Dynamical Features of the Lorenz Model of Atmospheric Circulation

F.A.M. Al-Simary and I.A. Al-Saidi
Department of Physics, College of Education, University of Basrah, Basrah, Iraq

Received 5/2/2005, Accepted 12/4/2005

Abstract

The nonlinear dynamical features of the Lorenz model of atmospheric circulation are investigated. It is found that, the Lorenz model is well suited to examine the theoretical predications of the atmospheric circulation system over a wide range of control parameters. The results show that, this system is capable of displaying various types of dynamical behaviors (instabilities) such as period-doubling sequences and self-pulsing instability leads to chaotic behavior under certain conditions.

Keywords: Periodic behavior, Instability, Chaos, Atmospheric, Lorenz equation.

Introduction

Many nonlinear systems exhibit a range of dynamical behaviors from smooth and regular to irregular and turbulent (chaotic) [1-5]. Some of these systems go through a sequence of transitions (or bifurcations) from a stationary state to periodic via period-doubling [6] and self-pulsing [7] routes, or via intermittency route [8] leads, (finally), to nonperiodic (or chaotic) state, when the (control) parameters involved are varied. Physicits have turned their attention to certain quite simple nonlinear differential equations describing the dynamical system. The analysis of these equations shows that their solutions exhibit very rich dynamics. A special kind of solutions known as "strange attractor" has been identified [9]. The discovery of chaos and strange attractors (or chaotic dynamics) is usually associated with the work of Lorenz [10] establishing a topology of strange solutions observed on analysis of convection and fluid flow. He used coupled first-order, nonlinear, ordinary differential equations describing the behaviors of the fluid-dynamical system. Lorenz model, which is concerned with the processes of convection in the atmosphere, has become standard example of chaos in large number of dynamical systems, such as, mechanical, chemical, biological and medical, electronic [11,12], and optical systems including lasers [13]. The characteristics and dynamics of chaos in these systems have been analysed using different methods [1,4]. In the present paper, we report a detailed numerical study of the Lorenz equations for different dynamical (control) parameters over a wide range of relating conditions.

Mathematical Description

We have used the fourth-order Runge-Kutta method for analysing the Lorenz equations govering the dynamics of the fluid convection system (or atmospheric flow). These equations can be written in the following modified form [14] :

$$\dot{x} = -y^2 - z^2 - a\,x + a\,f \qquad \text{(a)}$$
$$\dot{y} = xy - bzx - y + g \qquad \text{(b)} \qquad (1)$$
$$\dot{z} = bxy + xz - z \qquad \text{(c)}$$

where x is the intensity of westerly-atmospheric circulation, y and z are the sine and consine components of a travelling wave. The parameters f and g are forcing terms due to the average north-south temperature contrast and earth-sea temperature contrast, respectively.

a and *b* are other system control parameters (commonly taken fixed). The values of the system parameters are taken in arbitrary units.

In order to study the dynamical behaviors of the atmospheric flow system, we have solved the Lorenz equations (Eqs.(1)) numerically for several selected values of the control parameters. We present below selected results, which we consider as representative.

Results and Discussion

The control parameters *f* and *g* play important roles on the dynamical behaviors of the fluid system. Let us first consider the case of varying *f* with *a,b,* and *g* fixed at 0.25,4, and 1, respectively. The effect of this variation is illustrated in Fig.1, where the variable x is plotted as a function of time (the time-evolution) in Fig.1 (A) for different values of *f*. the phase-space portrait (or system trajectory), (in x,z plane), corresponding to the x time-series (or time-evolution) is also plotted (as shown in Fig.1 (B)) in order to obtain additional informations about the dynamical behavior features of the fluid system. This can be done through the examination (tracking) time-evoluton of the generated attractors (trajectories).

Fig.1 A(a) shows stable steady-state solution, when *f*=4.295 (or smaller than this value). The phase-space portrait (or the system trajectory) corresponding to this behavior is shown in Fig.1 B(a). We can see that the trajectory of the system spirals towards a single fixed point, therefore the resulting attractor represents the fixed point attractor [1,4].

As *f* varies to 4.3, the behvior changes to periodic pulsation with pulses of equal amplitudes, as shown in Fig. A(b). Such behavior represents the period-one (P1) oscillations. The structure of the phase-space portrait corresponding to this behavior is illustrated in Fig. B(b) and represents a single limit cycle (or closed loop). The periodic nature does not change when *f* is increased to 4.5, and the dynamical behavior remains qualitatively has same structure. The period-one oscillation is still observed and the system trajectory still limit-cycle, but the separation between successive pulses in the time-evolution reduces, i.e., the pulses in the wave train come closer to each other.When *f* is increased to values beyond 4.5 (here over the range *f*=4.58-4.66), the behavior changes through periodic bifurcation to period (2×3)-oscillation [15] and then to period-four (P4) oscillation, as shown in Figs. A(d,e) and the corresponding phase-space trajectories Figs. B(d,e). At *f*=4.66, we note that the system exhibits interesting behavior, where two coexisting sets of stable oscillations appear in the time-evolution these are period-four (P4) and period-six (P6), as illustrated in Fig. A(f). The trajectory corresponds to this behavior is illustrated in Fig. B(f). The dynamical behavior changes to two-frequency oscillation state, as *f* slightly increases (*f*=4.67), and this is illustrated in Fig. A(g). We note here that the system has two identical locked frequencies oscillating simultaneously. This behavior which is reflected in the shape of the trajectory of the attractor and two loops (or two limit cycles) appear in the phase-space plot, as seen in Fig. B(g).Increasing the value of *f* beyond 4.67 causes a considerable changes in the manifestation of the system behavior, the stable regular pulsations convert gradually to irregular pulsations. When *f* is increased to 4.682, the system starts to display quasi-periodic chaos or the so-called weak chaos, as shown in Fig. A(h). This behavior varies to strong chaos as *f* increases to 4.69, Fig. A(i). The phase-space trajectories correspond to these two cases are illustrated in Figs. B(h) and B(i), respectively, and showing strange attractor feature. This attractor (or the chaotic behavior) begins to return back to the previous stable sate via the inverse bifurcation route as *f* increases more further. This situation is clearly illustrated in the time-series. Figs. A(j,k), and the phase-space plot, Figs. B(j,k). In Fig. A(j), we can recognize two sets of pulsations, irregular and regular. Fig. A(k) represents two-frequency pulsations whose nature is similar to those in Fig. A(g). These two figures show clearly and nicely the conversion of the system attractor from the chaotic state to periodic state. For *f* somewhat is larger than 4.985 (namely, *f*=5), the system again starts to exhibit quasi-periodic behavior (Fig. A(l)), changes to chaotic behavior (as *f* increases to 5.1), but the system does not reach (or show) the strong chaos state (Fig. A(m)). We can see that the general feature of the behavior in Fig. A(m) is bursts of noise, they occur as a result of the sudden change in the pulsations or what we call crisis [16]. This phenomenon (crisis) is also apparently seen in Fig. A(n), where some intervals of periodic behavior appear (exist) between the bursts. The behavior here is a type of aperiodic behavior and usually named intermittency [8]. These dynamical behaviors might be understood from the study of the corresponding phase-space portraits (Figs. B(l,m,n)). The figure B(l) reveals several overlaping limit cycles, gives indication of the periodic-oscillation behavior, whereas the figures B(m,n) show that these limit cycles (or trajectories) are randomly interrupted by "turbulent" bursts (as *f* increases) and these in turn causes distortion in the wave-train envelope (see Figs. A(m,n)). The noise nature in the behavior will reduce significantly if we start to increase the value of *f* (for example from 5.25 to 5.50). We note that the attractor loses its irregularity and starts recovering and returns back to the periodic-oscillation structure (state) after passing through several transions, as illustrated in Figs. A(o,p,q), and the corresponding phase-space trajectories, Figs. B(o,p,q). Fig. (o) represents noise limit cycles, while Fig. B(q) represents clean limit cycle (after the noise completely disappears), and this behavior corresponds to period-one (P1) oscillation (Fig. A(q)). We note that, as increase *f* to values greater than the previous one, for instance in the range *f*=6.725-8.0, the system appears to be undergo sequence of transitions and the system displays dynamical behaviors analogous to those in the preceding cases. These change to rather complicated structures (Fig. A(x)), when the system begins to lose its stability, and the oscillatory attractor becomes unstable. In the end, at *f*=8, the system reaches the strong chaos state (or strong attractor), as shown in Fig. A(y).

We notice that, when *f* varies from ~6.50 to ~6.82, the transition of behavior of the system does not lead to chaos as before (as we expected) but the system reveals noisy (nonperiodic) pulsations, change gradually to periodic pulsations, as shown in Figs. (r-t).

The phase-space trajectories correspond to the above cases are shown in Figs. B(r-y). The behavior in Fig. A(y) is confirmed by analysing the time-series using the fast Fourier transformation (FFT) technique, and we find that the obtained power spectrum reveals a broad-band spectrum which is the main feature of the chaotic behavior. The detailed study of this behavior using FFT is the subject of a forthcoming paper.

We now consider the case of variation of the control parameter *g*. To examine the effect of varying this parameter, we have changed the value of *g* over the range 0.8-1.2, with *a*,*b*, and *f* fixed at 0.25,4, and 6. Fig.2 illustrates the resulting effects.

When *g*=0.8, the system displays fully developed chaos (or turbulent behavior), Fig. A(a). The trajectory of the attractor is shown in Fig. B(a), and it has a complicated structure which represents the strange attractor. With increasing the value of *g*, the shape of the wave-train pattern changes to stable steady-state solution (Fig. A(h)), and the system attractor converts to the fixed-point attractor (Fig. B(h)). This occurs through a sequence of transitions. We notice when *g*=0.9, the behavior is in the form of periodic chaos (or metastable chaos), as shown in Fig. A(b). The phase picture of the resulting attractor is illustrated in Fig. B(b). Over the range *g*=0.952-0.960, the noisy periodic pulsations convert to form of stable periodic oscillations of period-one, as shown in Figs. A(c-e). The corresponding noise-limit cycles in Figs. B(c,d) convert to clean single limit cycle, as shown in Fig. B(e). If *g* keeps being increased (>0.96), then the pulses (or the peaks) of the wave train start to move away from each other, but the behavior remains to be stable period-one pulsations, and this is clearly evident in Figs. A(f,g). The phase-state trajectory, also, remains has a limit cycle shape, as illustrated in Figs. B(f,g). In order to obtain more information about the features of the attractor of the chaotic behavior, we have chosen the case of strange attractor in Fig.1 (y) and we tried to analyse (examine) the variation of the attractor structure when the parameter *g* varies. Here we have allowed *g* to change over the range *g*=1.0-0.7. Fig.3 shows the results obtained from the analysis. In this figure, we can regonize considerable changes in the dynamical behavior. The chaotic pulsations in Fig. A(a) settles into a stable steady state period-one pulsations (Fig. 3 A(c)), and the attractor converts from strange chaotic (Fig. B(a)) to clean single limit cycle (orbit), (Fig. B(c)). It is clear evidence that the dynamical system we have can move easily from the chaotic state to the stable steady state through the period-three oscillations route analogous to that observed in Refs.[15,17], as shown in Fig. A(b). The phase-space trajectory belongs to these period-three oscillations consists of three overlaping limit cycles, since there are three different groups of oscillations in the time-series plot.In all the discussion above we have assumed that the values of the parameters *a* and *b* in Eq. (1) are fixed. Now let us try to test the effect of the variation of these parameters. Starting with *a*, if we fix *b* at 4 and try to change the parameter *a* over a selected range (here *a*=0.150-0.298), we will obtain the bifurcations in Fig.4. They show clearly how effectively and nicely the variation of a can bring the system from the stable steady-state (fixed point attractor) to chaos (chaotic attractor) and verse vice, as shown in Figs. A(a-d) and B(a-d). This allows us to control the behavior of the dynamical system simply by varying the control parameter.

We notice that the variation of b plays also an effective role in the features of the dynamical system as the parameter a does. We have analysed the dynamical behavior for selected values of b (namely over the range b=1.55-4.50), the results we have obtained are presented in Fig.5. It is clearly apparent from this figure that the stable two-frequency pulsations (Fig. A(a)) transfer to irregular pulsations and chaotic behavior as b varies. It is seen that the two-frequency pulsations pattern results two overlaping clean limit cycles (orbits), each one belongs to one set of pulsations in the time-series, as shown in Fig. B(a). When the pulsations convert to form of period-one pulsations (Fig. A(b)), the phase-space trajectory changes to a single limit cycle attractor smilar to those we have already observed, as illustrated in Fig. B(b).It may be useful to close the paper by listing in table 1 the ranges of the system control parameter values used in the present work (study).

Parameter	Range of values
f	4.295-6.82
g	0.8-1.2
a	0.150-0.298
b	1.55-4.50

Table 1. Range of the parameter values.

Conclusion

We have reported a theoretical study of the dynamical characteristics of the atmospheric circulation system using the Lorenz model. Our study is carried out by analysing (solving) the Lorenz equations for different conditions. A variety of instabilities including periodic and chaotic behaviors and range of attractors have observed. We find that these behaviors are exteremely dependent on the system control parameters and can be controlled simply by varying these parameters.

The results we have obtained are useful to establish the suitable conditions for the stability of the atmospheric circulation system, and also useful to gain better understanding of this complicated system (specially the transient to

References

[1] H.G. Schuster, "Deterministic Chaos" (Physik-Verlag GMbH, Germany, 1984). Or 2nd edition. (VGH, Weinheim, 1988).
[2] J.M. Thompson and H.B. Stewart, "Nonlinear Dynamics and Chaos"; (John Wiley and Son's, London, 1986).
[3] Pysica D, Nonlinear phenomena, "Order in Chaos", Eds. D. Campbell and H. Rose, 7D (1983), N.B. Abraham, J.P. Gollub, and H.L. Swinney, Physica 11D, 252 (1984).
[4] "Universilty of Chaos", Ed. P. Cvitanovic, (Adam-Hilger, Bristol, 1984).
[5] S. Wiggins, "Introduction to applied nonlinear dynamical systems and chaos", (Springer-Verlag, Berlin/Heidelberg, 1990).
[6] M. J. Feigenbaum, J. Stat, Phys. 19, 259(1978).
[7] A.N. Oraevskil, Sov. J. Quantum Electron., 11, 71 (1981).
[8] P. Manneville and Y. Pomeau, Phys. Lett., 75A, 1 (1979), Y. Pomeau and P. Manneville, Commun. Math. Phys., 74, 189 (1980).
[9] E. Ott., Rev. Mod. Phys., 53, 655 (1981).
[10] E.N. Lorenz, J. Atmos. Sci., 20, 130 (1963).
[11] "Bifurcation theory and Applications in Scientific Disciplines", Eds. O. Gurel and O. E. Rossler, (Annals of the New York Academy of Sciences, Vol. 316, 1979).
[12] W.L. Ditto and L.M. Pecora, Scientific Am. August, 101 (1993).
[13] "Instabilities and Chaos in Quantum Optics", Vol. 34, Eds, F.T. Arecchi and R.G. Harrison, (Springer-Verlag, Berlin/Heidelberg, 1987).
[14] E.N. Lorenz, Telluse 36A, 98 (1984).
[15] H.L. Swinney, Physica 7D, 3 (1983).
[16] C. Grebogi, E. Ott, and J.A. Yorke, Physica 7D, 181 (1983).
[17] T. Y. Li and J. A. Yorke, Am. Math. Month. 82, 985 (1975).

Fig.1. (A) Time-evolution of x for different values of *f*, when *a*=0.25, *b*=4.0, and *g*=1.

(B) The Corresponding phase-space portrait in the (x,z) plane.

Continue

Continued

Fig.2. (A) Time-evolution of x for different values of *g*, when *a*=0.25, *b*=4.0, and *f*=6.0.
(B) The Corresponding phase-space portrait.
Continue

Continued

Fig.3. Effect of the g variation on the strange attractor in Fig.1 (y).

Fig.4. Effect of the variation of the parameter *a* on the time-evolution of atmospheric dynamical behavior.
(1st) x time-series. (B) Phase-space portrait (z against x). when *b*=4, *f*=6, and *g*=1.

Fig.5. Effect of the variation of the parameter *b* on the time-evolution of atmospheric dynamical behavior.
(1st) x time-series. (B) Phase-space portrait (z against x). when *a*=0.25, *f*=6, and *g*=1.

الخلاصة:

تم فحص واستقصاء الخصائص الحركية (الديناميكيةDynamic) اللاخطية لنموذج لورنس (Lorenz model) لدورة الغلاف الجوي (Atmospheric circulation) . لقد وجد ان نموذج لورنس مناسبا لفحص التنبؤات النظرية لنظام دورة الغلاف الجوي على مدا واسعا من معاملات التحكم . تظهر النتائج التي تم الحصول عليها امكانية (قابلية النظام على اظهار انواع مختلفة من التصرفات الحركية (اللااستراريات), مثل سلاسل تضاعف الزمن المتعاقب ولااستقرارية التنبيض الذاتي التي تؤدي الى التصرف الفوضوي تحت ظروف محددة .