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New values for $m_r(2,29)$ and $t_r(2,29)$ in PG(2,29)

DR. Nada Yassen Kasm Yahya
Department of Mathematics
College of Education
University of Mosul

Shua'a M. Aziz
Department of Mathematics
College of Comp.& Maths. Science
University of Mosul

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الملخص

في هذا البحث تم إيجاد بعض الأقـواس التامـة القريبـة مـن **$m_r(2,29)$** عنـدما $2 \le r \le 28$ وأيضاً قمنا بإيجاد الأقـواس الأقـرب لأصـغر قـوس تـام **$t_r(2,29)$** عنـدما $2 \le r \le 28$ فضلاً عن ذلك فقد تبينت لنا بعض الأسئلة المفتوحة المتعلقة بعدم وجود بعـض الأقواس- **(n,r)** التامة. تم ذلك باستخدام برنامج حاسوبي، من الجدير بالذكر أنه لا توجد قيم منشورة لحد الآن عندما $8 \le r \le 28$ وكذلك لا توجد قيم منـشورة لحـد الآن لـ **$t_r(2,29)$** عندما $3 \le r \le 28$ وذلك للمستوي الاسقاطي **PG(2,29)** (حسب إطلاع الباحثين).

ABSTRACT

In this paper we find the nearest complete(n,r)-arcs of the maximum value for $m_r(2,29)$-arc when $2 \le r \le 28$. Also, we find the nearest minimum size of complete $t_r(2,29)$-arcs when $2 \le r \le 28$,as well as we set some open questions concerning for non-existence of complete (n,r)-arcs. All this done by a computer program, noteworthy that there is no results published till now when $8 \le r \le 28$ for $m_r(2,29)$ and there are no results published till now for $t_r(2,29)$ when $3 \le r \le 28$ in PG(2,29) (according to the knowledge of the authors).

1. Introduction:

Let PG(2,q) be a finite projective plane Π of order q, where q= p^h ,$h \ge 1$ this plane consist of q^2+q+1 lines and the same number of points, q+1 points on every line and q+1 lines passing through every point.

An (n,r)–arc K in the projective plane is a set of k points such that some n, but no n+1 of them are collinear. An (n,r)-arc K is complete if there is no (n+1,r)-arc containing it. A line L of the plane containing precisely i points of K, called an i-secant. Let T_i denote the total number of i-secants to K in PG(n,q).

2. The projective plane PG(2,29):-

Let $f(x)=x^3-4x^2-x-1$ be an irreducible monic polynomial over GF(29) then companion matrix T of f(x) is

$$T=\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ 1 & 1 & 4\end{bmatrix}$$

Which a cyclic projectivity on PG(2,29).

Let p_0 be the point $U_0=(1,0,0)$ then $p_i=p_0T^i$, i=0,…,870, are the 871 points of PG(2,29). (see Table(1.1))

Table(1.1) Points of PG(2,29)

i	P_i		
0	1	0	0
1	0	1	0
2	0	0	1
3	1	1	4
.		.	
869	1	8	10
870	1	4	28

Let L_1 be the line which contains the points:-

0, 1, 139, 146, 148, 180, 218, 236, 280, 307, 362, 367, 392, 478, 502, 569,577, 600, 683, 702, 705, 742, 752, 763, 778, 806, 820, 826, 855 and 859.

then $L_i=L_1T^{i-1}$, i=1,…,871, are the lines of PG(2,29), the 871 lines L_i are given by the rows in (Table(2.1)).

Table(2.1) Lines of PG(2,29)

LINES	Points
Line 1	0 1 139 146 148 180 218 236 280 307 362 367 392 478 502 569 577 600 683 702 705 742 752 763 778 806 820 826 855 859
Line 2	1 2 140 147 149 181 219 237 281 308 363 368 393 479 503 570 578 601 684 703 706 743 753 764 779 807 821 827 856 860
Line 3	2 3 141 148 150 182 220 238 282 309 364 369 394 480 504 571 579 602 685 704 707 744 754 765 780 808 822 828 857 861
Line 4	3 4 142 149 151 183 221 239 283 310 365 370 395 481 505 572 580 603 686 705 708 745 755 766 781 809 823 829 858 862
Line 5	4 5 143 150 152 184 222 240 284 311 366 371 396 482 506 573 581 604 687 706 709 746 756 767 782 810 824 830 859 863
…	…
Line 869	868 869 136 143 145 177 215 233 277 304 359 364 389 475 499 566 574 597 680 699 702 739 749 760 775 803 817 823 852 856
Line 870	869 870 137 144 146 178 216 234 278 305 360 365 390 476 500 567 575 598 681 700 703 740 750 761 776 804 818 824 853 857
Line 871	870 0 138 145 147 179 217 235 279 306 361 366 391 477 501 568 576 599 682 701 704 741 751 762 777 805 819 825 854 858

Definition 2-1:[٤]

Let $m_r(2,q)$ be the maximum value such that an n-arc exists in PG(2,q).

Theorem 2-1-1:-[٣]

$$m_2(2,q) = \begin{Bmatrix} q+2 & \text{for} & q & \text{even} \\ q+1 & \text{for} & q & \text{odd} \end{Bmatrix}$$

3. The maximum value $m_r(2,q)$:

It is clear that a lot of researchers have done some work like [6], so the latest data related with the plane PG(2,29) is updated from theorems and lemmas appeared in [6],[7],[8] and [2], so, from them together we can give the following table (Table(3.1)), for q=29:

Table(3.1) latest updated $m_r(2,29)$

r	2	3	4	5	6	7	8
$m_r(2,29)$	24-30	42-59	70-89	94-119	126-149	154-179	?-209
r	9	10	11	12	13	14	15
$m_r(2,29)$	?-239	?-269	?-299	?-329	?-359	?-389	407-419
r	16	17	18	19	20	21	22
$m_r(2,29)$	436-449	?-479	?-509	?-539	?-569	574-599	?-269
r	23	24	25	26	27	28	29
$m_r(2,29)$	?-659	?-689	696-719	?-749	754-779	784-809	841

In this paper we find new numbers instead of the sign (**?**) which appeared in [8], as well as, we find some complete arcs which their lengths are near to $m_r(2,29)$, see table (3.2):

Table(3.2) Our improvement of table(3.1) (written in bold face)

r	2	3	4	5	6	7	8
$m_r(2,29)$	24-30	42-59	70-89	94-119	126-149	154-179	**164**-209
r	9	10	11	12	13	14	15
$m_r(2,29)$	**191**-239	**219**-269	**247**-299	**275**-329	**303**-359	**334**-389	407-419
r	16	17	18	19	20	21	22
$m_r(2,29)$	436-449	**421**-479	**457**-509	**489**-539	**520**-569	574-599	**570**-629
r	23	24	25	26	27	28	29
$m_r(2,29)$	**602**-659	**631**-689	696-719	**704**-749	754-779	784-809	841

3.1 Existence of complete (n,2)-arcs in PG(2,29):-

By computer search we could find 3 different talls of complete (n,2)-arcs for n=17, 18, 19. which are near to $m_2(2,29)$. Let $\{T_i : i = 0,...,r\}$ denote the total number of i-secant a (n, r)-arc to n in PG(2,29), see Tabled below:-

the Complete (n,r)- arc; n= 17, r= 2
0 1 2 3 4 6 13 59 99 209 253 295 419 462 486 583 642
$\{T_i$:i=0,...,r$\}$= {497 238 136}
the Complete (n,r)- arc , n=18 , r=2
0 2 4 5 6 13 20 75 84 96 132 148 186 202 552 653 675 751
$\{T_i$:i=0,...,r$\}$ = {484 234 153}
the Complete (n,r)- arc , n=19, r=2
0 2 3 4 6 103 132 180 245 256 292 351 485 562 574 657 669 743 797
$\{T_i$:i=0,...,r$\}$]= {472 228 171}

3.2. Existence of complete (n,3)-arcs in PG(2,29):-

By computer search we find 4 different talls of complete(n,3)-arcs for n=37,…,40, which are near to $m_3(2,29)$. Tabled below:-

the Complete (n,r)- arc , n=37, r=3
0 2 3 4 12 25 41 69 84 115 139 145 163 167 189 196 227 243 250 265 266 285 286 365 390 397 407 458 461 469 507 513 553 683 764 839 852
$\{T_i$:i=0,...,r$\}$= {281 216 228 146}
the Complete (n,r)- arc , n=38, r=3
0 1 2 3 4 6 13 59 75 88 99 126 131 151 208 209 253 282 295 308 311 316 329 358 380 419 462 470 486 519 538 548 583 642 679 733 826 840
$\{T_i$:i=0,...,r$\}$ = {277 205 232 157}
the Complete (n,r)- arc , n=39, r=3
0 2 3 4 6 7 63 66 83 113 159 162 173 192 195 239 258 261 266 267 280 281 294 300 359 385 388 397 456 463 582 596 622 637 644 690 739 857 868
$\{T_i$:i=0,...,r$\}$= {274 192 237 168}
the Complete (n,r)- arc , n=40 , r=3
0 2 4 5 6 13 17 20 45 46 75 79 84 96 132 148 182 186 202 211 218 224 273 292 294 306 351 370 431 461 476 552 564 653 675 690 722 751 788 845
$\{T_i$:i=0,...,r$\}$ = {263 204 216 188}

3.3. Existence of complete (n,4)-arcs in PG(2,29):-

By computer search we find 4 different talls of complete (n,4) -arcs for n= 60,…, 63. which are near to $m_4(2,29)$. Tabled below:-

the Complete (n,r)- arc , n=60 , r=4
0 1 2 3 4 8 13 19 27 37 49 51 60 70 85 97 98 99 100 109 120 123 143 146 161 185 234 244 251 256 279 287 289 294 299 302 311 321 338 351 362 412 430 462 503 563 570 571 573 594 616 636 637 655 663 738 749 786 790 845
$\{T_i$:i=0,...,r$\}$ = {153 175 153 241 149}
the Complete (n,r)- arc , n=61, r=4
0 2 3 4 5 23 55 56 57 79 82 89 92 111 124 138 139 144 146 147 172 183 209 234 245 250 294 306 312 325 327 365 379 399 409 410 412 413 414 416 419 440 468 478 511 529 535 544 570 595 609 626 629 650 677 678 726 727 754 800 821
$\{T_i$:i=0,...,r$\}$ = {157 151 171 231 161}

the Complete (n,r)- arc , n=62, r=4
0 2 3 4 9 12 14 25 29 41 69 75 84 93 115 139 145 148 156 159 163 167 189 196 199 201 227 243 250 256 265 266 267 285 286 306 311 325 365 372 375 377 390 397 399 406 407 451 458 461 469 507 513 537 553 664 665 683 764 839 852 857
{ T_i:i=0,...,r} ={ 151 158 157 232 173}
the Complete (n,r)- arc , n=63, r=4
0 1 2 3 4 6 9 13 43 59 75 88 91 99 109 118 126 131 151 190 208 209 210 249 253 265 282 295 308 311 316 329 358 367 380 419 457 458 462 466 470 486 519 537 538 548 583 592 611 642 645 652 664 679 733 751 786 787 825 826 830 840 854
{ T_i:i=0,...,r} ={ 149 153 156 227 18}

3.4. Existence of complete (n,5)-arcs in PG(2,29):-

By computer search we find 5 different talls of complete (n,5) -arcs for n= 84 ,…, 88 . which are near to $m_5(2,29)$. Tabled below:-

the Complete (n,r)- arc is , n=84, , r=5
0 2 3 4 5 12 14 25 40 47 48 57 67 78 81 99 102 108 109 131 147 154 157 171 182 237 242 249 253 275 282 295 301 321 328 329 338 339 345 355 360 370 386 412 417 425 434 446 450 467 469 486 501 503 513 534 548 569 570 575 585 615 624 633 650 653 675 678 680 689 696 700 737 741 751 755 780 791 796 797 801 831 832 838
{T_i :i=0,...,r} = {88 118 126 150 245 144}
the Complete (n,r)- arc is , n=85, , r=5
0 2 3 4 6 7 18 21 22 35 42 45 50 53 62 78 101 108 132 136 145 158 180 192 195 201 204 210 228 232 238 263 267 275 281 289 297 299 303 307 310 322 345 355 390 406 422 433 461 464 466 467 476 484 487 488 503 522 535 550 559 565 568 582 585 586 587 599 610 615 619 623 650 652 661 678 697 700 705 710 714 780 847 853 870
{T_i:i=0,...,r} = {80 124 137 125 248 157}
the Complete (n,r)- arc is , n=86, , r=5
0 1 2 3 4 6 9 13 15 18 43 59 67 75 79 88 91 99 109 118 126 131 151 157 158 175 190 202 208 209 210 216 218 222 249 253 265 282 295 303 308 311 316 329 342 351 358 367 380 386 400 419 447 456 457 458 462 466 470 486 519 522 537 538 548 583 592 611 642 645 652 664 679 713 733 751 786 787 801 808 817 825 826 830 840 854
{T_i:i=0,...,r} = {72 140 115 140 230 174}
the Complete (n,r)- arc is , n=87, , r=5
0 2 3 4 9 12 14 21 25 29 41 69 75 84 93 106 115 119 133 139 145 148 150 156 159 163 167 180 189 191 196 199 201 209 227 233 241 243 250 256 265 266 267 285 286 306 311 325 365 372 375 377 390 395 397 399 406 407 416 430 451 453 458 461 469 476 495 499 501 507 513 537 553 586 596 664 665 683 709 764 776 785 813 839 852 857 863
{T_i:i=0,...,r} = {80 122 116 131 247 175}
the Complete (n,r)- arc is , n=88, , r=5

0 1 2 3 6 7 21 34 38 44 50 53 75 91 96 110 131 133 135 159 163 164 167 168 171 179 195 201 229 255 264 267 276 283 284 291 298 305 307 308 317 342 358 360 361 362 369 374 378 384 400 406 413 417 423 442 452 453 462 470 498 500 502 513 514 516 524 526 537 546 563 582 585 635 638 669 685 691 698 711 747 757 777 786 811 812 823 834
$\{T_i$:i=0,...,r$\}$ = { 84 106 120 142 227 192}

3.5. Existence of complete (n,6)-arcs in PG(2,29):-

By computer search we find 5 different talls of complete (n,6) -arcs for n=109 ,…,114 . which are near to $m_6(2,29)$. Tabled below:-

the Complete(n,r)-arc is, n=109, r= 6
0 1 2 3 7 9 11 18 24 33 48 58 60 84 88 94 100 104 106 109 115 131 133 163 169 182 185 192 218 235 240 247 248 250 254 263 267 272 277 285 287 292 321 338 343 356 371 373 374 376 383 389 391 410 421 425 426 435 446 457 459 465 504 505 506 509 517 540 553 574 593 594 601 602 603 621 625 628 639 656 660 667 689 696 705 709 715 724 734 735 743 758 767 773 781 784 791 797 800 804 807 829 831 833 850 853 856 869 870
$\{T_i$:i=0,...,r$\}$ = {38 96 98 113 144 229 153}
the Complete (n,r)- arc is, n=110, r= 6
0 2 3 4 5 12 15 17 23 31 45 55 56 57 60 74 79 80 82 86 89 92 97 111 121 124 138 139 141 142 144 146 147 150 154 158 172 183 185 197 209 229 230 234 245 250 265 286 294 295 306 312 325 327 330 333 346 358 365 371 376 379 385 399 409 410 412 413 414 416 419 422 423 434 440 451 453 468 475 478 501 503 511 522 524 529 535 544 570 595 609 614 626 629 650 653 677 678 690 726 727 737 746 754 800 815 821 822 826 847
$\{T_i$:i=0,...,r$\}$ = {50 74 98 113 143 239 154}
the Complete (n,r)- arc is , n=111, r= 6
0 1 2 3 4 6 9 13 15 18 43 49 59 67 69 75 79 81 88 91 99 109 116 118 126 131 151 157 158 175 184 190 202 208 209 210 216 218 222 234 249 253 265 282 295 302 303 308 311 316 329 340 342 351 358 367 370 378 380 383 386 388 390 400 419 421 447 456 457 458 462 466 470 486 498 519 522 537 538 543 548 578 583 592 599 604 611 621 630 642 645 652 664 679 702 713 723 733 751 753 776 786 787 801 808 817 825 826 830 840 854
$\{T_i$:i=0,...,r$\}$ = {40 94 87 102 152 228 168}
the Complete (n,r)- arc is , n=112, r= 6
0 2 3 4 9 12 14 21 25 29 39 41 66 69 75 77 84 93 106 115 119 133 139 145 148 150 156 159 163 167 173 180 181 189 191 196 199 201 206 207 209 227 229 233 235 241 243 250 256 265 266 267 285 286 296 306 311 325 345 365 372 375 377 390 395 397 399 402 406 407 409 416 430 451 453 458 461 469 476 495 497 499 501 507 513 516 537 545 553 559 564 577 586 596 626 629 647 664 665 683 709 712 715 764 776 785 813 839 840 852 857 863
$\{T_i$:i=0,...,r$\}$ = {43 83 95 99 139 238 174}
the Complete (n,r)- arc is ,n=113,n=6
0 1 2 3 5 7 11 13 15 17 18 21 54 58 62 77 79 80 83 84 90 107 109 115 150 154 167 169 171 179 185 204 207 211 216 221 222 231 243 259 275 285 291 295 299 334 335 336 345 347 349 353 366 372 376 395 404 413 425 432 440 445 447 451 452 453 472 492 495 509 510 513 514 515 518 520 527 547 556 557 571 575 578 588 593 594 600 605 609 636 646 649 654 667 675 687 692 701 704 720 736 753 758 779 781 790 791 826 845 858 859 860 862
$\{T_i$:i=0,...,r$\}$ ={46 74 92 103 152 209 195}

the Complete (n,r)- arc is,n=114.r=6
0 1 2 3 5 6 8 11 17 19 33 38 41 45 49 58 68 101 102 110 112
116 129 132 136 160 180 182 183 197 202 210 224 231 240 244 251 254 266 295
301 304 314 327 337 338 341 344 352 360 366 368 372 374 380 382 385 394 398
406 427 433 452 467 478 481 483 489 501 505 522 524 537 542 548 550 553 565
568 569 604 606 607 617 622 635 636 637 638 647 661 698 702 712 715 725 729
741 743 756 758 768 776 789 792 801 804 818 825 827 830 835 850 852
$\{T_i$:i=0,...,r$\}$ = {50 74 72 116 136 228 195}

3.6 Existence of complete (n,7)-arcs in PG(2,29):-

By computer search we find 5 different talls of complete (n,7)-arcs for n=135 ,…,140. Which are near to $m_7(2,29)$. Tabled below:-

the Complete (n,r)- arc is , n=135, r= 7
0 2 3 4 5 6 7 10 13 17 20 32 34 35 37 45 46 50 67 70 73 75
76 79 82 84 86 91 95 96 99 102 118 125 126 132 140 146 148 162 165 170
177 182 186 190 194 202 210 211 214 218 219 223 224 231 236 253 257 258 259
269 273 279 287 289 292 293 294 302 306 314 320 325 343 348 350 351 352 356
365 367 370 372 380 399 400 402 411 425 427 431 448 450 456 459 461 465 468
476 489 495 503 516 528 552 564 566 574 583 593 626 636 653 675 690 722 726
730 745 751 765 770 783 788 792 797 798 805 809 818 835 844 845 851
$\{T_i$:i=0,...,r$\}$ = {26 48 75 97 100 138 238 149}
the Complete (n,r)- arc is , n=136, r= 7
0 1 2 3 4 5 7 9 12 20 29 37 42 47 49 56 59 61 63 68 76 77 80
85 91 92 95 115 117 135 136 144 147 148 152 158 159 169 172 173 175 199 202
208 212 215 217 225 230 232 240 241 242 243 244 246 258 259 260 262 265 268
281 283 294 297 301 302 307 309 319 322 330 339 341 347 355 356 360 361 366
380 382 393 397 417 425 434 460 463 469 470 483 491 496 502 507 510 522 529
534 535 536 564 574 580 608 614 628 651 659 684 692 695 704 709 720 724 725
727 731 732 739 751 759 760 767 769 780 784 802 807 813 829 833 868
$\{T_i$:i=0,...,r$\}$ = {24 52 75 85 103 138 237 157}
the Complete (n,r)- arc is , n=137, r= 7
0 2 3 4 9 12 14 18 21 25 27 29 39 41 55 63 66 69 75 77 84 93 106
115 119 128 130 133 139 145 148 150 156 159 163 167 173 180 181 189 191 196
199 201 206 207 209 227 229 233 235 240 241 243 250 254 256 265 266 267 285
286 296 306 311 314 325 345 347 365 372 375 377 390 395 397 399 402 406 407
409 414 416 430 439 451 453 458 460 461 464 469 476 481 495 497 499 501 507
513 516 524 537 545 553 559 561 564 577 586 596 620 626 629 641 647 664 665
683 709 712 715 733 756 764 776 785 802 809 813 822 839 840 852 857 863 870
$\{T_i$:i=0,...,r$\}$ = {23 59 66 83 91 148 241 160}
the Complete (n,r)- arc is , n=138, r= 7
0 1 2 3 4 6 9 11 13 15 18 43 49 59 67 69 75 79 81 88 91 99 109
116 118 126 131 151 157 158 175 184 190 197 202 208 209 210 216 218 222 234
249 253 260 265 281 282 295 297 302 303 308 311 316 329 340 342 351 354 358
367 370 378 380 383 386 387 388 390 400 419 420 421 434 447 456 457 458 462
463 466 470 486 491 498 505 519 522 526 527 530 531 537 538 543 548 574 578
583 584 592 598 599 604 611 621 622 630 642 645 652 664 679 688 700 702 713
723 733 751 753 776 786 787 788 801 804 808 817 825 826 828 830 840 845 854
859
$\{T_i$:i=0,...,r$\}$ = {25 54 60 90 100 129 240 173}

the Complete (n,r)- arc is , n=139, r= 7
0 2 3 4 5 13 20 27 29 37 38 39 44 46 54 58 68 72 77 83 86 88 91 100 104 117 124 130 131 132 143 144 147 155 158 164 178 187 189 190 194 197 213 214 220 227 229 237 240 248 250 254 256 258 263 270 276 282 284 297 300 301 316 323 324 327 328 329 334 337 375 389 393 396 428 431 441 443 449 453 467 468 470 471 472 473 488 491 497 500 511 519 522 526 534 538 545 548 553 558 561 566 570 577 595 597 598 599 615 631 637 649 653 658 661 709 712 717 721 729 733 739 743 752 755 759 765 774 786 795 797 803 807 809 818 838 853 864 865
$\{T_i:i=0,...,r\}$ = {27 45 74 80 86 142 236 181}
the Complete (n,r)- arc is , n=140, r= 7
0 1 2 3 4 7 9 13 14 21 28 32 34 36 39 44 50 53 63 68 70 90 98 99 100 101 102 106 110 112 132 138 143 160 165 166 168 174 186 187 197 202 212 230 233 236 242 251 265 277 288 290 291 292 293 302 321 347 355 363 366 374 385 395 398 400 402 405 409 412 413 432 454 458 460 466 472 475 476 479 483 492 495 502 507 509 517 538 542 545 552 557 564 565 566 572 573 575 576 577 579 582 603 604 608 610 619 621 628 647 653 663 673 677 686 690 702 715 726 728 736 746 770 771 774 780 786 794 801 811 812 816 825 832 837 840 841 842 854 855
$\{T_i:i=0,...,r\}$ = {30 43 63 81 98 136 224 196}

4. The minimum value $t_r(2,q)$:-

Definition:-[٤]

Let $t_r(2,q)$ be the minimum value for n such that an (n,r)-arc is complete in PG(2,q).

Despite the maximum value is studied from a lot of researchers, the minimum value studied from few researchers like (Leo storme[2], J.W.P. Hirschfeld,[5], S.Ball[8] and G.Keri[6]). Known values for $t_r(2,29)$ are known only for t2(2,29) which is equals 13.

In this paper we put new numbers instead of the sign (**?**) which appeared in table(4.1)as well as, we find some complete arcs which their lengths are near to $t_r(2,29)$ as appears here in table(4.2):

Table(4.2) Our improvement of table(4.1) (written in bold face)

r	2	3	4	5	6	7	8
$t_r(2,29)$	?-13	?-**28**	?-**46**	?-**62**	?-**82**	?-**100**	?-**125**
r	9	10	11	12	13	14	15
$t_r(2,29)$	?-**152**	?-**177**	?-**203**	?-**230**	?-**254**	?-**282**	?-**310**
r	16	17	18	19	20	21	22
$t_r(2,29)$	?-**337**	?-**363**	?-**390**	?-**422**	?-**453**	?-**484**	?-**515**
r	23	24	25	26	27	28	29
$t_r(2,29)$	?-**548**	?-**585**	?-**616**	?-**653**	?-**691**	?-**730**	?-**776**

4.1 Existence of complete (n,2)-arcs in PG(2,29):-

By computer search we find 3 different talls of complete (n,2)-arcs for n=14 ,…, 16. which are near to $t_2(2,29)$. Tabled below:-

the Complete (n,r)- arc is , n=14, r= 2
0 1 2 3 7 22 38 360 379 470 484 509 829 863
$\{T_i$:i=0,...,r$\}$ = {542 238 91}
the Complete (n,r)- arc is , n=15, r= 2
0 1 2 3 12 101 114 225 260 326 328 370 434 554 564
$\{T_i$:i=0,...,r$\}$ = {526 240 105}
the Complete (n,r)- arc is , n=16, r= 2
0 1 2 5 22 91 107 112 178 212 240 275 477 561 721 792
$\{T_i$:i=0,...,r$\}$ = {511 240 120}

4.2 Existence of complete (n,3)-arcs in PG(2,29):-

By computer search we could find 5 different talls of complete (n,3) -arcs for n=28 ,…,32 . which are near to $t_3(2,29)$. Tabled below:-

the Complete (n,r)- arc is , n=28, r= 3
0 1 12 18 20 27 34 40 82 113 132 142 144 148 271 317 323 374 389 391 491 564 565 597 615 794 843 870
$\{T_i$:i=0,...,r$\}$= {322 345 117 87}
the Complete (n,r)- arc is , n=29, r= 3:
0 1 6 37 39 65 72 87 107 110 129 139 213 229 240 284 313 343 402 429 458 496 541 548 654 671 772 785 802
$\{T_i$:i=0,...,r$\}$]= { 311 346 118 96}
the Complete (n,r)- arc is , n=30, r= 3
0 1 7 9 15 18 67 107 171 201 215 242 280 322 370 401 432 480 505 581 610 623 625 633 693 751 773 832 858 863
$\{T_i$:i=0,...,r$\}$ = {303 339 126 103}
the Complete (n,r)- arc is , n=31, r= 3
0 1 5 12 23 29 31 51 78 111 143 146 175 177 204 210 225 252 262 356 360 379 402 429 454 493 517 522 554 558 595
$\{T_i$:i=0,...,r$\}$ = {300 318 147 106}
the Complete (n,r)- arc is , n=32, r= 3:
0 1 2 3 4 5 12 41 88 93 98 107 114 181 196 290 356 358 367 386 394 405 495 510 526 551 563 630 654 670 717 813
$\{T_i$:i=0,...,r$\}$ = {293 310 154 114}

4.3 Existence of complete (n,4)-arcs in PG(2,29):-

By computer search we find 4 different talls of complete (n,4)-arcs for n=46 ,…,49. which are near to $t_4(2,29)$. Tabled below:-

the Complete (n,r)- arc is , n=46, r= 4
0 1 4 5 37 47 67 86 93 116 129 161 165 196 212 218 226 233 249 258 264 278 299 341 374 384 386 391 394 400 439 443 459 529 587 588 602 611 654 669 699 705 745 786 807 829
$\{T_i$:i=0,...,r$\}$= {162 310 219 88 92}
the Complete (n,r)- arc is , n=47, r= 4

0 1 3 5 17 24 47 49 56 73 90 91 111 119 121 148 156 172 178 185 194 214 258 280 284 322 349 359 363 394 482 505 507 508 531 544 602 677 690 701 737 791 817 835 848 851 860
$\{T_i$:i=0,...,r$\}$ = {165 287 226 101 92}
the Complete (n,r)- arc is , n=48, r= 4
0 1 2 4 7 12 13 14 27 32 39 42 57 64 157 198 265 278 282 295 309 317 320 374 386 389 399 401 413 427 446 458 469 528 535 542 543 556 583 610 611 647 721 789 801 824 849 860
$\{T_i$:i=0,...,r$\}$ = {155 299 207 113 97}
the Complete (n,r)- arc is , n=49, r= 4
0 1 8 12 15 17 41 57 67 70 75 86 108 130 136 146 156 174 183 191 216 228 297 312 316 335 347 350 370 382 396 405 410 421 442 445 540 588 595 709 711 716 737 754 766 818 820 830 841
$\{T_i$:i=0,...,r$\}$ = {158 274 222 116 101}

4.4 Existence of complete (n,5)-arcs in PG(2,29):-

By computer search we find 10 different talls of complete (n,5)-arcs for n=62 ,…,72 . which are near to $t_5(2,29)$. Tabled below:-

the Complete (n,r)- arc is , n=62, r= 5
0 1 5 15 21 39 43 57 66 78 82 87 89 91 103 175 204 248 262 289 313 371 373 378 389 398 402 445 446 455 460 470 479 495 496 506 520 566 567 583 589 594 596 601 628 636 661 679 686 690 692 719 739 748 775 790 800 834 841 854 864 869
$\{T_i$:i=0,...,r$\}$ = {82 237 249 172 46 85}
the Complete (n,r)- arc is , n=63, r= 5
0 1 2 7 9 13 15 45 56 65 69 95 106 113 122 129 139 146 151 153 158 187 213 239 244 262 315 322 324 372 391 400 411 421 500 522 529 532 535 551 566 571 602 630 636 650 671 684 685 688 708 725 733 755 769 782 794 811 817 826 845 861 867
$\{T_i$:i=0,...,r$\}$ = {83 222 259 164 57 86}
the Complete (n,r)- arc is , n=64, r= 5
0 1 4 9 15 32 39 41 43 46 51 57 58 66 114 121 124 128 133 138 165 169 196 270 277 287 291 301 338 357 362 375 379 383 384 388 425 433 446 458 461 471 507 515 543 565 567 570 587 588 591 594 611 647 671 691 721 727 731 733 752 801 820 857
$\{T_i$:i=0,...,r$\}$={81 220 250 166 68 86}
the Complete (n,r)- arc is , n=65, r= 5
0 1 3 6 20 27 35 39 41 50 55 57 63 74 80 82 107 131 132 162 170 177 180 184 188 244 246 305 351 355 369 398 408 413 430 452 465 467 468 480 484 494 502 503 524 548 555 556 565 613 645 657 664 678 696 697 720 722 725 727 746 776 820 842 864
$\{T_i$:i=0,...,r$\}$ = {78 211 259 167 60 96}
the Complete (n,r)- arc is , n=66, r= 5
0 1 8 27 28 32 35 39 49 54 62 67 75 76 98 103 155 173 184 186 311 312 334 341 345 361 367 370 386 394 401 433 451 456 462 467 468 474 483 497 499 502 523 533 537 576 597 609 614 686 689 697 718 738 742 750 771 783 804 812 817 821 831 832 837 867
$\{T_i$:i=0,...,r$\}$ = {77 207 251 170 69 97}

the Complete (n,r)- arc is , n=67, r= 5
0 1 7 12 13 14 27 42 49 54 56 59 62 64 81 82 83 101 107 109 110 113 119 124 151 154 155 166 173 205 224 231 240 261 300 310 320 322 350 364 374 403 410 427 434 459 504 523 553 557 567 571 593 631 636 666 683 693 706 719 735 740 750 773 784 787 848
$\{T_i$:i=0,...,r$\}$= {64 233 224 171 79 100}
the Complete (n,r)- arc is , n=68, r= 5
0 1 3 4 8 13 17 19 22 34 41 54 59 65 73 77 78 82 83 102 105 112 116 117 122 156 192 204 230 235 243 256 283 297 304 335 336 351 357 358 362 375 392 408 410 440 441 470 473 478 511 531 544 563 593 607 613 641 649 674 707 735 772 804 812 817 827 829
$\{T_i$:i=0,...,r$\}$ = {73 202 239 169 87 101}
the Complete (n,r)- arc is , n=69, r= 5
0 1 4 7 15 19 23 31 38 39 41 47 86 95 106 125 132 136 151 180 223 268 273 275 313 318 342 352 357 371 376 395 396 413 421 428 433 435 437 448 449 471 472 487 505 506 536 599 619 622 639 643 652 654 663 669 698 702 729 746 752 758 788 816 819 821 842 845 858
$\{T_i$:i=0,...,r$\}$ = {79 185 231 183 91 102}
the Complete (n,r)- arc is , n=70, r= 5:
0 1 5 14 26 34 37 41 47 57 58 62 67 78 96 102 107 127 134 143 156 163 198 213 218 224 225 264 272 278 361 365 384 392 393 398 403 406 427 429 433 437 457 467 474 506 525 545 548 574 583 600 604 619 636 653 662 664 672 707 708 710 716 744 765 785 792 810 811 861
$\{T_i$:i=0,...,r$\}$= {69 188 251 154 97 112}
the Complete (n,r)- arc is , n=71, r= 5:
0 1 13 14 20 30 33 49 67 70 104 112 115 130 152 164 165 171 174 176 200 214 247 258 263 282 289 294 307 310 313 318 370 374 404 405 409 410 419 448 469 472 482 484 499 501 543 547 548 579 597 600 624 626 658 667 671 693 697 699 711 729 747 769 775 781 796 804 820 825 834
$\{T_i$:i=0,...,r$\}$ = {66 197 218 179 95 116}
the Complete (n,r)- arc is , n=72, r= 5:
0 1 8 11 12 20 26 27 54 78 86 104 120 134 139 149 153 167 169 171 190 209 224 231 254 286 297 309 373 379 389 394 398 412 445 447 457 489 496 528 531 532 535 540 554 591 599 613 624 631 645 663 672 675 684 691 693 704 705 722 723 748 749 763 813 829 836 839 840 843 845 857
{Ti:i=0,...,r} = {79 158 237 177 103 117}

4.5 Existence of complete (n,6)-arcs in PG(2,29):-

By computer search we find 10 different talls of complete (n,6)-arcs for r=82 ,…,92 . which are near to $t_6(2,29)$. Tabled below:-

the Complete (n,r)- arc is , n=82 , r= 6
$\{T_i$:i=0,...,r$\}$ = {31 151 219 231 104 48 87}
0 1 7 13 14 15 20 23 28 31 37 49 51 53 55 59 67 78 84 85 94 104 119 127 149 152 156 164 180 197 221 226 237 252 253 256 288 299 308 321 342 356 358 363 401 405 407 417 425 427 444 467 470 485 525 531 537 551 563 572 573 579 583 584 590 595 606 620 623 627 660 662 684 708 733 754 761 773 809 814 825 837

New values for mr(2,29) and tr(2,29) in PG(2,29).

the Complete (n,r)- arc is , n=83 , r= 6
0 1 3 4 6 27 29 33 35 39 40 47 48 51 60 62 63 76 82 85 96 102 106 110 117 130 148 159 160 162 169 170 176 185 195 210 226 238 265 292 303 307 350 351 367 409 413 416 445 447 452 456 465 479 485 494 506 518 524 560 566 568 579 589 590 609 610 619 630 632 659 678 679 720 723 727 735 754 762 774 842 850 855
$\{T_i:i=0,...,r\}$ = {31 145 230 197 132 50 86}
the Complete (n,r)- arc is , n=84 , r= 6
0 1 4 12 13 24 25 31 37 41 51 72 103 114 130 151 155 158 162 168 202 204 206 229 231 232 239 240 256 262 267 271 301 326 357 366 370 374 402 420 421 439 449 469 482 488 522 525 529 548 554 574 577 582 587 588 592 603 606 613 617 648 649 668 680 683 697 720 721 748 765 771 782 784 793 800 803 808 816 843 858 859 864 869
$\{T_i:i=0,...,r\}$ = {31 132 234 217 111 51 95}
the Complete (n,r)- arc is , n=85, r= 6
0 1 2 4 19 32 49 65 67 75 82 85 98 104 118 119 127 133 140 144 149 175 180 181 191 193 197 199 206 218 219 221 284 293 296 299 316 333 342 344 348 354 355 356 358 364 365 370 415 441 453 470 474 480 495 496 497 502 521 542 546 564 574 588 598 600 606 607 611 624 626 669 671 693 694 709 711 712 776 783 799 818 862 867 870
$\{T_i:i=0,...,r\}$ = {36 141 190 213 156 44 91}
the Complete (n,r)- arc is , n=86, r= 6
0 1 3 37 47 56 58 62 73 97 105 108 113 124 125 128 133 134 135 140 142 177 180 184 193 198 200 209 234 283 284 301 335 338 354 355 357 359 361 366 367 369 371 397 402 403 408 421 428 429 442 457 459 466 476 487 497 503 551 578 592 600 601 603 606 618 629 643 645 647 649 666 672 707 728 746 751 752 754 764 770 792 808 828 839 847
$\{T_i:i=0,...,r\}$ = {27 143 207 200 143 55 96}
the Complete (n,r)- arc is , n=87 , r= 6
0 1 3 8 11 14 27 28 29 30 31 35 36 37 39 40 46 47 57 58 73 79 93 94 95 96 113 116 124 125 144 149 155 159 162 171 176 177 224 228 249 273 275 277 290 299 326 329 338 340 347 359 384 388 430 444 459 474 479 481 486 501 509 513 530 582 588 594 604 611 617 627 650 658 661 664 674 691 697 715 717 723 739 751 756 777 861
$\{T_i:i=0,...,r\}$ = {33 123 213 209 126 72 95}
the Complete (n,r)- arc is , n=88 , r= 6
0 1 4 15 20 21 23 34 35 65 72 77 83 92 102 104 105 113 114 120 124 129 135 141 142 146 147 154 155 167 181 182 187 191 201 207 213 215 237 266 280 297 309 324 340 343 355 368 394 408 410 411 413 427 429 442 460 474 483 487 509 546 556 563 567 588 590 593 601 602 609 620 625 639 643 690 693 701 721 722 726 731 740 744 766 823 831 846
$\{T_i:i=0,...,r\}$ = {28 137 197 188 157 67 97}
the Complete (n,r)- arc is , n=89 , r= 6
0 1 2 3 12 14 24 30 35 56 60 71 83 86 91 99 101 104 106 116 122 125 133 134 148 166 173 188 193 206 216 233 239 244 254 256 265 266 271 282 284 291 317 330 333 354 357 389 394 457 468 470 474 475 478 485 486 506 527 528 536 545 553 563 600 604 622 637 655 667 687 689 693 707 709 715 725 738 741 751 766 788 790 802 826 860 863 867 868
$\{T_i:i=0,...,r\}$ = {35 120 181 224 148 54 109}

the Complete (n,r)- arc is , n=90 , r= 6
0 1 4 7 12 13 17 35 36 51 56 58 69 77 96 110 118 151 155 158 161 180 187 188 196 197 224 237 240 247 266 283 299 300 302 317 335 352 353 364 366 378 379 398 423 466 469 498 505 513 516 529 532 534 539 541 560 564 572 576 598 608 614 624 637 670 675 683 687 715 719 730 731 732 740 742 749 753 777 778 780 784 789 791 792 813 834 845 850 865
{T_i:i=0,...,r} = {35 110 196 210 139 74 107}
the Complete (n,r)- arc is , n=91 , r= 6
0 1 6 8 11 16 31 43 49 53 77 101 105 110 113 114 130 132 133 145 155 160 204 208 210 216 229 231 242 247 249 265 292 300 301 308 310 317 325 326 346 352 355 356 365 369 375 380 397 401 406 430 433 435 444 445 469 479 482 484 505 506 507 530 544 553 555 561 568 581 606 607 615 622 632 685 697 701 702 709 722 742 752 753 758 796 798 810 820 824 829
{T_i:i=0,...,r} = {34 120 172 201 165 71 108}
the Complete (n,r)- arc is , n=92 , r= 6
0 1 6 9 11 28 29 33 43 45 46 50 52 53 57 66 67 72 81 112 113 139 141 163 177 188 216 221 226 227 240 250 252 254 262 267 268 269 273 277 280 286 296 298 320 321 327 333 339 358 359 360 366 408 421 428 459 462 469 471 472 474 490 507 515 517 519 521 547 577 579 594 629 651 659 665 671 691 694 711 715 729 749 758 762 773 798 800 830 835 863 864
{T_i:i=0,...,r} = {30 114 190 191 148 87 111}

4.6 Existence of complete (n,7)-arcs in PG(2,29):-

By computer search we find 12 different talls of complete (n,7)-arcs for r=100 ,103,104…,113 . which are near to $t_7(2,29)$.Tabled below:-

the Complete (n,r)- arc is, n=100 , r= 7,
0 1 2 5 9 29 32 33 44 45 46 56 65 77 78 79 85 93 96 107 115 117 119 120 126 141 147 157 162 164 168 175 177 181 182 184 206 210 214 218 227 233 236 258 268 276 277 293 300 306 309 312 314 330 339 346 366 379 385 388 395 398 402 405 432 448 452 459 461 505 510 515 520 547 552 553 567 598 633 646 658 659 664 678 685 693 696 702 704 744 760 763 784 801 806 821 831 832 842 849
{T_i:i=0,...,r} = {16 78 174 239 153 102 28 81}
the Complete (n,r)- arc is , n=103 , r= 7:
0 1 4 23 24 28 33 37 42 48 51 71 75 76 77 81 97 102 104 108 109 115 129 163 164 167 169 170 177 189 191 203 220 224 227 228 236 237 257 259 265 267 281 295 310 318 320 322 338 367 388 403 414 417 420 425 431 470 472 478 483 494 500 513 529 530 531 552 559 579 580 601 614 615 624 638 644 652 675 678 683 687 700 708 711 715 718 723 728 734 735 754 776 778 789 803 814 823 833 852 857 866 868
{T_i:i=0,...,r} = {7 88 168 209 163 114 37 85}
the Complete (n,r)- arc is , n=104 , r= 7
0 1 3 21 25 34 35 59 68 74 84 86 90 110 111 112 120 134 135 136 146 151 153 158 176 177 194 196 199 202 207 210 216 222 224 236 245 250 254 266 271 291 314 317 329 333 343 347 349 352 354 358 398 399 406 410 434 440 443 447 449 452 468 476 481 485 496 505 522 533 539 556 577 578 579 588 619 636 640 646 648 651 654 657 660 661 673 700 703 705 708 724 732 735 747 761 764 767 781 820 835 848 849 857
{T_i:i=0,...,r} = {12 76 172 196 173 120 34 88}

the Complete (n,r)- arc is , n=105 , r= 7
0 1 2 8 14 17 21 65 67 68 73 74 75 85 98 104 140 147 149 181 182 184 188 191 200 205 212 218 229 253 256 257 288 291 293 300 312 316 332 338 344 347 352 358 370 384 387 395 399 405 420 441 448 449 457 471 476 481 487 494 516 518 519 520 528 537 543 557 560 561 567 575 585 586 594 598 603 619 632 639 657 678 687 692 717 725 735 749 758 766 770 771 779 785 791 799 802 809 811 816 824 830 831 854 864
{T_i:i=0,...,r} = {12 76 155 220 159 112 51 86}
the Complete (n,r)- arc is , n=106 , r= 7
0 1 3 12 15 22 23 37 47 58 63 66 73 83 91 101 103 104 105 106 109 118 137 151 162 203 205 206 217 223 229 234 237 240 244 264 268 277 280 283 288 317 318 319 320 322 358 369 384 398 399 413 439 447 468 472 476 483 487 499 502 509 518 523 529 536 560 573 576 579 588 589 591 593 599 608 609 611 627 643 653 664 682 686 691 699 702 708 716 718 741 756 761 770 782 785 790 792 809 813 818 821 824 847 859 870
{T_i:i=0,...,r} = {12 71 151 223 172 95 54 93}
the Complete (n,r)- arc is , n=107 , r= 7
0 1 4 6 15 27 30 34 42 43 48 55 57 63 81 92 93 97 99 102 103 120 122 134 144 154 155 161 165 167 170 171 174 176 180 182 183 187 196 205 236 244 251 252 266 289 299 304 306 314 338 342 354 362 374 375 378 383 388 395 424 434 446 451 456 459 461 465 469 502 510 516 533 535 537 540 541 556 561 572 573 580 598 608 613 627 636 640 655 664 674 684 694 695 727 733 751 762 763 766 772 800 808 818 829 837 844
{T_i:i=0,...,r} = {14 75 150 187 176 133 47 89}
the Complete (n,r)- arc is , n=108 , r= 7
0 1 4 12 13 21 25 30 31 34 35 44 51 53 55 56 59 71 73 76 81 82 83 86 97 103 110 119 122 126 134 146 151 158 160 174 182 187 199 203 205 210 213 229 242 245 257 262 264 272 274 283 287 303 307 325 332 336 337 365 374 385 399 400 422 429 436 458 462 466 481 486 498 504 562 565 568 569 583 600 602 606 608 615 639 654 682 689 720 736 737 738 743 755 766 767 770 773 778 783 785 803 840 841 849 856 857 864
{T_i:i=0,...,r} = {15 71 143 188 190 121 47 96}
the Complete (n,r)- arc is, n=109 , r= 7
0 1 3 6 11 24 35 39 45 51 52 54 69 74 78 87 115 126 127 128 162 166 170 179 207 216 217 221 260 265 276 279 288 294 301 302 308 309 312 340 345 351 354 368 378 397 408 413 414 430 433 434 438 442 444 452 463 468 474 480 483 492 502 514 515 516 522 524 534 540 543 548 560 562 584 605 630 641 643 666 678 685 688 689 695 697 699 722 723 727 729 730 749 759 763 769 770 779 782 784 796 800 816 821 822 825 827 856 859
{T_i:i=0,...,r} = {15 68 132 219 155 123 67 92}
the Complete (n,r)- arc is , n=110 , r= 7
0 1 2 4 12 13 26 27 48 51 56 75 77 80 87 91 96 102 113 117 145 151 158 160 164 182 183 191 205 211 222 234 236 237 238 258 260 279 286 315 326 359 370 379 381 400 432 440 445 453 454 455 463 478 483 492 501 504 505 508 517 523 525 530 532 534 537 539 545 556 558 560 568 574 579 588 591 594 595 597 599 606 620 621 636 646 649 691 692 699 701 705 715 719 720 724 725 728 734 736 743 749 776 783 797 817 822 829 840 861
{T_i:i=0,...,r} = {17 60 145 190 168 135 59 97}

the Complete (n,r)- arc is , n=111, r= 7
0 1 13 15 19 24 29 31 43 53 55 57 59 63 73 90 92 95 102 112 113 114 120 123 126 127 134 146 150 190 198 200 206 208 212 232 237 240 242 246 269 279 284 286 292 300 316 319 333 337 340 353 354 357 366 368 374 376 380 384 397 400 404 407 414 429 448 455 464 466 470 472 476 478 490 499 519 522 533 542 545 553 567 568 575 577 591 597 599 600 608 638 641 661 669 692 713 729 734 769 782 783 784 788 805 822 843 849 855 866 868
$\{T_i:i=0,...,r\}$ = {15 71 125 179 198 118 65 100 }
the Complete (n,r)- arc is , n=112 , r= 7
0 1 3 5 7 11 16 25 26 28 35 40 50 52 61 64 69 75 76 89 90 107 130 132 135 141 161 162 190 199 204 222 236 241 278 285 286 288 293 305 331 335 351 354 359 373 390 397 402 411 430 448 456 457 458 459 464 467 470 471 486 495 503 510 516 536 544 545 565 566 580 584 589 601 615 618 623 646 654 662 663 664 667 670 674 679 684 685 700 702 705 713 724 730 763 764 766 772 779 792 793 802 807 816 824 826 828 830 831 839 851 862 here n=112 , here r= 7 ,
$\{T_i:i=0,...,r\}$ = {11 66 132 202 153 135 67 105}
the Complete (n,r)- arc is , n=113 , r= 7
0 1 5 9 11 16 20 23 24 26 34 42 44 58 61 65 71 80 82 83 95 119 120 125 126 131 134 137 138 147 156 163 173 177 180 188 202 216 235 241 269 275 286 295 300 312 315 344 345 351 356 366 386 390 393 409 420 427 428 433 451 452 455 456 463 471 474 475 478 496 505 509 517 529 546 547 548 569 575 577 599 605 621 636 637 638 650 666 674 676 677 684 697 706 718 727 728 730 744 761 770 798 799 803 805 807 811 821 834 837 846 855 868
$\{T_i:i=0,...,r\}$ = {9 67 141 165 186 127 65 111}

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