Degenerate Four Wave Mixing in Photonic Crystal Fibers

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Abstract

In this study, Four Wave Mixing (FWM) characteristics in photonic crystal fibers are investigated. The effect of channel spacing, phase mismatching, and fiber length on FWM efficiency have been studied. The variation of idler frequency which obtained by this technique with pumping and signal wavelengths has been discussed. The effect of fiber dispersion has been taken into account; we obtain that the influence of FWM can be reduced by irregular channel spacing. We use three wavelengths in the calculations (580, 780, and 1040nm) which are the zero dispersion wavelengths, all the results reported in this work are simulated using MATLAB 7.

الخلاصـــــة:

في هذا البحث, تم دراسة دمج أربع موجات في الأليـاف الضـوئية البلوريـة. حيث تم دراسة تـأثير كل من التردد الفاصل بين الموجات, عدم تطابق طور الموجات و طول الليف المستخدم علـى كفاءة دمج الموجات. و كذلك تم دراسة التغير في تردد الموجة الناتجة من هذه التقنية مع كل من الطول الموجي لمصدر الضخ و للموجة المنتشرة مع الأخذ بنظر الاعتبار تأثير التشتت،تم استخدام ثلاثة اطوال موجية في الحسابات و هي (580,780, 1040nm)، كل النتائج المثبتة في هذا البحث تمت باستخدام برنامج محاكاة الماتلاب (٧).

1. Introduction

Photonic crystal fibers (PCFs) are fibers in which the light is guided by a periodic array of air holes in a glass matrix surrounding the core. While in early work the core was characterized by the absence of an air hole, perhaps the most intriguing type of fiber is that for which the core consists of an air hole that is larger than the others. In some of the modes of these fibers the energy is mostly in the air core, which immediately suggests a number of applications, including low-loss propagation, for example at wavelengths where the glass absorbs [1].

PCFs have two different guiding mechanisms. The first mechanism uses a defect mode in a two-dimensional photonic band gap; the second is analogous to conventional guiding, and relies on a form of total internal reflection. The former utilizes structure which stops propagation in any transverse direction, is typically narrowband, but, in principle, allows light to propagate in the air core. The latter achieves a total internal reflection condition because the effective index of the cladding is lower that the dielectric core. This type of PCF, which we consider in this paper, does not need the strict periodicity of air holes or the high air filling ratio required for the existence of a photonic band gap [2].

Index-guiding PCFs, also called holey fibers or microstructured optical fibers, possess the specially attractive property of great controllability in chromatic dispersion by varying the hole diameter and hole-to-hole spacing. Control of chromatic dispersion in PCFs is a very important problem for practical applications to optical communication systems, dispersion compensation, and nonlinear optics. So far, various PCFs with remarkable dispersion properties as, for example, zero dispersion wavelengths shifted to the visible and near-infrared wavelengths, an ultra-flattened chromatic dispersion, and a large positive dispersion with a negative slope in the 1.55 μm wavelength range, have been reported. However, in conventional PCFs, the chromatic dispersion is controlled by using air-holes with same diameter in a cladding region. Using a conventional design technique, it is difficult to control the dispersion slope in wide wavelength range [3]. Photonic bandgap guiding occurs by surrounding the core of an optical fiber with the photonic crystal structure. Wavelengths that fall within the photonic crystal's bandgap cannot propagate out and are thus confined to the core. As a result, the core can even have a lower index of refraction than the cladding [4].

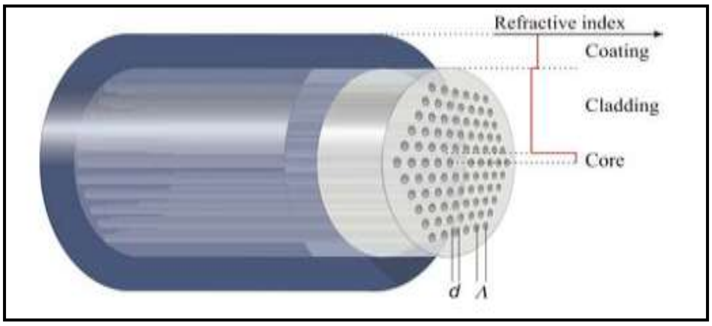


Fig.1: Photonic crystal fiber [2]

2. Four Wave Mixing

The concept of three electromagnetic fields interacting to produce a fourth field is central to the description of all four-wave mixing processes.

The traditional method of modeling an optical material's nonlinear response is to expand the induced polarization as a power series in the electric field strength

$$\vec{P} = \chi^{(1)} \cdot \vec{E} + \chi^{(2)} \cdot .\vec{E}\vec{E} + \chi^{(3)} \cdot \vec{E}\vec{E}\vec{E} + ... \qquad (1)$$

The expansion coefficients are known as susceptibilities in analogy to classical linear electromagnetic theory. The third order nonlinear susceptibility $\chi^{(3)}$ is responsible for four-wave mixing processes.

When a high-power optical signal is launched into a fiber, the linearity of the optical response is lost. One such nonlinear effect, which is due to the third-order electric susceptibility, is called the optical Kerr effect. FWM is a type of optical Kerr effect, and occurs when light of two or more different wavelengths is launched into a fiber. Generally speaking FWM occurs when light of three different wavelengths is launched into a fiber, giving rise to a new wave (know as an idler), the wavelength of which does not coincide with any of the others. FWM is a kind of optical parametric oscillation [5].

Several experimental results on observation of four-wave mixing in photonic crystal fibers (PCFs) have been published over the last several years [2]. This new kind of fibers represents an ideal system for investigating the optical nonlinearities in fused-silica, because of their unique dispersive and nonlinear properties. In particular, the enhanced nonlinearity due to the smallness of the effective core area can increase dramatically every nonlinear effect.

The phase matching conditions for these fibers are found to be quantitatively different with respect to ordinary fibers: phase matching in PCFs can be achieved for a long range of pump wavelengths, because the strong waveguide contribution to the overall dispersion permits a compensation of the material dispersion for a broad window of frequencies. On the other hand, the improved nonlinearity can generate a nonlinear coefficient which can further improve the compensation in the phase [3].

3. Theory

Figure (2) is a schematic diagram that shows four-wave mixing in the frequency domain. As can be seen, the light that was there from before launching, sandwiching the two pumping waves in the frequency domain, is called the probe light (or signal light). The idler frequency f_{idler} may then be determined by

$$f_{idler} = f_{p1} + f_{p2} - f_{probe} \qquad (2)$$

Where: f_{p1} and f_{p2} are the pumping light frequencies, and f_{probe} is the frequency of the probe light.This condition is called the frequency phase-matching condition. When the frequencies of the two pumping waves are identical, the more specific term "degenerated four-wave mixing" (DFWM) is used, and the equation for this case may be written

$$f_{idler} = 2f_p - f_{probe} \tag{3}$$

where: f_p is the frequency of the degenerated pumping wave.

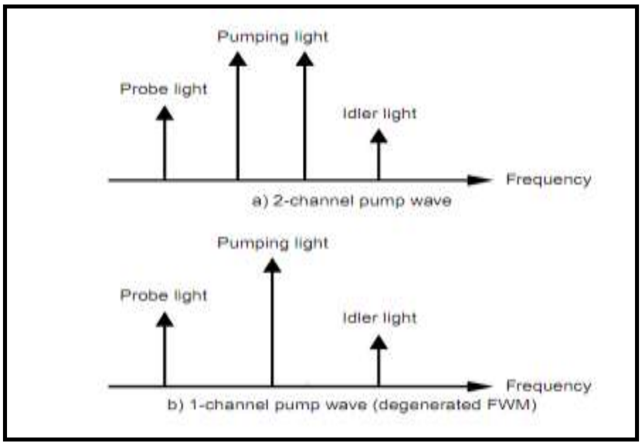


Fig. (2) Schematic of four-wave mixing in Frequency domain

γ is the nonlinear coefficient, and is obtained by

$$\gamma \equiv \frac{2\pi f_P}{c} \cdot \frac{n_2}{A_{eff}} \tag{4}$$

Where: n_2 is the nonlinear refractive index, A_{eff} is the effective area of the fiber and c is the speed of light in a vacuum.

The term $\Delta\beta$ in (eq.3) represents the phase mismatch of the propagation constant, and may be defined as

$$\Delta\beta = \beta_{probe} + \beta_{idler} - 2\beta_{pupm} = -\frac{8\pi f_p^2}{c} D(f_p)(f_{probe} - f_p) \tag{5}$$

where: D is the chromatic dispersion coefficient.

To generate FWM efficiently, it is required that pump wavelength coincides with the fiber zero-dispersion wavelength.

4. Four Wave Mixing Efficiency

To see the origin of FWM, we studied the case of a multichannel light wave system and write the total optical field A(z,t) in the Nonlinear Schrödinger (NLS) equation as

$$A(z,t) = \sum_{m=1}^{M} A_m(z,t)\exp(-i\Omega_m t) \tag{6}$$

where $\Omega_m = \omega_m - \omega_0$, ω_m, is the carrier frequency of the m^{th} channel, and ω_0 is the reference carrier frequency that was used in deriving the NLS equation. Now substituting eq.(5) in the following equation and collect all terms oscillating at a specific frequency.

$$\frac{\partial A}{\partial z} + \frac{i\beta_2}{2}\frac{\partial^2 A}{\partial t^2} = i\gamma|A|^2 A - \frac{\alpha}{2}A$$

The resulting equation for the rn[th] channel takes the form

$$\frac{\partial A_m}{\partial z}+\Omega_j\beta_2\frac{\partial A_m}{\partial t}+\frac{i\beta_2}{2}\frac{\partial^2 A_m}{\partial t^2}=\frac{i}{2}\beta_2\Omega_m^2 A_m-\frac{\alpha}{2}A_m+i\gamma\left(\left|A_m\right|^2+2\sum_{j\neq m}^{\cdots}\left|A_j\right|^2\right)A_m+i\gamma\sum_i\sum_j\sum_k A_i A_j A_k^* \quad (7)$$

In the last term that takes into account FWM among various channels, the triple sum is restricted to only those frequency combinations that satisfy the FWM condition $\omega_m = \omega_i + \omega_j - \omega_k$. Fiber losses have been added to this equation for completeness.

An exact analysis of the FWM process in optical fibers requires a numerical approach. However, considerable physical insight can be gained by considering a single FWM term in the triple sum in eq. (7) and focusing on the quasi-CW case so that time-derivative terms can be set to zero. If we neglect the phase shifts induced by SPM and XPM, assume that the three channels participating in the FWM process remain nearly undepleted, and eliminate the remaining β_2 term through the transformation [7]

$$A_m = B_m \exp(i\beta_2\Omega_m^2 z/2-\alpha z/2)$$

the amplitude B_m, of the FWM component is governed by

$$\frac{dB_m}{dz}=i\gamma B_i B_j B_k^* \exp(-\alpha z - i\Delta k z) \quad (8)$$

where the linear phase mismatch depends on the dispersion parameter as

$$\Delta k=\beta_2(\Omega_m^2+\Omega_k^2-\Omega_i^2-\Omega_j^2) \quad (9)$$

Equation (8) can be easily integrated to obtain B_m(z). The power transferred to the FWM component in a fiber of length L is given by

$$\left|A_m(L)\right|^2=\eta_{FWM}(\gamma L)^2 P_i P_j P_k e^{-\alpha L} \quad (10)$$

where $P_j = |A_j(0)\,^2|$ is the power launched initially into the *jth* channel and η_{FWM} is the FWM efficiency defined as

$$\eta_{FWM}=\left|\frac{1-\exp\left[-(\alpha+i\Delta k)L\right]}{(\alpha+i\Delta k)L}\right|^2 \quad (11)$$

The FWM efficiency η_{FWM} depends on the channel spacing through the phase mismatch Δk given in eq. (9). Using the FWM condition $\Omega_m = \Omega_i + \Omega_j - \Omega_k$, this mismatch can also be written as

$$\Delta k=\beta_2(\Omega_i-\Omega_k)(\Omega_j-\Omega_k)\equiv\beta_2(\omega_i-\omega_k)(\omega_j-\omega_k) \quad (12)$$

In the case of degenerate FWM for which both pump photons come from the same channel ($\Omega_i = \Omega_j$), the phase mismatch is given by $\boldsymbol{\Delta k= \beta_2 (2\pi\Delta \upsilon_{ch})^2}$, where $\Delta\upsilon_{ch}$ is the channel spacing. Figure 3 shows how η_{FWM} varies with $\Delta\upsilon_{ch}$ for several values of dispersion parameter D, related to β_2 as $D = (-2\pi c/\lambda_0^2)\,\beta_2$, using α= 0.2 dB/km and λ_0 = 1.55 μm for a 25km long fiber. The FWM efficiency is relatively large for low dispersion fibers even when channel spacing exceeds 100 GHz. In contrast, it nearly vanishes even for $\Delta\upsilon_{ch}$ = 50 GHz when D > 2 ps/ (km-nm).

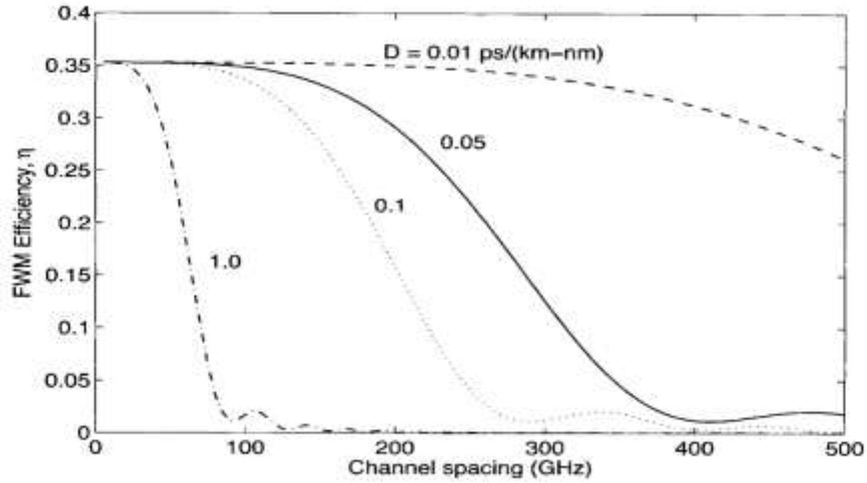


Fig. (3): FWM efficiency plotted as a function of channel spacing for 25km-long fibers with different dispersion characteristics. Fiber loss is assumed to be 0.2 dB/km in all cases [7].

Four-wave mixing is relevant in a variety of different situations. Some examples are:

- It can be involved in strong spectral broadening in fiber amplifiers e.g. for nanosecond pulses. For some applications, this effect is made very strong and then called supercontinuum generation.
- The parametric amplification by four-wave mixing can be utilized in fiber-based optical parametric amplifiers (OPAs) and oscillators (OPOs). Such fiber-based devices have a pump frequency between that of signal and idler.
- Four-wave mixing can have important deleterious effects in optical fiber communications, particularly in the context of wavelength division multiplexing, where it can cause cross-talk between different wavelength channels, and/or an imbalance of channel powers. One way to suppress this is avoiding equidistant channel spacing.
- Four-wave mixing is applied for spectroscopy, most commonly in the form of *coherent anti-Stokes Raman spectroscopy* (CARS), where two input waves generate a detected signal with slightly higher optical frequency.
- Four-wave mixing can also be applied for phase conjugation, holographic imaging, and optical image processing.

5. Simulation Results

Simulation results are presented here to characterize degenerate FWM in PCF. In this section we will study the effect of channel spacing, phase mismatch, pump wavelength, pumping power, fiber length, and fiber dispersion on the degenerate FWM efficiency.

5.1 Effect of Channel Spacing on FWM Efficiency

In this subsection the effect of channel spacing on FWM was studied with three different values of fiber dispersion using eq.(11), as shown in Fig. (4).
From this figure we note that the efficiency of degenerate FWM is inversely proportional to channel spacing, and increasing fiber dispersion will decrease this efficiency. So we can say that FWM efficiency is relatively large for low dispersion wavelength, even when channel spacing exceeds (100 GHz). In this figure the value of fiber losses is (0.1 dB/km), fiber length is (25 km), and results obtained are relatively in a good approximation with that in ref. (7)

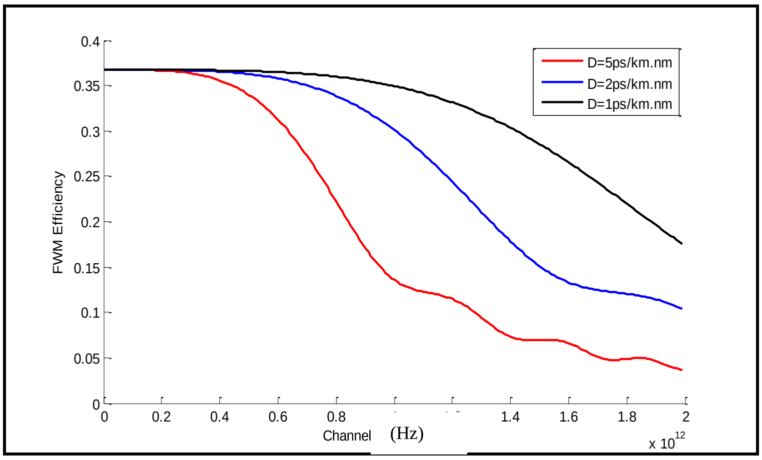
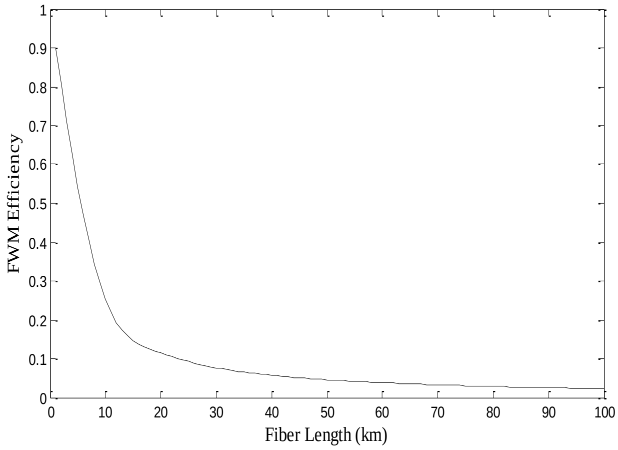


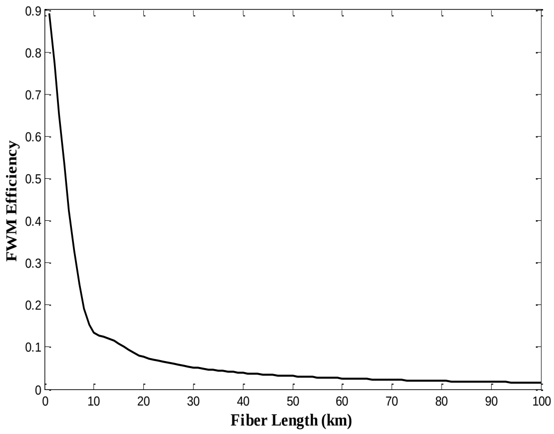
Fig. (4): Variation of FWM efficiency with channel spacing

5.2 Effect of Fiber Length on FWM Efficiency

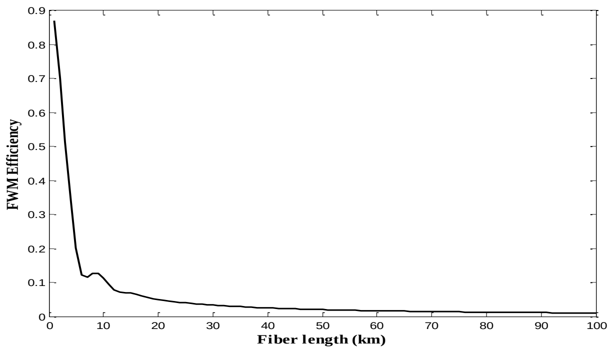
The effect of fiber length on FWM efficiency was studied here, for three different values of phase mismatch, as shown in Fig.(5). From this figure we can note that the efficiency decreases with increasing the length of fiber, and when the phase mismatch increase the efficiency also decreases since it inversely proportional to both fiber length and mismatch values as it quite clear from eq.(11).



(a)



(b)

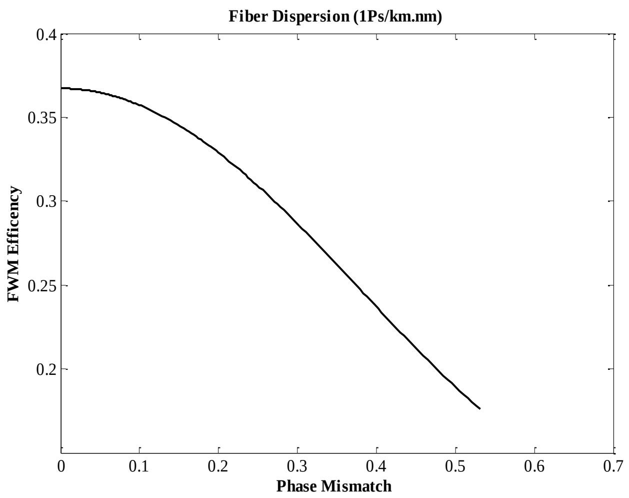


(c)

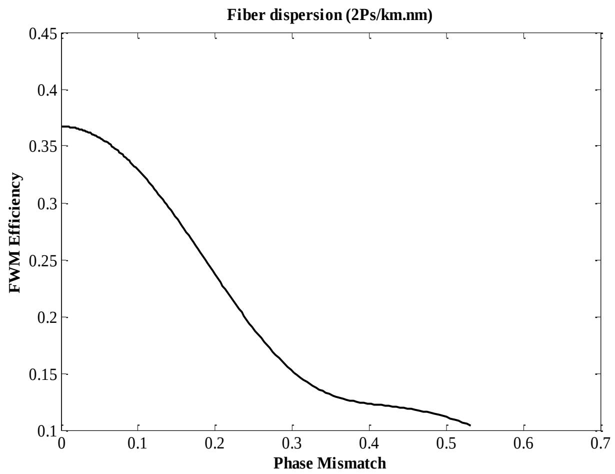
Fig. (5): Variation of FWM efficiency with fiber length. (a) Δk= -0.3875, (b)) Δk= -0.6131, (c) Δk= -0.9774

5.3 Effect of Phase-Mismatch on FWM Efficiency

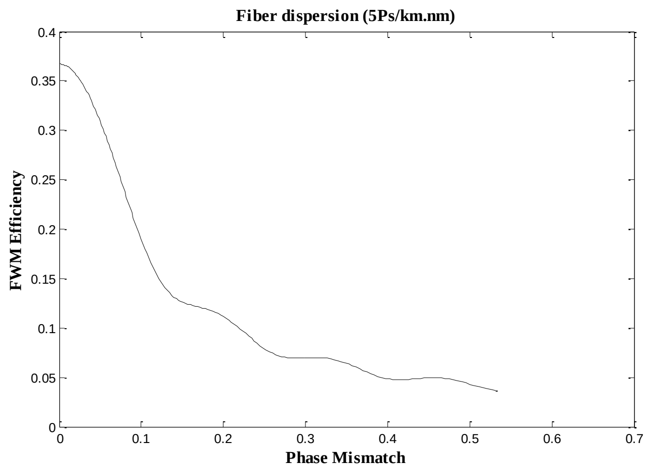
Figure (6) shows the simulation results presented to assess the effect of phase mismatch on FWM efficiency by using eq.(11). The results are displayed for D=1,2, and 5 ps/(nm.km), respectively, and assuming L=25 km. The main result drawn from this figure is that fiber dispersion introduces pulse broadening which is an increasing function of fiber length and fiber dispersion, and this will increase the phase mismatch between waves so, the efficiency will decrease.



(a)



(b)



(c)

Fig. (6): Variation of FWM efficiency with phase mismatch.

5.4 Effect of Channel Spacing on Phase-Mismatch

In this subsection, the effect of channel spacing on phase mismatch characteristics is discussed for three different values of pumping wavelength and dispersion of fiber. The calculations are carried out for fiber dispersion of D=1,2, and 5 ps/(nm.km), assuming the zero dispersion wavelengths that the pumping wavelength should coincides with it are λ = 580nm, 780nm, and 1040nm respectively. From Fig.(7) one can reveal that increasing channel spacing will increase the value of mismatch coefficient. While increasing pumping wavelengths will decrease the phase mismatch. Hence one should balance between these values to obtain the best vale of phase mismatch to enhance FWM efficiency.

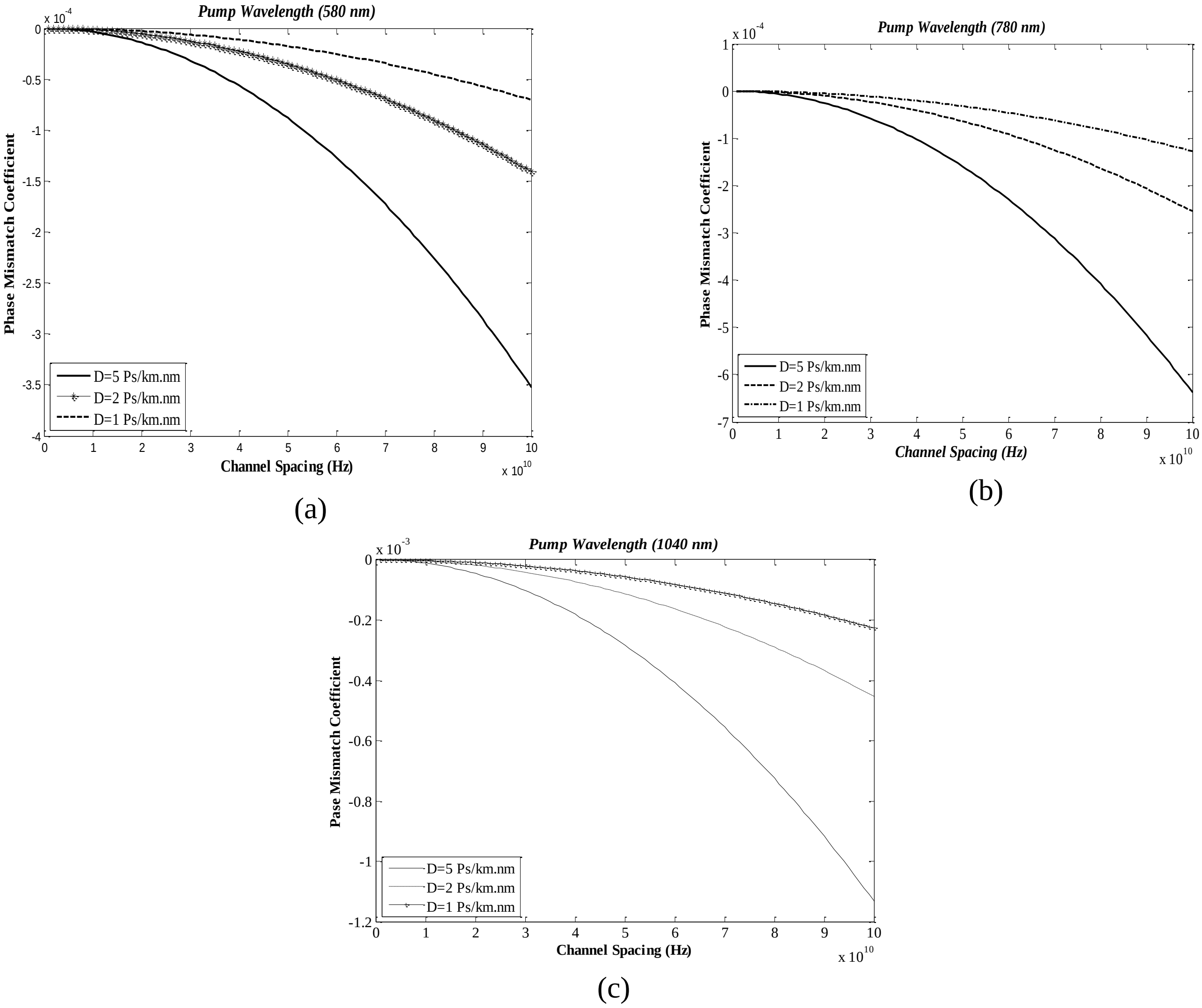


Fig. (7): Variation of phase mismatch with channel spacing.

5.5 Effect **of Pumping Wavelength on Idler Frequency**

In this subsection we will study the variation of idler frequency which obtained by the technique of four wave mixing with pumping wavelengths for two different signal wavelengths as shown in Fig.(8). From this figure it is clear that the idler frequency decreases with increasing the pump wavelength. When signal wavelength increases this frequency will increase, the results are obtained using eq.(3). This is because the pumping frequency should be near the zero dispersion wavelengths.

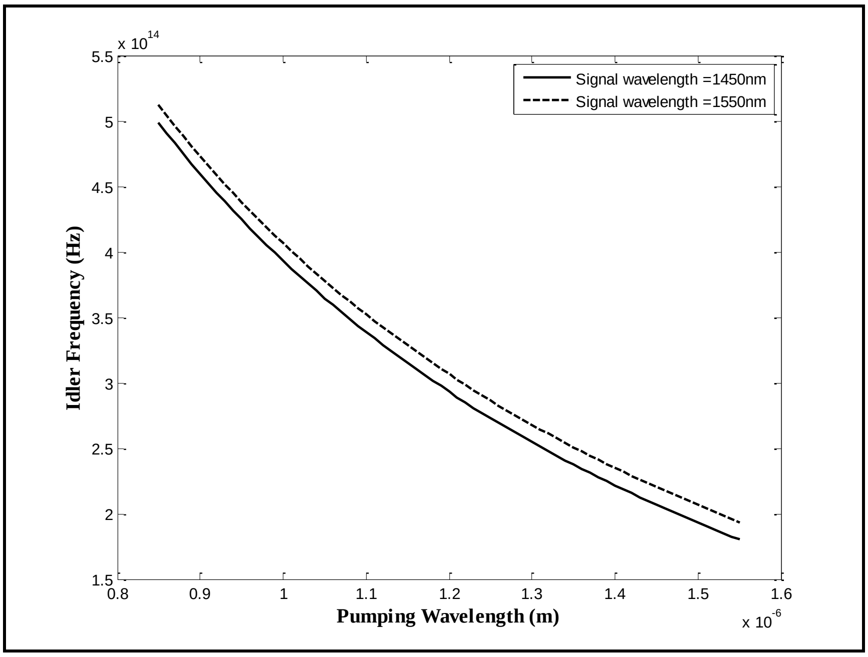


Fig. (7): Variation of idler frequency with pumping wavelength.

6. Conclusions

From this study we obtain that FWM is dependent on signal power, the effective fiber area, phase mismatching, channel spacing, fiber length and fiber type. FWM is therefore an issue of system design and type of fiber used. Different wavelengths with the same propagation speed – or group velocity – traveling at a constant phase over a long period of time, increase the effect of FWM. The effects of FWM are greatest near the zero dispersion point of the fiber, a certain amount of chromatic dispersion leads to different group velocities resulting in a reduction of FWM. The influence of FWM can be reduced by irregular channel spacing.

7. References

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