Application of the Operational Matrices for Solving Nonlinear Volterra Integral Equations System of the Second Kind

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Abstract:

In this paper, we use the Operational Matrices generated from Haar Wavelets to solve Nonlinear Volterra Integral Equations System of the Second Kind. We found that high accuracy of the results in this method in the solution of nonlinear integral equations is realized even in case of the small capacity matrices, but the accuracy of the solutions increases when the capacity of the matrices being used gets larger.

As for its efficiency it is tested by solving two examples for which the exact solution is known. This allows us to estimate the More Accuracy of the obtained numerical results.

Keywords: Operational Matrices, Nonlinear Volterra Integral Equations, Haar Wavelets.

تطبيقات لمصفوفة العوامل لحل نظام من معادلات Volterra التكاملية غير الخطية من النوع الثاني

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ملخص البحث:

في هذا البحث تمكنا من استخدام مصفوفات العوامل المتولدة من موجات Haar القصيرة في حل نظام من معادلات Volterra التكاملية غير الخطية من النوع الثاني. فوجدنا أن درجة عاليــــة من الدقة في نتائج هذه الطريقة في حل المعادلات غير الخطية التكاملية حتى في حالة مــصفوفات ذات سعة صغيرة، ولكن دقة الحلول تزداد عند زيادة سعة المصفوفات المستخدمة.

إن كفاءة الطريقة العددية اختبرت بواسطة حل مثالين ومقارنتها مع الحلـــول المــضبوطة وهذا سمح لنا بتقدير دقة اكبر للحلول العددية التي حصلنا عليها باستخدام هذه الطريقة.

1-Introduction:

Many problems from physics and other disciplines lead to linear or nonlinear integral equations, these equations have applications in physics, chemistry or biology.

In recent years, many different methods have been used to approximate the solution of linear or nonlinear Volterra integral equations system. Tricomi [7], in his book introduced the classical method of successive approximations for nonlinear Volterra integral equations. Brunner [1] applied a collocation-type method to nonlinear Volterra equations and integro-differential equations and discussed its connection with the iterated collocation method. Maleknjad, Sohrab, Rostam [5] are applied Chebyshev polynomials for solving of nonlinear Volterra integral equations of the second kind. Fard and Tahmasbi, [2] are used a numerical method based upon power series to solve nonlinear Volterra integral equations system of the second kind, this method gives an approximate solution as the Taylor expansion.

Wu and Chen (2003) [8] are studied the numerical solution for partial differential equations of first order via operational matrices , they are using the Haar wavelets in the solution with constant initial and boundary conditions.

Wu and Chen (2004) [9] are studied the numerical solution for fractional calculus and the fractional differential equations by using the operational matrices of orthogonal functions. The fractional derivatives of the four typical functions and two classical fractional differential equations solved by the new method and they are compared the results with the exact solutions, they are found the solutions by this method is simple and computer oriented.

Lepik and Tamme (2007) [4] are derived the solution of nonlinear Fredholm integral equations via the Haar wavelet method, they are find that the main benefits of the Haar wavelet method are sparse representation, fast transformation, and possibility of implementation of fast algorithms especially if matrix representation is used.

Lepik Uio (2009) [3] is studied application of the Haar wavelet transform to solving integral and differential equations, he was to demonstrate that the Haar wavelet method is a powerful tool for solving different types of integral equations and partial differential equations. The method with far less degrees of freedom and with smaller CPU time provides better solutions classical ones.

In this paper, we will study the numerical solution for nonlinear Volterra integral equations system of the second kind by the operational matrices of Haar wavelets method and we will compare the results of this method with the exact solution.

In this search, we consider the second kind Volterra integral equations system of the from:

$$\begin{cases} y_1(x) = g_1(x) + \sum_{j=1}^{n} \int_0^x \lambda_{1j}\, k_{1j}(x,t)\,(y_j(t))^{P_{1j}}\, dt, \\[2mm] y_2(x) = g_2(x) + \sum_{j=1}^{n} \int_0^x \lambda_{2j}\, k_{2j}(x,t)\,(y_j(t))^{P_{2j}}\, dt, \\[2mm] \qquad\qquad\vdots \\[2mm] y_n(x) = g_n(x) + \sum_{j=1}^{n} \int_0^x \lambda_{nj}\, k_{nj}(x,t)\,(y_j(t))^{P_{nj}}\, dt, \end{cases} \qquad \dots(1)$$

where, is a real constant, and is a nonnegative integer. Moreover, in equation (1) the function and the kernel are given and assumed to be sufficiently differentiable with respect to all their arguments on the interval . Also, is the solution to be determined.

Haar wavelets have become an increasingly popular tool in the computational sciences. They have had numerous applications in a wide range of areas such as signal analysis, data compression and many others[8].

Using the operational matrix of an orthogonal function to perform integration for solving, identifying and optimizing a linear dynamic system has several advantages: (1) the method is computer oriented, thus solving higher order differential equation becomes a matter of dimension increasing; (2) the solution is a multi-resolution type and (3) the answer is convergent, even the size of increment is very large [8,9].

2- The operational matrices and Haar wavelets:

The main characteristic of the operational method is to convert a differential equation into an algebraic one, and the core is the operational matrix for integration. The integral property of the basic orthonormal matrix, $\phi(t)$. We write the following approximation:

$$\int_0^t \int_0^t \int_0^t \dots \int_0^t \phi(t)(dt)^k \cong Q_\phi^k \phi(t) \qquad \dots(2)$$

where $\phi(t) = [\vec{\phi}_0(t) \quad \vec{\phi}_1(t) \quad \dots \quad \vec{\phi}_{m-1}(t)]^T$ in which the elements $\vec{\phi}_0(t), \vec{\phi}_1(t), \dots, \vec{\phi}_{m-1}(t)$ are the discrete representation of the basis functions which are orthogonal on the interval [0,1) and Q_ϕ is the operational matrix for integration of $\phi(t)$ [8,9].

The operational matrix Q_ϕ of an orthogonal matrix $\phi(t)$ can be expressed by:

$$[Q_\phi] = [\phi]\cdot[Q_B]\cdot[\phi]^{-1} \qquad \dots(3)$$

where $[Q_B]$ is the operational matrix of the block pulse function:

$$Q_{B_m} = \frac{1}{m} \begin{bmatrix} 1/2 & 1 & \dots & \dots & 1 \\ 0 & 1/2 & 1 & \dots & 1 \\ 0 & \dots & 1/2 & \dots & 1 \\ 0 & \dots & 0 & 1/2 & 1 \\ 0 & \dots & \dots & 0 & 1/2 \end{bmatrix}_{m \times m} \qquad \dots(4)$$

If the transform matrix $[\phi]$ is unitary, that is $[\phi]^{-1} = [\phi]^T$, then the equation (3) can be rewritten as [8,9]:

$$[Q_\phi] = [\phi] \cdot [Q_B] \cdot [\phi]^T \qquad \dots(5)$$

The Haar functions are an orthogonal family of switched rectangular waveforms where amplitudes can differ from one function to another. They are defined in the interval [0,1] by [8,9]:

$$h_0(t) = \frac{1}{\sqrt{m}}$$

$$h_i(t) = \frac{1}{\sqrt{m}} \begin{cases} 2^{\frac{J}{2}} & \dfrac{k-1}{2^J} \le t < \dfrac{k-1/2}{2^J} \\[2mm] -2^{\frac{J}{2}} & \dfrac{k-1/2}{2^J} \le t < \dfrac{k}{2^J} \\[2mm] 0 & otherwise \quad in \quad [0,1) \end{cases} \qquad \dots(6)$$

where i=0,1,2,.....,m-1, $m = 2^\alpha$ and α is a positive integer. J and k represent the integer decomposition of the index i , i.e. $i = 2^J + k - 1$.
Theoretically, this set of functions is complete. $h_0(t)$ is called the scaling function and $h_1(t)$ the mother wavelet, such that from the mother wavelet $h_1(t)$, compression and translation are performed to obtain $h_2(t)$ and $h_3(t)$.

Any function $u(x,t)$ which is square integrable in the interval $0 \le t < 1$ and $0 \le x < 1$ can be expanded into Haar series by:

$$u(x,t) = \sum_{i=0}^{m-1} \sum_{J=0}^{m-1} c_{iJ} h_i(x) h_J(t) \qquad \dots(7)$$

where $c_{iJ} = \int_0^1 u(x,t) h_i(x) dx \cdot \int_0^1 u(x,t) h_J(t) dt$.

The equation (7) can be written into the matrices form by:

$$u(x,t) = H^T(x) \cdot C \cdot H(t) \qquad \dots(8)$$

where

$$[C] = \begin{bmatrix} C_{0,0} & C_{0,1} & \dots & \dots & C_{0,m-1} \\ C_{1,0} & C_{1,1} & \dots & \dots & C_{1,m-1} \\ \vdots & \vdots & \dots & \dots & \vdots \\ \vdots & \vdots & \dots & \dots & \vdots \\ C_{m-1,0} & C_{m-1,1} & \dots & \dots & C_{m-1,m-1} \end{bmatrix}$$

is the coefficient matrix of $u(x,t)$ calculated by:

$$[C] = [H] \cdot [u] \cdot [H]^T \qquad \dots(9)$$

For deriving the operational matrix of Haar wavelets, we let $[\phi] = [H]$ in the equation (5), and obtain:

$$[Q_H] = [H] \cdot [Q_B] \cdot [H]^T \qquad \ldots(10)$$

where $[Q_H]$ is the operational matrix for integration of $[H]$.

For example, the operational matrix of the Haar wavelet in the case of m=4 is given by:

$$[Q_H] = [H]_{4*4} \cdot [Q_B] \cdot [H]^T_{4*4}$$

$$= \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix} \cdot \frac{1}{4} \begin{bmatrix} \frac{1}{2} & 1 & 1 & 1 \\ 0 & \frac{1}{2} & 1 & 1 \\ 0 & 0 & \frac{1}{2} & 1 \\ 0 & 0 & 0 & \frac{1}{2} \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}^T$$

$$= \begin{bmatrix} 0.5 & -0.25 & -0.0884 & -0.0884 \\ 0.25 & 0 & -0.0884 & 0.0884 \\ 0.0884 & 0.0884 & 0 & 0 \\ 0.0884 & -0.0884 & 0 & 0 \end{bmatrix}$$

Now, by using the equation (2) and the integration of equation (8) with respect to variable t yields [8]:

$$\int_0^t u(x,t)\,dt = \int_0^t H^T(x) \cdot Cu \cdot H(t)\,dt = H^T \cdot Cu \cdot \int_0^t H(t)\,dt \qquad \ldots(11)$$

$$= [H]^T \cdot [Cu] \cdot [Q_H] \cdot [H]$$

Further integration with respect to variable x gives:

$$\int_0^x u(x,t)\,dx = \int_0^x H^T(x) \cdot Cu \cdot H(t)\,dx = \int_0^x H^T(x)\,dx \cdot Cu \cdot [H] \qquad \ldots(12)$$

$$= [H]^T \cdot [Q_H]^T \cdot [Cu] \cdot [H]$$

3- Numerical solutions:

We will use the operational matrices of the Haar wavelets to solve nonlinear system (1).

Let

$$\begin{cases} u_{1j}(x,t) = k_{1j}(x,t) \cdot (y_j(t))^{P_{1j}} \\ u_{2j}(x,t) = k_{2j}(x,t) \cdot (y_j(t))^{P_{2j}} \\ \quad\vdots \\ u_{nj}(x,t) = k_{nj}(x,t) \cdot (y_j(t))^{P_{nj}} \end{cases} \qquad \ldots(13)$$

such that $j = 1,2,\ldots,n$.

by the equation (8), we can write the equation (13) as Haar matrix, that is:

$$\begin{cases} u_{1j}(x,t) = H^T(x) \cdot C_{1j} \cdot H(t) \\ u_{2j}(x,t) = H^T(x) \cdot C_{2j} \cdot H(t) \\ \vdots \\ u_{nj}(x,t) = H^T(x) \cdot C_{nj} \cdot H(t) \end{cases} \qquad \ldots(14)$$

such that $j = 1,2,\ldots,n$, $C_{1j}, C_{2j}, \ldots, C_{nj}$ are the coefficient matrices of $u_{1j}, u_{2j}, \ldots, u_{nj}$ respectively and calculated by using the equation (9), we get:

$$\begin{cases} C_{1j} = H(x) \cdot U_{1j}(x,t) \cdot H^T(t) \\ C_{2j} = H(x) \cdot U_{2j}(x,t) \cdot H^T(t) \\ \vdots \\ C_{nj} = H(x) \cdot U_{nj}(x,t) \cdot H^T(t) \end{cases} \qquad \ldots(15)$$

where

$$U_{1j} = \begin{bmatrix} k_{1j}(x_0,t_0)(y_j(t_0))^{P_{1j}} & k_{1j}(x_0,t_1)(y_j(t_1))^{P_{1j}} & \cdots & k_{1j}(x_0,t_{m-1})(y_j(t_{m-1}))^{P_{1j}} \\ k_{1j}(x_1,t_0)(y_j(t_0))^{P_{1j}} & k_{1j}(x_1,t_1)(y_j(t_1))^{P_{1j}} & \cdots & k_{1j}(x_1,t_{m-1})(y_j(t_{m-1}))^{P_{1j}} \\ \vdots & \vdots & \cdots & \vdots \\ k_{1j}(x_{m-1},t_0)(y_j(t_0))^{P_{1j}} & k_{1j}(x_{m-1},t_1)(y_j(t_1))^{P_{1j}} & \cdots & k_{1j}(x_{m-1},t_{m-1})(y_j(t_{m-1}))^{P_{1j}} \end{bmatrix}_{m \times m} \vdots$$

$$U_{nj} = \begin{bmatrix} k_{1j}(x_0,t_0)(y_j(t_0))^{P_{nj}} & k_{nj}(x_0,t_1)(y_j(t_1))^{P_{nj}} & \cdots & k_{nj}(x_0,t_{m-1})(y_j(t_{m-1}))^{P_{nj}} \\ k_{nj}(x_1,t_0)(y_j(t_0))^{P_{nj}} & k_{nj}(x_1,t_1)(y_j(t_1))^{P_{nj}} & \cdots & k_{nj}(x_1,t_{m-1})(y_j(t_{m-1}))^{P_{nj}} \\ \vdots & \vdots & \cdots & \vdots \\ k_{nj}(x_{m-1},t_0)(y_j(t_0))^{P_{nj}} & k_{nj}(x_{m-1},t_1)(y_j(t_1))^{P_{nj}} & \cdots & k_{nj}(x_{m-1},t_{m-1})(y_j(t_{m-1}))^{P_{nj}} \end{bmatrix}_{m \times m}$$

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$$t_i = \frac{1}{2m} + \frac{i}{m}$$
$$\qquad\qquad i=0,1,2,3,\ldots \qquad \ldots(16)$$
$$x_i = \frac{1}{2m} + \frac{i}{m}$$

Now, by using the equation (13), the equation (1) becomes:

$$\begin{cases} y_1(x) = g_1(x) + \sum_{j=1}^{n} \int_0^x \lambda_{1j} \, u_{1j}(x,t) \, dt, \\ y_2(x) = g_2(x) + \sum_{j=1}^{n} \int_0^x \lambda_{2j} \, u_{2j}(x,t) \, dt, \\ \vdots \\ y_n(x) = g_n(x) + \sum_{j=1}^{n} \int_0^x \lambda_{nj} \, u_{nj}(x,t) \, dt, \end{cases} \qquad \ldots(17)$$

such that $j = 1,2,\ldots,n$.

We transform the system (17) into the matrices form by using the equation (14), we get:

$$
\begin{cases}
Y_1 = G_1 + \sum_{j=1}^{n} \int_{0}^{x} \lambda_{1j} \, H^T(x) \cdot C_{1j} \cdot H(t) \, dt, \\[2mm]
Y_2 = G_2 + \sum_{j=1}^{n} \int_{0}^{x} \lambda_{2j} \, H^T(x) \cdot C_{2j} \cdot H(t) \, dt, \\[2mm]
\qquad\qquad \vdots \\[2mm]
Y_n = G_n + \sum_{j=1}^{n} \int_{0}^{x} \lambda_{nj} \, H^T(x) \cdot C_{nj} \cdot H(t) \, dt,
\end{cases}
\qquad \dots(18)
$$

such that $H^T(x)$ and $H(t)$ are the Haar matrices and:

$$
Y_1 = \begin{bmatrix}
y_1(t_0) & y_1(t_1) & \cdots & y_1(t_{m-1}) \\
y_1(t_0) & y_1(t_1) & \cdots & y_1(t_{m-1}) \\
\vdots & \vdots & \cdots & \vdots \\
y_1(t_0) & y_1(t_1) & \cdots & y_1(t_{m-1})
\end{bmatrix}_{m\times m}
$$

$\vdots$

$$
Y_n = \begin{bmatrix}
y_n(t_0) & y_n(t_1) & \cdots & y_n(t_{m-1}) \\
y_n(t_0) & y_n(t_1) & \cdots & y_n(t_{m-1}) \\
\vdots & \vdots & \cdots & \vdots \\
y_n(t_0) & y_n(t_1) & \cdots & y_n(t_{m-1})
\end{bmatrix}_{m\times m}
$$

$G_1, G_2, ..., G_n$ are matrices such that the diagonal elements of each matrix $G_i \quad i=1,2,...,n$ are known that is when $x_i = t_i$, but other elements are unknown:

$$
G_1 = \begin{bmatrix}
g_1(x_0,t_0) & g_1(x_0,t_1) & \cdots & g_1(x_0,t_{m-1}) \\
g_1(x_1,t_0) & g_1(x_1,t_1) & \cdots & g_1(x_1,t_{m-1}) \\
\vdots & \vdots & \cdots & \vdots \\
g_1(x_{m-1},t_0) & g_1(x_{m-1},t_1) & \cdots & g_1(x_{m-1},t_{m-1})
\end{bmatrix}_{m\times m}
$$

$\vdots$

$$
G_n = \begin{bmatrix}
g_n(x_0,t_0) & g_n(x_0,t_1) & \cdots & g_n(x_0,t_{m-1}) \\
g_n(x_1,t_0) & g_n(x_1,t_1) & \cdots & g_n(x_1,t_{m-1}) \\
\vdots & \vdots & \cdots & \vdots \\
g_n(x_{m-1},t_0) & g_n(x_{m-1},t_1) & \cdots & g_n(x_{m-1},t_{m-1})
\end{bmatrix}_{m\times m}
$$

The functions $g_1(x), g_2(x), ..., g_n(x)$ are found by integrating the functions $u_{1j}(x,t), u_{2j}(x,t), ..., u_{nj}(x,t)$ from (0 to x) with respect to (t) and adding the functions $y_1(x), y_2(x), ..., y_n(x)$, therefore, the functions $g_1(x), g_2(x), ..., g_n(x)$ are consisting of two variables (t) and (x) $g_1(x,t), g_2(x,t), ..., g_n(x,t)$ but by substituting the integration boundaries they become one variable functions; that is:

$$g_1(x_i, t_i) = g_1(x_i) \qquad \forall x_i = t_i$$
$$\vdots$$
$$g_n(x_i, t_i) = g_n(x_i) \qquad \forall x_i = t_i$$

...(19)

Now, by using the equation (11), the system (18) becomes:

$$
\begin{cases}
Y_1 = G_1 + \sum_{j=1}^{n} \lambda_{1j} \, H^T \cdot C_{1j} \cdot Q \cdot H \\[2mm]
Y_2 = G_2 + \sum_{j=1}^{n} \lambda_{2j} \, H^T \cdot C_{2j} \cdot Q \cdot H \\[2mm]
\qquad\qquad \vdots \\[2mm]
Y_n = G_n + \sum_{j=1}^{n} \lambda_{nj} \, H^T \cdot C_{nj} \cdot Q \cdot H
\end{cases}
$$

...(20)

$$j = 1, 2, ..., n$$

by substitute the equation (15) in the system (20), then:

$$
\begin{cases}
Y_1 = G_1 + \sum_{j=1}^{n} \lambda_{1j} \cdot U_{1j} \cdot H^T \cdot Q \cdot H \\[2mm]
Y_2 = G_2 + \sum_{j=1}^{n} \lambda_{2j} \cdot U_{2j} \, H^T \cdot Q \cdot H \\[2mm]
\qquad\qquad \vdots \\[2mm]
Y_n = G_n + \sum_{j=1}^{n} \lambda_{nj} \cdot U_{nj} \cdot H^T \cdot Q \cdot H
\end{cases}
$$

...(21)

To find the values of the matrices $y_1, y_2, ..., y_n$ which have 2^m of the elements respectively, we solve the system (21) which gives nonlinear system of the equations such that the equations number is equal to the variables number and we can solved them by Newton-Raphson system method .

4- Illustrating Examples:

For illustrating the numerical solution for the above system, consider the following example.

Example 1:

We consider the following nonlinear integral equations:

$$
\begin{cases}
y_1(x) = g_1(x) + \displaystyle\int_0^x k_{11}(x,t)\, y_1^2(t)\, dt + \int_0^x k_{12}(x,t)\, y_2^2(t)\, dt \\[4mm]
y_2(x) = g_2(x) + \displaystyle\int_0^x k_{21}(x,t)\, y_1^2(t)\, dt + \int_0^x k_{22}(x,t)\, y_2^2(t)\, dt
\end{cases}
$$

...(22)

where

$$
\begin{bmatrix} g_1(x) \\ g_2(x) \end{bmatrix} = \begin{bmatrix} x - \dfrac{x^5}{4} - \dfrac{11x^6}{30} \\[3mm] x^2 - \dfrac{x^4}{12} - \dfrac{x^5}{6} \end{bmatrix}
$$

...(23)

and

$$\begin{bmatrix} k_{11}(x,t) & k_{12}(x,t) \\ k_{21}(x,t) & k_{22}(x,t) \end{bmatrix} = \begin{bmatrix} xt & x+t \\ x-t & \dfrac{t}{x} \end{bmatrix} \qquad \ldots(24)$$

The exact solution for equation (22) are:
$$\begin{bmatrix} y_1(x) \\ y_2(x) \end{bmatrix} = \begin{bmatrix} x \\ x^2 \end{bmatrix}$$

the system (22) can be solved by using numerical solution described in this paper and we get:

X	$y_1(x)$	Exact=x	Error	X	$y_2(x)$	Exact=x^2	Error
1/8=0.125	0.125029138072377	0.125	2.9138e-005	1/8	0.015630106212422	0.015625	5.1062e-006
3/8=0.375	0.376704539897345	0.375	0.0017	3/8	0.141231766539293	0.140625	6.0677e-004
5/8=0.625	0.637207738582420	0.625	0.0122	5/8	0.395383102043036	0.390625	0.0048
7/8=0.875	0.932483521157945	0.875	0.0575	7/8	0.784288165537893	0.765625	0.0187

Table 1: Comparison between the real solution and the numerical solution and to found $y_1(x)$, $y_2(x)$ the amount of error for Example(1) when m=4.

when the dimension of the matrices increase then the numerical solution converges towards the exact solution as shown in the table (2) such that the dimensions of the matrices are m=8.

X	$y_1(x)$	Exact=x	Error	X	$y_2(x)$	Exact=x^2	Error
1/16	0.062500812605673	0.0625	8.1261e-7	1/16	0.003905772929912	0.00390625	4.77070088e-7
3/16	0.187538573673104	0.1875	3.8574e-5	3/16	0.035153537434621	0.03515625	2.712565379e-6
5/16	0.312728525731551	0.3125	2.2853e-4	5/16	0.097726399249249	0.09765625	7.0149249249e-5
7/16	0.438267582560184	0.4375	7.6758e-4	7/16	0.191708324679084	0.19140625	3.02074679084e-4
9/16	0.564476892276982	0.5625	0.0020	9/16	0.317204874929301	0.31640625	7.98624929301e-4
11/16	0.691925427919539	0.6875	0.0044	11/16	0.474373022641958	0.47265625	1.71677264195e-3
13/16	0.821751345767716	0.8125	0.0093	13/16	0.663481937615570	0.66015625	3.32568761557e-3
15/16	0.956518518319132	0.9375	0.0190	15/16	0.885044973971101	0.87890625	6.13872397110e-3

Table 2: The numerical solution to found $y_1(x)$, $y_2(x)$ and Comparison with the exact solution and the amount of error for Example(1) when m=8.

Example 2:

Solve a system of nonlinear integral equations:

$$\begin{cases} y_1(x) = g_1(x) + \int_0^x k_{11}(x,t)\, y_1^2(t)\, dt - \int_0^x k_{12}(x,t)\, y_2^2(t)\, dt \\ y_2(x) = g_2(x) - \int_0^x k_{21}(x,t)\, y_1^2(t)\, dt + \int_0^x k_{22}(x,t)\, y_2^2(t)\, dt \end{cases} \quad \dots(25)$$

where

$$\begin{bmatrix} g_1(x) \\ g_2(x) \end{bmatrix} = \begin{bmatrix} \sec(x) - x \\ 3\tan(x) - x \end{bmatrix} \quad \dots(26)$$

and

$$\begin{bmatrix} k_{11}(x,t) & k_{12}(x,t) \\ k_{21}(x,t) & k_{22}(x,t) \end{bmatrix} = \begin{bmatrix} xt & x+t \\ x-t & \dfrac{t}{x} \end{bmatrix} \quad \dots(27)$$

The exact solution for equation (26) are:

$$\begin{bmatrix} y_1(x) \\ y_2(x) \end{bmatrix} = \begin{bmatrix} \sec(x) \\ \tan(x) \end{bmatrix}$$

the system (25) can be solved by using numerical solution described in this paper and we get:

X	$y_1(x)$	Exact= sec(x)	Error	X	$y_2(x)$	Exact= tan(x)	Error
1/8	1.007971079548726	1.007863687880318	1.0739166840e-4	1/8	0.123071377092145	0.125655136575131	2.583759482986e-3
3/8	1.076148143268895	1.074682223392077	1.46591987681e-3	3/8	0.384819092485131	0.393626575925633	8.807483440502e-3
5/8	1.242362033442251	1.233101698397970	9.26033504428e-3	5/8	0.700797540748446	0.721484440990904	2.068690024245e-2
7/8	1.615750611392769	1.560070049119026	5.56805622737e-2	7/8	1.136500967619520	1.197421629234348	6.092066161482e-2

Table 3: The numerical solution to found $y_1(x)$, $y_2(x)$ and Comparison with the exact solution and the amount of error for Example(2) when m=4.

X	$y_1(x)$	Exact= sec(x)	Error	X	$y_2(x)$	Exact= tan(x)	Error
1/16	1.001959202221292	1.001956308972237	2.893249055e-6	1/16	0.062257135583389	0.062581507566275	3.24371982886e-4
3/16	1.017873025426493	1.017839351629227	3.3673797266e-5	3/16	0.188730758236102	0.189728610718059	9.97852481957e-4
5/16	1.051049586224071	1.050897103093027	1.52483131044e-4	5/16	0.321326127396855	0.323086244351746	1.760116954891e-3
7/16	1.104446257276087	1.103979789991509	4.66467284578e-4	7/16	0.465008108863209	0.467730025452392	2.721916589183e-3
9/16	1.183325126267815	1.182138596185575	1.18653008224e-3	9/16	0.626343630602862	0.630437673835885	4.094043233023e-3
11/16	1.296715970919937	1.293937347137912	2.778623782025e-3	11/16	0.814831774602729	0.821141801589894	6.310026987165e-3
13/16	1.460551998217057	1.454152966031519	6.399032185538e-3	13/16	1.045374140380485	1.055727639411920	1.0353499031435e-2
15/16	1.705103947959705	1.689745563340660	1.535838461904e-2	15/16	1.343292552101465	1.362071976376228	1.8779424274763e-2

Table 4: The numerical solution to found $y_1(x)$, $y_2(x)$ and Comparison with the exact solution and the amount of error for Example(2) when m=8.

Figure 1: Comparison between the exact solution and the numerical solution of y1(x) when m=4 and m=8 for Example 1.
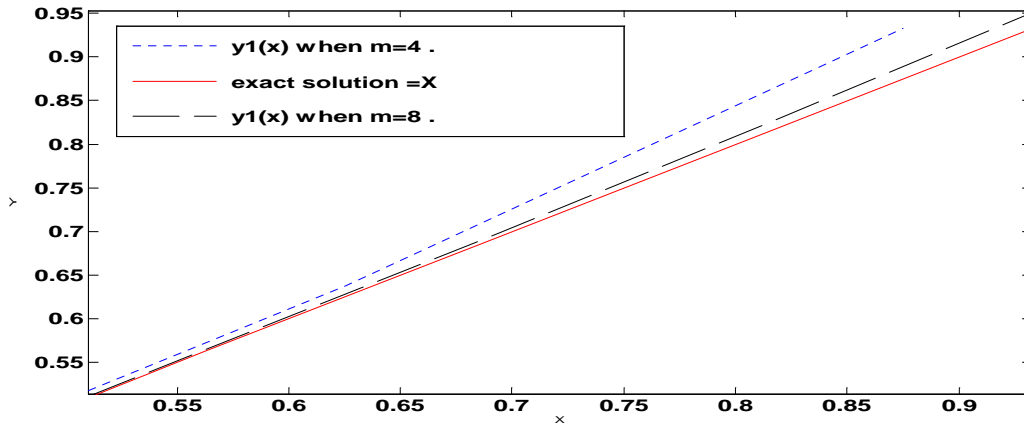
Figure 2: Comparison between the exact solution and the numerical solution of y2(x) when m=4 and m=8 for Example 1.
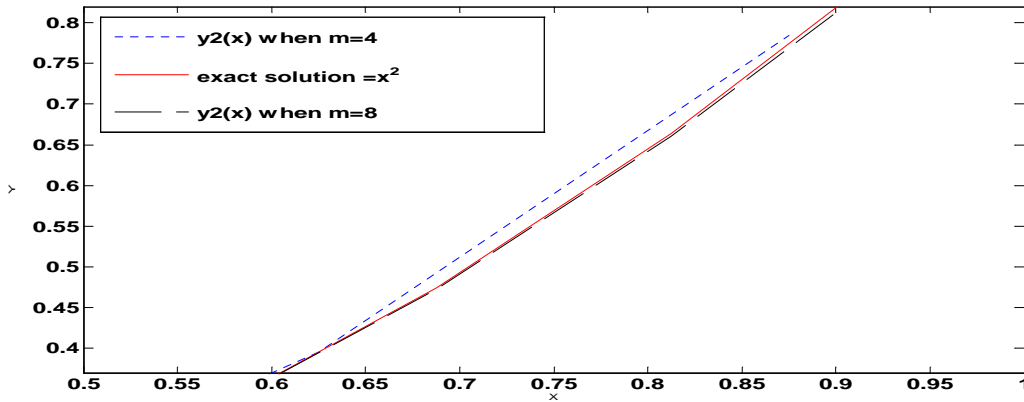
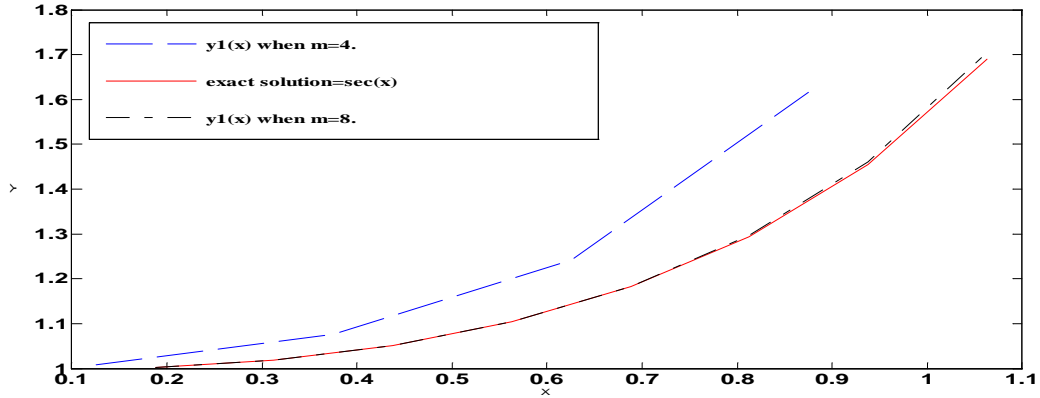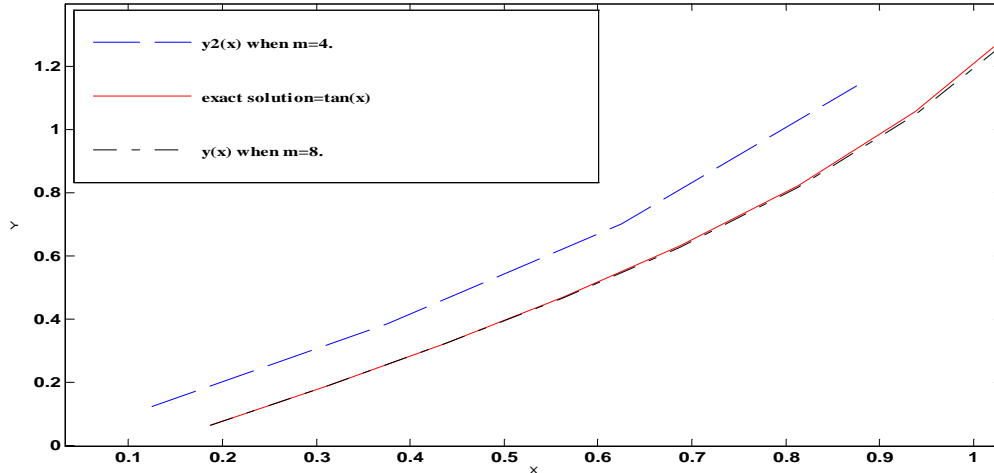Figure 3: Comparison between the exact solution and the numerical solution of y1(x) when m=4 and m=8 Example 2.

Figure 4: Comparison between the exact solution and the numerical solution of y2(x) when m=4 and m=8 Example 2.

5-Conclusions:

In this paper, We are using the operational matrices of the Haar wavelets method for solving non linear volterra integral equations system of the second kind. And compare the results with the exact solutions, by solving two examples when m=4 and m=8. Note that the high accuracy of numerical solution increases is directly proportional to increase in dimensions of operational matrices and note that converges towards the exact solution as shown in the tables (1),(2),(3)&(4)and figures(1),(2),(3) &(4). Then for better results, using the greater m is recommended. The present method reduces an numerical integral equations system into a set of algebraic equations which its solve by using Newton-Raphson method.

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