{98}Ru	0.5019	0.000	0.0114	0.000	0.000	0.0445	0.000	1.000
^{98}Sr	0.000	0.000	0.0184	- 0.0058	0.000	0.000	- 1.320	1.000
^{98}Mo	0.6780	0.000	0.0011	0.000	0.0587	0.011	0.000	1.000
^{100}Pd	0.6569	0.000	0.0093	- 0.028	0.000	0.000	0.000	1.000
^{100}Zr	0.000	0.0611	0.0301	0.000	0.0219	0.000	0.000	1.000
^{100}Ru	0.2920	0.000	0.0181	0.000	0.0022	0.0761	0.000	1.000

Table 2. The Hamiltonian parameters set used in the present study for the IBM-2 calculations of some nuclei with mass numbers (A=98,100)

Nucleus	N_π	N_ν	ε (MeV)	κ (MeV)	χ_ν	χ_π	$C_{L\nu}$ (L=0,2,4) (MeV)	$C_{L\pi}$ (L=0,2,4) (MeV)
^{98}Ru	3	2	0.799	- 0.064	- 0.57	- 0.6	0.10,-0.5,0	-0.6,-0.35,0
^{98}Sr	6	5	0.686	- 0.165	0.21	-1.05	0.94,-0.44,0	0.82,0,0
^{98}Mo	4	3	0.953	- 0.047	- 1.00	- 1.00	-1.0,0.2,0.06	0.6,0. 5,-0.25
^{100}Pd	2	2	0.795	- 0.083	- 1.20	0.60	0.2,-0.4,0	0.2,-0.4, 0
^{100}Zr	5	5	0.577	- 0.118	0.27	- 0.26	- 0.68,-0.38,0.1	0.6,0.7,0
^{100}Ru	3	3	0.748	- 0.074	-0.51	- 0.6	0.15, 0,0	-0.6,0.25,0.2

Also, we calculated the mixing ratios of some transitions in this nuclei by evaluation The reduced E2 and M1 matrix elements for a selected transitions .
The ratio δ(E2/M1) is defined as the ratio of the reduced E2 matrix element to the reduced M1 matrix element. This quantity is related to the usual δ- mixing ratio .
We have estimated multipole mixing ratios (δ(E2/M1)) of some transitions for this nuclei and then compared them with some previous experimental results. in Tables (3-5) There is no reference for transitions in some nuclei . The variations in sign of the E2/M1 mixing ratios from nucleus to nucleus for the same class transitions and within a given nucleus for transitions from different spin states suggest that a microscopic approach is needed to explain the data theoretically. For that reason, we did not take into consideration the sign of mixing ratios.

The mixing ratios values are given in following tables :

Table 3 . The mixing ratios values of ^{98}Mo and ^{98}Ru

$I_i \rightarrow I_f$	^{98}Mo		$I_i \rightarrow I_f$	^{98}Ru	
	Exp.	IBM-2		Exp.[11]	IBM-2
$2_1^+ \rightarrow 2_2^+$	----	0.0103	$2_1^+ \rightarrow 2_2^+$	13	12.5734
$2_1^+ \rightarrow 2_3^+$	----	0.9798	$2_1^+ \rightarrow 2_3^+$	----	0.2292
$4_1^+ \rightarrow 4_2^+$	----	0.0706	$4_1^+ \rightarrow 4_2^+$	----	9.3824
$3_1^+ \rightarrow 2_1^+$	----	1.3847	$3_1^+ \rightarrow 2_1^+$	< - 0.2	0.6390
$1_1^+ \rightarrow 2_1^+$	----	3.6156	$1_1^+ \rightarrow 2_1^+$	----	0.1916

Table 4 . The mixing ratios values of ^{98}Sr and ^{100}Zr

$I_i \rightarrow I_f$	^{98}Sr		$I_i \rightarrow I_f$	^{100}Zr	
	Exp.	IBM-2		Exp.	IBM-2
$2_1^+ \rightarrow 2_2^+$	----	1.2903	$2_1^+ \rightarrow 2_2^+$	----	0.8700
$2_1^+ \rightarrow 2_3^+$	----	0.0006	$2_1^+ \rightarrow 2_3^+$	----	0.0520
$4_1^+ \rightarrow 4_2^+$	----	0.8918	$4_1^+ \rightarrow 4_2^+$	----	11.5679
$3_1^+ \rightarrow 2_1^+$	----	2.9903	$3_1^+ \rightarrow 2_1^+$	----	0.4884
$1_1^+ \rightarrow 2_1^+$	----	0.2588	$1_1^+ \rightarrow 2_1^+$	----	0.1393

Table 5 . The mixing ratios values of ^{100}Pd and ^{100}Ru

$I_i \rightarrow I_f$	^{100}Pd		$I_i \rightarrow I_f$	^{100}Ru	
	Exp.	IBM-2		Exp.[11]	IBM-2
$2_1^+ \rightarrow 2_2^+$	----	5.5831	$2_1^+ \rightarrow 2_2^+$	3.2	14.468
$2_1^+ \rightarrow 2_3^+$	----	0.7701	$2_1^+ \rightarrow 2_3^+$	----	0.1362
$4_1^+ \rightarrow 4_2^+$	----	0.9083	$4_1^+ \rightarrow 4_2^+$	----	6.7203
$3_1^+ \rightarrow 2_1^+$	----	1.0001	$3_1^+ \rightarrow 2_1^+$	----	1.0612
$1_1^+ \rightarrow 2_1^+$	----	1.0790	$1_1^+ \rightarrow 2_1^+$	----	0.0754

4- Conclusion :

We have determined some the energy levels for a nuclei have mass numbers (98,100) within the framework of the Interacting Boson Model. The figures (1-6) suggests a satisfactory agreement between the presented IBM-1 and IBM-2 results, and experimental data for the studied energy levels .some of the energy values that are still not known so far are stated. Also, the validity of the presented parameters in IBM formulations was investigated and it was seen that they are the best approximation which has been carried out so far. It is also concluded that the presented results in this work confirms the adequacy of the approximation in this model. Also , the mixing ratios was evaluated well.

5- References :

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The energy levels which calculated in the present study is in the following figures :

Fig.(1): Comparing between expermental,IBM-1 and IBM-2 energy levels of ^{98}Ru

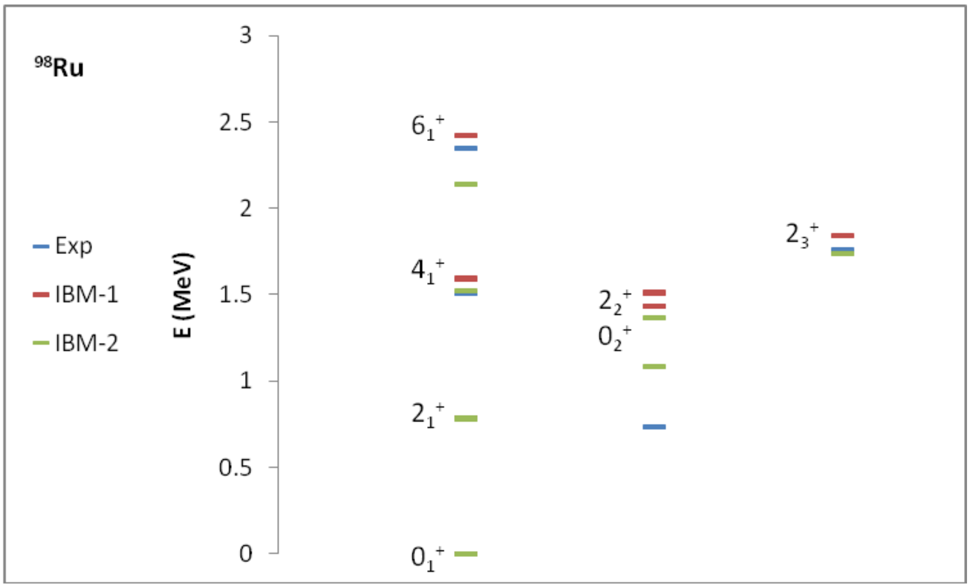

Fig.(2): Comparing between expermental,IBM-1 and IBM-2 energy levels of ^{98}Sr

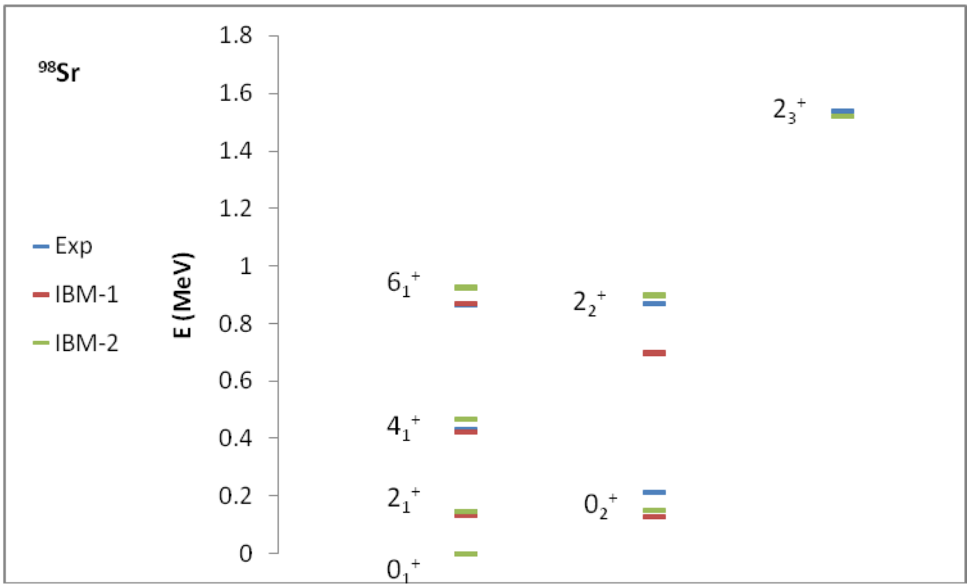

Fig.(3):Comparing between expermental,IBM-1 and IBM-2 energy levels of ^{98}Mo

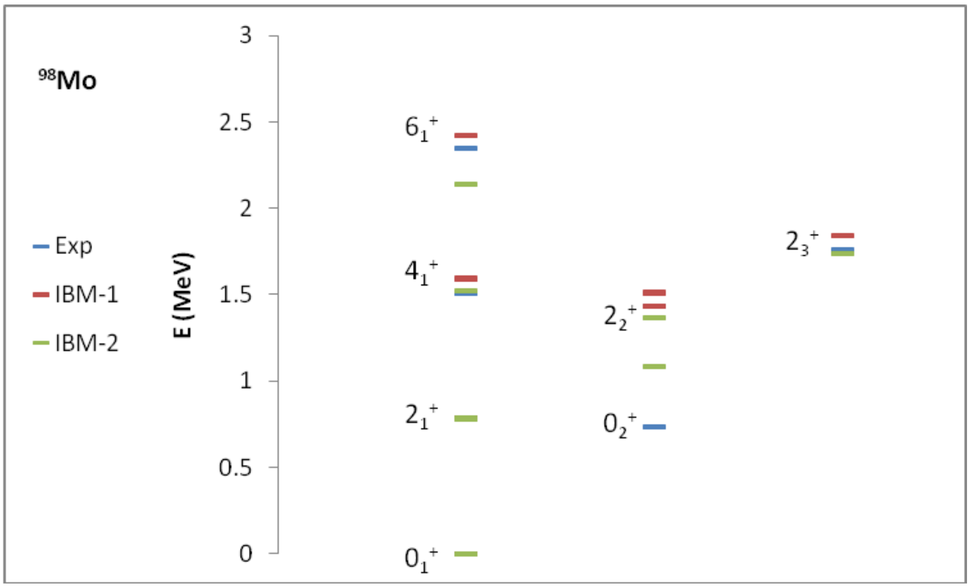

Fig.(4): Comparing between expermental,IBM-1 and IBM-2 energy levels of ^{100}Pd

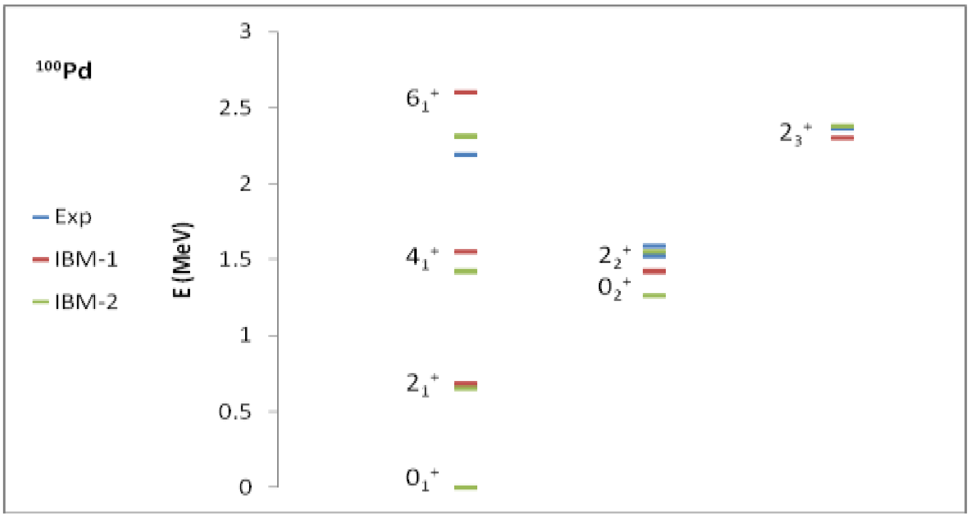

Fig.(5):Comparing between expermental,IBM-1 and IBM-2 energy levels of ^{100}Zr

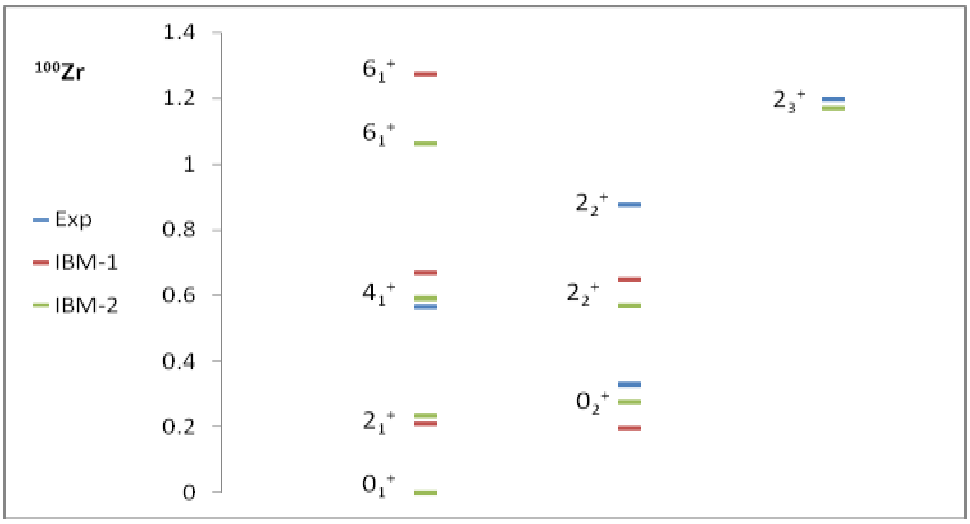

Fig.(6): Comparing between expermental,IBM-1 and IBM-2 energy levels of ^{100}Ru