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MONOPOLE TRANSITIONS IN DEFORMED NUCLIE OF Hf ISOTOPES

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Abstract

The structure and monopole transitions of sum neutron rich deformed Hf isotopes have been studied within the framework of the interacting boson model. The level structure for two selected isotopes $Hf^{176\text{-}178}$ and B(E2), ρ(E0) and the X(E0/E2) ratios have been calculated. The numerical results obtained have been compared with experimental data. Satisfactory results for comparison were obtained.

Keywords

Nuclear structure, Energy levels, Transition Probability, Monopole Transition, Interacting Boson model.

Introduction

Experimental nuclear physicists, with improved equipment and techniques, continue to challenge nuclear theorists with interesting new phenomena. The measurement of new levels, mostly lying below 3 MeV, raises questions about the nature of collective excitations and their decay properties in atomic nuclei. Even though heavey deformed nuclei with $A \geq 150$ are god candidates for probing such degree of freedom, microscopic calculations (like the interacting boson calculations) are very important to gaining a deeper understanding of the nature on nuclear properties, for example the monopole transitions between $I_i = I_f$ states, where the E0 component exist , also the transition when $I_i = I_f = 0^+$ in which it is pure E0 transition. This transition in forbidden in the models which suggest in advance that the nucleus has a spherical shape. The study of excited 0^+ in even-even nuclei can provide important facts on the nuclear structure in a given region of the nuclear chart, which is in the case of Hf isotopes is the rotational limit. A variety of the theoretical descriptions, for the existence of the first excited 0+, have been suggested during the last four decays. In the vibrational model; the low lying excited 0^+ level is consider as a member of the two phonon triplet. Configuration mixing of two particle in the shell model produces excited 0^+ levels [1]. In the collective models [2,3] the excited 0^+ states are described as a band head of the vibrational beta bands. While thw gamma-soft model, the first 0^+ state is member of the two phonon triplet [4].

The Interacting Boson Approximation has been rather successful at describing the collective properties of several medium and heavy nuclei specialy te existence of the excited 0^+ suaetes. The interacting boson model (IBM-1) firstly introduced by Arima and Iachello [5] and Casten [6] has been enjoyed considerable success in resent year [7]. In this model, the low – energy states of even - even nuclei are described in terms of interactions between s(j=0) and d(j=2) bosons. The corresponding Hamiltonian is diagonalized in this boson space by employing some rather powerful and efficient group theory methods. Later an IBM-2 version has been introduced, in which there is two types of bosons; Proton bosons and Neutron bosons [8]. IBM-1 has been successfully applied to the strongly deformed nucleus Er^{168} by Casten et al [9] and to nucleus Hf^{178} by Hyque et al [10]. Calculated energy levels were in a good agreement with the experimental data and E2 transition probability as well. The branching ratios were also calculated. The question arises whether such nuclei could be well described with IBM-2 and this model can reproduced the E2 and E0 properties of the strongly deformed Hafnium isotopes.

The Hf two isotopes (A=176, A=178) under consideration have Z=72, and N=104 to 106, which means that we have 22 proton particles outside the closed shell at 50 or 10 proton holes related to the closed shell at the magic number 82. The neutrons numbers are 104, means that we have 22 neutrons outside the major closed shell at 82, or we have 22 holes related to the closed shell at 126, and 106, which means that we have 24 neutrons outside the number 82 or 20 holes outside the closed shell at magic number 126.The large numbers of nucleons out side the major shell make the nucleus closed to heavy deformed nuclei more like Gd, Er and Sm nuclei[11-13].

The Model Hamiltonian:

For a given nucleus, the boson number N_ν *and* N_π are found by counting neutrons and protons from the nearest closed shell related to the magic numbers. The vector space of IBM-2 is then just the product of all possible states $(s,d)^{N\pi}$ with those of $(s,d)^{N\nu}$, where each factor the set of states is the same as in IMB-1. The model Hamiltonian is written as[14]

$$H = H_\pi + H_\nu + H_{\pi\nu} \tag{1}$$

The Hamiltonian generally used in phenomenological calculations can be written as

$$H = \varepsilon_d (n_{d\nu} + n_{d\pi}) + \kappa(Q_\nu . Q_\pi) + V_{\nu\nu} + V_{\pi\pi} + M_{\nu\pi} \tag{2}$$

The first term represents the single-boson energies for neutron and protons, ε_d is the energy difference between s- and d- boson and $n_{d\rho}$ is the number of d-bosons, where ρ correspond to π (proton) or ν (neutron) bosons. The second term denotes the main part of the boson-boson interaction, i.e. the quadrupole-quadrupole interaction between neutron and proton bosons with the strength κ. The quadrupole operator is

$$Q_\rho = [d_\rho{}^+ s_\rho + s_\rho{}^+ d_\rho]^{(2)} + \chi_\rho [d_\rho{}^+ d_\rho]^{(2)} \tag{3}$$

where χ_ρ determines the structure of the quadrupole operator and is determined empirically. The square bracket in Eq. (3) denotes angular momentum coupling.

The term $V_{\pi\pi}$ *and* $V_{\nu\nu}$,in equation (2) which correspond to interaction between like-boson, are sometimes included in order to improve the fit to experimental energy spectra. They are of the form

$$V_{\rho\rho} = \frac{1}{2} \sum_{L=0,2,4} C_L{}^\rho ([d^+{}_\rho d^+{}_\rho]^{(L)} . [d_\rho d_\rho]^{(L)}). \tag{4}$$

However, their effects are usually considered minor and often neglected.

The Majorana term, $M_{\nu\rho}$, which contains three parameters ξ_1 , ξ_2 *and* ξ_3 may be written as

$$M_{\nu\pi} = \frac{1}{2}\xi_2 ([s_\nu{}^+ d_\pi{}^+ - d_\nu{}^+ s_\pi{}^+]^{(2)} . [s_\nu d_\pi - d_\nu s_\pi]^{(2)}) - \sum_{k=1,3} \xi_k ([d_\nu{}^+ d_\pi{}^+]^{(k)} . [d_\nu d_\pi]^{(k)}). \tag{5}$$

This work aimed at two thinks: first, to give the Hamiltonian of IBA-2 in terms of the formalism; second is to study the monopole transitions probability and mixing between E0 and E2 transitions in deformed Hf isotopes by use of this Hamiltonian.

Electromagnetic Transitions

The general one body E2 transition operator in the IBM-2 is

$$T^{(E2)} = e_\pi Q_\pi + e_\nu Q_\nu \tag{6}$$

where Q_ρ is the same as in equation (3) and e_π *and* e_ν are boson effective charges depending on the boson number N_ρ and they can take any value to fit the experimental results.

The M1 transition operator obtained by letting $l = 1$ in the single boson operator of the IBM-1 and can be written as

$$T^{(M1)} = \left[\frac{3}{4\pi}\right]^{1/2} (g_\pi L_\pi^{(1)} + g_\nu L_\nu^{(1)}) \qquad (7)$$

where g_π, g_ν are the boson g-factors in units of μN and $L^{(1)} = \sqrt{10}(d^+ x\tilde{d})^{(1)}$. This operator can be written as

$$T^{(M1)} = \left[\frac{3}{4\pi}\right]^{1/2} \left[\frac{1}{2}(g_\pi + g_\nu)(L_\pi^{(1)} + L_\nu^{(1)}) + \frac{1}{2}(g_\pi - g_\nu)(L_\pi^{(1)} - L_\nu^{(1)})\right] \qquad (8)$$

The first term on the right hand side, in the above equation, is diagonal and therefore for M1 transitions the previous equation may be written as

$$T^{(M1)} = 0.77\left[(d^+\tilde{d})_\pi^{(1)} - (d^+\tilde{d})_\nu^{(1)}\right](g_\pi - g_\nu) \qquad (9)$$

The monopole matrix elements in the IBM-2 are written as:

$$\rho(E0) = \frac{Z}{R^2}\langle I_f \left| T(E0) \right| I_i \rangle = \frac{Z}{R^2}\sum \beta_s n_s \qquad (10)$$

Where s=proton (π) or neutron (ν), n_s is a number operator for proton (neutron) and β_π, β_ν are coefficient which must be determined from experimental data. In the deformed limit of the IBM-2, the E0 matrix elements were calculated using th technique explained in reference [15]

The results and Discussion

To reintroduce the monolpole matrix elements one has to obtain the best fit for energy level and the reproduced the reduced transition matrix elements. So, fit to experimental energy spectrum of the Hf^{176} and Hf^{178}. The required boson numbers are $N_\pi = 5 \, and \, N_\nu = 11$ for A=176 and $N_\pi = 5 \; and \; N_\nu = 10$ for A=178. After several iterations it is found that the following values of the parameters of the Hamiltonian in equation (2) gave the best fit to experimental energies[16] of the ground state band and of beta and gamma bands.

Table-1 The IBM-2 parameters for energy levels reproduce.

Isotope	Є	χ_π	χ_ν	K	ξ_1	ξ_2	ξ_3
176	0.522	-1.181	-0.221	-0.079	-0.506	-0.032	-0.397
178	0.526	-1.150	-0.222	-0.087	-0.506	-0.023	-0.385

$C_\pi^{\,L} = 0.0, \; C_\nu^{\,L} = -0.25, -0.13, 0.0 \, MeV$

The calculated values of the parameters $\chi_\pi \; and \; \chi_\nu$ are closed to the values calculated by Pittle et al. [17]. The calculated level schemes are presented in figure-1 and 2. The agreement with three lower energy bands is good.

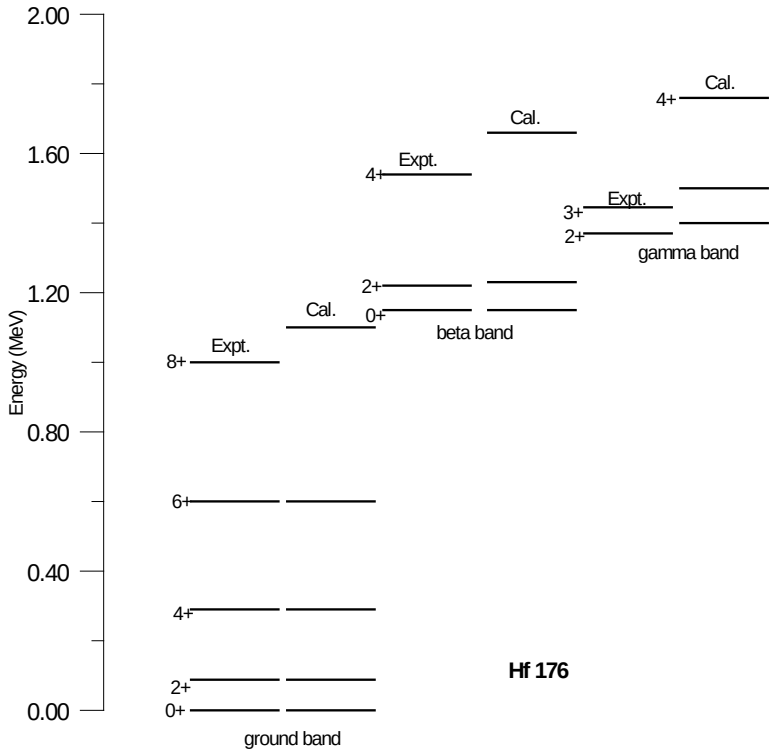

Figure-1 The calculated and experimental low lying levels in Hf^{176}

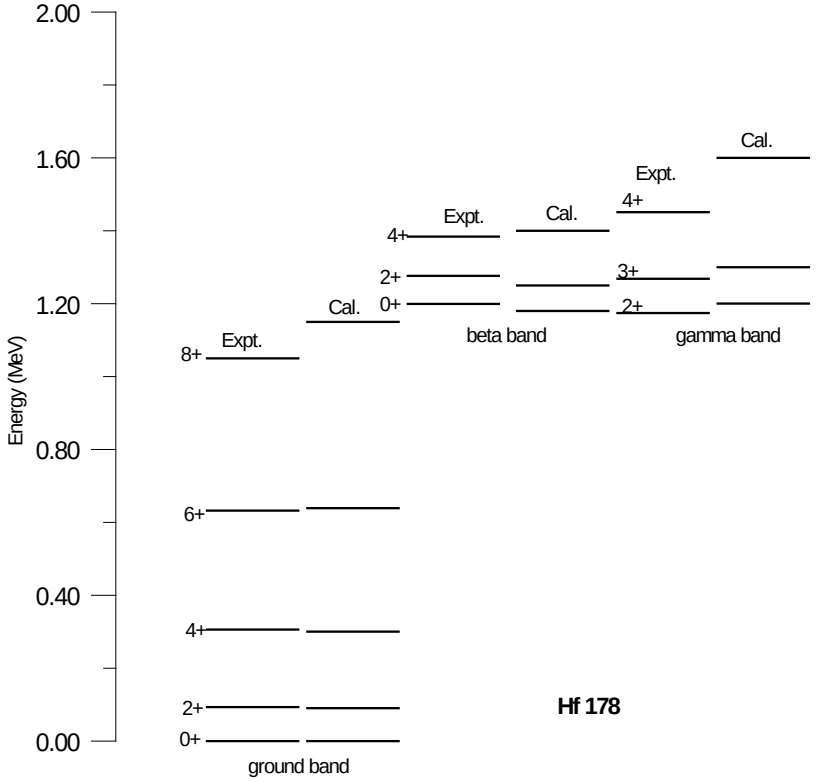

Figure-2 The calculated and experimental low lying levels in Hf^{178}

Then one has to choose the parameter for the transition probability calculations which is a sensitive test for our procedure. The method of estimate the best fitting parameters is discussed in reference [18].The parameters according to equation (6) are; $e_\pi = 0.115 e^2 b^2$ *and* $e_\nu = 0.34 e^2 b^2$ for Hf^{178} and $e_\pi = 0.118 e^2 b^2$ *and* $e_\nu = 0.30 e^2 b^2$ for Hf^{176}. The comparison between theoretical calculation and the experimental data are presented in Table-2.

Table-2. A comparison between experimental and theoretical values of B(E2) in $Hf^{176\text{-}178}$ isotopes.

Nucleus	$I_i - I_f$	$B(E2; I_i - I_f)$	
		Expt.	Cal.
Hf^{176}	2_1----- 0_1	5.21(5)	5.8000
	4_1----- 2_1		4.2000
	2_2----- 0_1	0.029(3)	0.0018
	2_2----- 2_1		0.0190
	0_2----- 2_1	0.119(8)	0.0980
	0_2----- 2_2		1.1300
	3_1----- 2_1		0.0030
	3_1----- 2_2		0.0000
	3_1----- 4_1		0.1320
Hf^{178}	2_1----- 0_1	4.86(5)	4.7800
	4_1----- 2_1		3.5000
	2_2----- 0_1	0.115(4)	0.1890
	2_2----- 2_1		0.0020
	0_2----- 2_1	0.0018(7)	0.0110
	0_2----- 2_2		0.9350
	3_1----- 2_1		0.0030
	3_1----- 2_2		0.0000
	3_1----- 4_1		0.0210

From the above table we can see that the calculated reduced quadrupole transitions agree with experimental data.

The monopole matrix elements

Monopole transitions (E0) are known to be pure penetration effect, where the transition is caused by an electromagnetic interaction between the nuclear charge and the atomic electron penetrating the nucleus. An E0 transition occurs between two states of the same spin and parity by transferring the energy and zero unit of angular momentum. Thus E0 has no competing gamma ray. These transitions are different from zero only in the case where the transition is accompanied by the nucleus surface change. For example in the nuclear models where the surface is assumed to be fixed E0 transitions are strictly forbidden. Electric monopole transitions can occur not only in 0^+--- 0^+ transition but also, in competition with gamma multipole transition and depending on transition selection rules may compete in any $\Delta I = 0$ decay such as a 2^+ --- 2^+. At transitions energies greater than $2m_o c^2$, monopole pair production is also possible.

The E0 reduced transitions probability written as [17]

$$B(E0; I_i - I_f) = e^2 R^4 \rho^2(E0) \qquad I_i = I_f \tag{11}$$

where e in the electronic effective charge, R is the nuclear radius and $\rho(E0)$ is the transition matrix element. However, there are only limited cases where $\rho(E0)$ can be measured directly. In most cases we have to determine the intensity ratio of E0 to the competing E2 transition calling this as $X(\frac{E0}{E2})$ value [18] which can be written as

$$X(E0/E2) = \frac{B(E0; I_i - I_f)}{B(E2; I_i - I_{f'})} \tag{12}$$

where $I_f = I_{f'}$ for $I_i \neq 0, and\ I_f = 0, I_{f'} = 2\ for\ I_i = 0.$

The $T^{(E0)}$ operator my be found by setting $l = 0$ on the IBM-2 operator [16]

$$\rho_{if}(E0) = \frac{Z}{R_0^{\,2}} \sum \tilde{\beta}_{0\rho} \left\langle f \left| d^{+}{}_{\rho} x d_{\rho} \right| i \right\rangle \qquad (13)$$

where $R_0=1.2A^{1/3}$ fm and $\rho(E0)$ is a dimensionless quantity. The two parameter $\tilde{\beta}_{0\pi}, \tilde{\beta}_{0\nu}$ in equation(14) may be estimated by fitting in isotope shift, which is the difference in the square radius $\delta\langle r^2 \rangle$ between neighboring isotopes in their ground state [17].i.e.

$$\delta\langle r^2 \rangle = \langle 2_1^{+} | T_0 | 2_1^{+} \rangle - \langle 0_1^{+} | T_0 | 0_1^{+} \rangle. \qquad (14)$$

In the case of Hafnium isotopes the measured isotopes shift [19] were used to fined the parameters used in the IBM-2 calculations and they are $\beta_{\pi} = 0.055\ fm^2$ and $\beta_{\nu} = 0.026\ fm^2$ produced the monopole matrix elements. The results of the calculation are listed in table-3.

Table-3. Calculated X(E0/E2) ratios compare with experimental .[19] data in even Hafnium isotopes

Isotope	Initial level (KeV)	I_i	I_f	X(E0/E2) Experimental	X(E0/E2) Theoretical
Hf^{176}		0^{+}_2	0^{+}_1		0.013
		2^{+}_3	2^{+}_1		0.009
		2^{+}_2	2^{+}_1	0.90(27)	0.500
		0^{+}_3	0^{+}_1		0.130
		2^{+}_3	2^{+}_2	0.097(16)	0.007
		0^{+}_3	0^{+}_2		1.780
Hf^{178}		0^{+}_2	0^{+}_1	0.18(4)	0.096
		2^{+}_3	2^{+}_1		0.067
		2^{+}_2	2^{+}_1	0.14(4)	0.190
		0^{+}_3	0^{+}_1	0.10(2)	0.0560
		2^{+}_3	2^{+}_2	0.12(5)	0.008
		0^{+}_3	0^{+}_2	1.38(8)	1.200

The above table shows that the model predicts well the monopole matrix elements compare with the quadrupole transition from the same states. However it is not easy to estimate this ratio because the smallness on the monopole matrix element and this is one of the reasons of not getting the exact ratio. A large X value for transition from 0_3 to 0_2 agrees well with experiment despite the band crossing transition, which means that the 0_3 has not collective structure.

Conclusion remarks

It is seen that for the nuclei $A \approx 170$ to 180 monopole matrix elements for transition from the beta band to the ground rotational band are mainly investigated with details, while $2_{\gamma} - -2_{g}$ E0 component are studied for nuclei $A \approx 190$. Unfortunately, the data available on E0 transitions are very rare and also the approximate nature of theory does not make it possible to settle the question of nuclear nonaxiality. Nerveless, the comparison made in the present work gives evidence against nuclear nonaxiality in the ground state. The E0 transition probability should depend upon the details of nuclear structure, shape and there is little hope to describe E0 transition with the framework of phenomenological models because the model contains rather rough approximations. More accurate calculations of $\rho(E0;\Delta I = 0)$ with the framework of microscopic model are needed.

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ألأنتقالات الصفرية للأنوية المشوهة من نظائر الهافنيوم

عبد الرضا حسين صبر

[illegible]

المستخلص:

تمت دراسة التركيب النووي والنتقالات الصفرية للأنوية الغنية بالنيوترونات في مجمعة نظائر الهافنيوم في مجال نموذج البوزونات المتفاعلة الثاني. مستويات الطاقة الخاصة باثنين من النظائر المختارة وهما 176و 178 وكذلك الأنتقالات الكهرومغناطيسية الرباعية القطب والصفرية تمت دراستها وحسابها ومقارنتها مع النتائج العملية وقد تم الحصول علبى نتائج مقبولة.