Solving Infinite Nonlinear Boundary Value Problems of Three Orders by Using Shooting Method

Ahmed Antsar Ghatheth
Department of Mathematics
College of Computer Science and Mathematics
Al-Mustansiriya University

الملخص

إن الهدف من هذا البحث هو تصميم خوارزمية لحل مسائل القيم الحدودية غير الخطية من الرتبة الثالثة ذات شروط حدودية غير منتهية حيث تم استخدام طريقة نيوتن رافسن لإيجاد قيمة (∞) ومن ثم حلها باستخدام طريقة القذف وقد تم الحصول على نتائج جيدة ونسبة خطأ صغيرة جداً.

ABSTRACT

The purpose of this research is to design an algorithm for solving nonlinear boundary value problems of third orders with an infinite number of boundary conditions. Newton-Raphson method was used for finding (∞) value then we solved it by using shooting method. good results were obtained with a very small error value.

1. Introduction

Studied the boundary layer flow over a continuous moving surface in another quiescent fluid medium.[2,12] have paid attention to the solution's existence and uniqueness, however, the approximate analytical solution remains unanswered. The goal to get it can be achieved by implementing the double-parameter transformation perturbation method which has been used in obtaining the approximate solutions of a wide class of differential equations.[13, 14, 15, 16] Engineers, mathematicians and other scientists systems who use mathematical models in order to describe and analyze physical often need to answer the question How

does the system response change as the system parameters change?" For example, how does the airow around an airplane wing change as the shape of the wing changes, and how does this eject drag Sensitivity analysis seeks to answer such questions. Techniques used for this analysis vary from those which focus on the original mathematical model to those which consider such questions only after the model has been discretized and a numerical method has been introduced. Continuous Sensitivity Equation Methods (CSEMs) examine the mathematical model-which usually consists of one or more partial deferential equations and appropriate boundary conditions-at the in_nite dimensional level. These methods seek to derive a (PDE) sensitivity equation, with boundary conditions, which describes how the state variables change with respect to small changes in the design variables. Up to this point, CSEMs have used formal techniques in order to derive the sensitivity equation. in this method the solution is considered as the summation of an infinite series which usually converges rapidly to the exact solution this simple method has been applied to solve linear and non-linear equations of heat transfer. Essentially, the PDE state equation and the boundary conditions are implicitly diaerentiated in order to derive the sensitivity PDE, For a specie class of state equations, one can construct an operator framework and use the Implicit Function Theorem in order to rigorously derive the sensitivity equation. [4, 6, 7, 8, 9]. The effects of blowing or suction on the free convection from a horizontal flat plate have been studied by [10].

2. Sensitivity Equations

The intent of this paper is to document the line of sight sensitivity equations for a Cassegrain telescope. These equations express the image motions at the focal plane in terms of the motions of primary, secondary, and focal plane. A numerical example is given for the Gemini F/16 IR configuration. The results are checked against those obtained from Code V Computer software for the same optical configuration.[5] Sensitivity analysis is used for a wide range of engineering problems, including characterization of complex ows, fast evaluation of nearby ows. of methods for fast evaluation of nearby ows provides a rigorous framework to answer some di_cult questions. For instance: what is the eject on the ow of changing the angle of attack, the thickness or the camber of an airfoil? First order Taylor series in parameter space yield quick and inexpensive estimates of the ow over the whole domain for nearby values of the shape parameter. Taylor series are very cost ejective because the sensitivities can be solved for a fraction of the cost of the ow computation Sensitivity analysis is also a scientifically rigorous tool for assessing the ejects of input data uncertainty such as manufacturing tolerances on the performance of an airfoil. In essence, sensitivity information allow us to

cascade input data uncertainty through the CFD code to obtain uncertainty estimates of 2 the ow response. Here, uncertainty analysis will be performed using first order sensitivity information combined to rot order statistics[11].

3. Shooting methods

Based on dividing the integration interval to n subintervals at the beginning of each subinterval values estimated for the given dependent variables then the ODE$_S$ of the problem integrated in the subinterval.[2] Methods for numerically solving stochastic initial-value problems have been under much study [1] and the references therein. However, the theory and numerical solution of stochastic boundary- value problems have received less attention [17]. Generally, these stochastic boundary-value problems cannot be solved exactly and numerical methods must be used to obtain an approximate solution.

4. Algorithm

In this part of the research we develop a new algorithm to Solve infinite nonlinear boundary value problem of third orders by using shooting method.

Let us consider boundary value problem

$$\frac{d^3 y}{dx^3} = f(x, y, y', y'')$$

With the boundary conditions
$$y(a) = A \quad , \quad y'(b) = B \quad , \quad y'(\infty) = C$$

Step 1: we change the boundary condition at infinity when
$$y'(L) = C$$

Step 2: we must change the boundary value problem into an initial value problem. Define the variables
$$y_1 = y \, , \, y_2 = y' \, , \, y_3 = y''$$
The equations are then
$$\frac{dy_1}{dx} = y_2 \quad , \quad \frac{dy_2}{dx} = y_3 \quad , \quad \frac{dy_3}{dx} = f(x, y, y', y'')$$
$$y_1(a) = A \quad , \quad y_2(b) = B \, , \, y_2(\infty) = C$$
We cannot impose the condition

$$y_2(\infty) = C$$
We must choose a value of
$$y_3(0) = k$$

and adjust k so that the condition is satisfied, the boundary condition are then

$$y_1(a) = A \quad , \quad y_2(b) = B \, , \, y_3(0) = k$$

Step 3: we use the Newton-Raphson method to adjust the value of **k**, and this requires we define some sensitivity equations. define

$$y_4 = \frac{\partial y_1}{\partial k} \; , \; y_5 = \frac{\partial y_2}{\partial k} \; , \; y_6 = \frac{\partial y_3}{\partial k}$$

Step 4 :

$$\frac{dy_4}{dx} = y_5 \; , \; \frac{dy_5}{dx} = y_6 \; ,$$

$$\frac{dy_6}{dx} = \frac{d}{dx}(y_6) = \frac{d}{dx}(\frac{\partial y_3}{\partial k}) = \frac{\partial}{\partial k}(\frac{dy_3}{dx}) = \frac{\partial}{\partial k} f(x, y, y', y'')$$

The conditions are

$$y_4(a) = A \; , \; y_5(b) = B \; , \; y_6(0) = C$$

Step 5:

$$y_2(L,k) = C \;\; , \;\; \phi(k) = y_2(L,k) - C = 0$$

and

$$\frac{d\phi}{dk} = y_5(L)$$

Step 6 :

$$k^{r+1} - k^r = -\frac{\phi(k^r)}{\phi'(k^r)}$$

5. Numerical applications

In this part we will solve two of nonlinear boundary value problems of third orders with an infinite number of boundary conditions using the algorithm that was mentioned in the third part of this paper and give the a mount of error between the exact solution and numerical solution with the drawing by using MATLAB program.
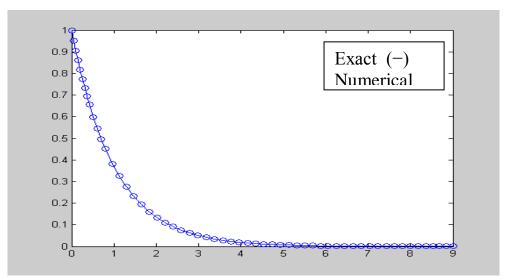
Problem(1):

$$y''' + yy'' + yy' + y = 0$$

with boundary conditions

$$y(0) = 1 \; , \; y'(0) = -1 \; , \; y'(\infty) = 0$$

Solution :
Step 1 :

$$y'(L) = 0$$

Step 2 :

$$y_1 = y \; , \; y_2 = y' \; , \; y_3 = y'' \quad \text{and the equations are then}$$

$$\frac{dy_1}{dx} = y_2 \; , \; \frac{dy_2}{dx} = y_3 \; , \; \frac{dy_3}{dx} = -y_1 y_3 - y_1 y_2 - y_1$$

$$y_1(0) = 1 \; , \; y_2(0) = -1 \; , \; y_2(\infty) = 0$$

change the boundary conditions at infinity when

$$y_1(0) = 1 \ , \quad y_2(0) = -1 \ , \quad y_3(0) = k$$

Step 3 :

$$y_4 = \frac{\partial y_1}{\partial k} \ , \quad y_5 = \frac{\partial y_2}{\partial k} \ , \quad y_6 = \frac{\partial y_3}{\partial k}$$

Step 4 :

$$\frac{dy_4}{dx} = y_5 \ , \quad \frac{dy_5}{dx} = y_6 \ ,$$

$$\frac{dy_6}{dx} = \frac{d}{dx}(y_6) = \frac{d}{dx}(\frac{\partial y_3}{\partial k}) = \frac{\partial}{\partial k}(\frac{dy_3}{dx}) = \frac{\partial}{\partial k}(-y_1 y_3 - y_1 y_2 - y_1)$$

$$= -y_1 y_6 - y_3 y_4 - y_1 y_5 - y_2 y_4 - y_4$$

are and the conditions

$$y_4(0) = 1 \ , \ y_5(0) = -1 \ , \ y_6(0) = 0$$

Step 5 :

$$y_2(L,k) = 0 \ , \quad \phi(k) = y_2(L,k) - 0 = 0$$

and

$$\frac{d\phi}{dk} = y_5(L)$$

Step 6 :

$$k^{r+1} - k^r = -\frac{\phi(k^r)}{\phi'(k^r)}$$

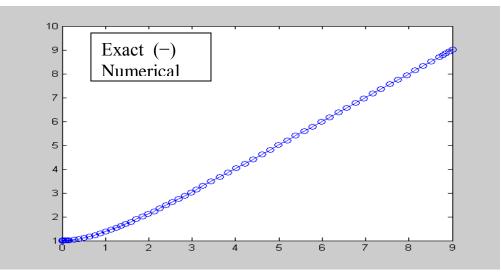We obtained on maximm error between numerical solution and exact solution of problem (1) (4.8697e-005)

Fig(1): comparison between numerical solution and exact solution of problem (1)

Problem(2) :

$$y'''+yy''+y''y'-xy''=0$$

with boundary conditions

$$y(0)=1 , \ y'(0)=0 , \ y'(\infty)=1$$

Solution :
Step 1:

$$y'(L)=0$$

Step 2:

$$y_1 = y , \ y_2 = y' , \ y_3 = y'' \quad \text{and the equations are then}$$

$$\frac{dy_1}{dx}=y_2 \ , \quad \frac{dy_2}{dx}=y_3 \ , \quad \frac{dy_3}{dx}=-y_1 y_3 - y_3 y_2 + xy_3$$

$$y_1(0)=1 , \quad y_2(0)=0 , \ y_2(\infty)=1$$

change the boundary conditions at infinity when

$$y_1(0)=1 , \quad y_2(0)=0 , \ y_3(0)=k$$

Step 3 :

$$y_4 = \frac{\partial y_1}{\partial k} , \ y_5 = \frac{\partial y_2}{\partial k} , \ y_6 = \frac{\partial y_3}{\partial k}$$

Step 4 :

$$\frac{dy_4}{dx}=y_5 \ , \quad \frac{dy_5}{dx}=y_6 \ ,$$

$$\frac{dy_6}{dx}=\frac{d}{dx}(y_6)=\frac{d}{dx}(\frac{\partial y_3}{\partial k})=\frac{\partial}{\partial k}(\frac{dy_3}{dx})=\frac{\partial}{\partial k}(-y_1 y_3 - y_3 y_2 + xy_3)$$

$$=-y_1 y_6 - y_3 y_4 - y_3 y_5 - y_2 y_6 + xy_6$$

are and the conditions

$$y_4(0)=1 , \ y_5(0)=0 , \ y_6(0)=1$$

Step 5:

$$y_2(L,k)=1 , \quad \phi(k)=y_2(L,k)-1=0$$

and

$$\frac{d\phi}{dk}=y_5(L)$$

Step 6 :

$$k^{r+1}-k^r=-\frac{\phi(k^r)}{\phi'(k^r)}$$

We obtained on maximm error between numerical solution and exact solution of problem (2) (1.4576e-005)

**Fig(2): comparison between numerical solution
and exact solution of problem (2)**

6. Conclusions

In this paper an algorithm is designed to solve of nonlinear boundary value problems of third orders with an infinite number of boundary conditions. where using Newton-Raphson method to find value (∞) and then solve this problem using shooting method, and obtained good results where error ratio is small, finally drawing the exact solution and numerical solution of two problems by using MATLAB program.

References

1) Arciniega, A., and Allen, E. Rounding error in numerical solution of stochastic differential equations. Stoch. Anal. Appl. (2003), 21(2):281–300.

2) A. J. Callegari and A. Nachman, Some singular non-linear differential equations arising in boundary layer theory, J. Math. Anal. APPL., 64 (1978), 96-105.

3) Bashir M. Khalaf, New parallel algorithms for BVP in ODE$_S$, college of education, Mosul university, (2006).

4) Brouwers, H.J.H., The effect of blowing or suction on laminar free convection heat transfer on (1993).

5) Eugene Huang, Line of Sight Sensitivity Equations, Arizona 85719 Phone: (602) 325-9329 Fax: (602) 322-8590, April 20, (1992).

6) J. T. Borggaard and J. A. Burns. A sensitivity equation approach to optimal design of nozzles. In Proc. of the 5th AIAA/Usaf/Nasa/Issmo Symposium on Multidisciplinary Analysis and Design, pages 232{241, AIAA Paper 94-4274 , (1994).

7) J. T. Borggaard and J. A. Burns. A sensitivity equation approach to Shape optimization in uid ows. In Flow Control, pages 49{78, M. Gunzburger, Editor ,(1995).

8) J. T. Borggaard and J. A. Burns. A PDE sensitivity equation method for optimal aerodynamic design. Journal of Computational Physics, 136:366{384, (1997).

9) J. T. Borggaard and D. P. Pelletier. Computing design sensitivities using an adaptive note element method. In 27th AIAA Fluid Dynamics Conference, AIAA Paper 96-1938, June (1996).

10) J. T. Borggaard and D. P. Pelletier. Observations in adaptive re_nement strategies for optimal design. In Computational Methods for Optimal Design and Control, Progress in Systems and Control Theory, pages 59{76. Birkhauser, Boston, (1998).

11) M.M. Putko, P.A. Newman, A.C. Taylor, and L.L. Green. Approach for uncertainty propagation and robust design in cfd using sensitivity derivatives. In 15th AIAA Computational Fluid Dynamics Conference, Anaheim, CA, AIAA Paper 2001-2528 ,June (2001).

12) K. Vajravelu and R. N. Mohapatra, On fluid dynamic drag reduction in some boundary layer flows, Acta Mech., 81 (1990), 59-68.

13) Y. Zhang, L. C. Zheng, X. X. Zhang, An Analytical Approximate Solution for Boundary Layer Flow with the Power Law Velocity, Chinese Journal of Computational Mechanics, 25 (2008), 20-22.

14) Y. Zhang, L. C. Zheng, X. X. Zhang, An analytical approximate solution for boundary layer flow of finite thickness Marangoni convection, Journal of University of Science and Technology Beijing, 31 (6) (2009), 799-803

15) Y. Zhang, L. C. Zheng, X. X. Zhang, The analytical approximate solution for Marangoni convection in a liquid layer with coupled boundary conditions, Acta Physica Sinica, 58 (8) (2009), 5501-5506

16) Y. Zhang, L. C. Zheng, X. X. Zhang, Approximate analytical solutions for boundary layer equations of Marangoni convection driven by a power-law temperature gradient, Proceedings of the third international conference on mechanical engineering and mechanics, (2009), 127-132.

17) Zeitouni, O., and Dembo, A. A change of variables formula for stratonovich integrals and existence of solutions for two-point stochastic boundary value problems. Probability Theory and Related Fields (1990), 84:411–425.