# The Effect of Evaporation and Condensation on Nonlinear Bubble Dynamics 

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#### Abstract

Mathematical formulations have been made of the physical behavior of a bubble in an acoustic field. A new model of nonlinear bubble dynamics is constructed including effects of compressibility of the liquid, and evaporation and condensation of water vapour at bubble wall. The kinetic equation of evaporation and condensation of water vapour at the bubble wall is calculated numerically in order to investigate the effect of evaporation and condensation. Comparison is given between this model and the isolated adiabatic model.

It is concluded that the effect of evaporation and condensation is considerable on bubble dynamics in acoustic field. It is also clarified that the partial pressure of water vapour is identical to the saturated vapour pressure except at bubble collapse. At a strong collapse, the partial pressure of water vapour differs from the saturated vapour pressure.

Comparison of radius-time curve is given between the calculated results and an experimental data by others. The calculated result by this model fits with the experimental data.


Keywords: Bubble Oscillation, Acoustic Cavitation, Water vapour

## 1- Introduction

The main interest in cavitation bubble dynamics arises from the destructive action due to the collapse of bubbles in liquids. Bubbles give high pressure and temperature during collapsing. The dynamics of a gas bubble in a liquid is strongly dependent on the pressure of the gas contained in it [1].After the discovery of bubble dynamics, many researchers study it theoretically [2-5]. However, the effect of evaporation and condensation of liquid (water) vapour is neglected in those studies [2-5]. The effect is known to be considerable on bubble dynamics in the study of the pressure wave produced by the collapse of a bubble in a liquid [6]. In this research, a simple model of bubble dynamics is constructed including the effect, which is a step of more accurate study of bubble dynamics. In many theories of bubble dynamics [7-10], it is assumed that the partial pressure of vapour inside a bubble is always identical to the saturated vapour pressure of the surrounding liquid. In addition, in these theories, the liquid temperature at the bubble wall is assumed to be always identical to the ambient liquid temperature. In order to investigate the validity of this assumption, the kinetic equation of evaporation and condensation at the bubble wall is calculated numerically as a function of time.In the present paper, we propose to give a systematic approximate theory of the radial motion of a spherical bubble in a compressible liquid in acoustic field. This problem was considered in connection with underwater explosions by (in 1941, Herring [11]; in 1948, Cole [12]; in 1952, Trilling [13]; in 1956, Keller and Kolodner [14]; and in 1980, Keller and Miksis [15]). It is desirable to have an equation of motion for the bubble boundary.In addition to the classic studies already cited, a number of papers have appeared in the last few years addressing the same problem (in 1998, Yasui [16] and in 2002, AL-Asady [17]). In the next section, the adiabatic model is described, which is frequently employed in the study of bubble dynamics. For example, the adiabatic model was used in the calibration of the experimental data of radius time curve [2, 3]. Also in this paper, the new model is described in which effect of evaporation and condensation of liquid vapor is taken into
account. Results of numerical calculations are shown both by the adiabatic model and the new model. The calculated result of the model is compared with an experimental data of radiustime curve for one acoustic cycle.

## 2-Adiabatic Model

In this model, as an equation of bubble radius, Keller equation is employed, in which compressibility of liquid is taken into account to the first order in the bubble wall Mach number relative to the speed of sound in the liquid [18, 19],

$$
\begin{equation*}
R \ddot{R}\left(1-\frac{\dot{R}}{c}\right)+\frac{3}{2} \dot{R}^{2}\left(1-\frac{\dot{R}}{3 c}\right)=\frac{1}{\rho_{\infty}}\left(1+\frac{\dot{R}}{c}\right)\left[\mathrm{P}_{\mathrm{B}}(\mathrm{t})-\mathrm{P}_{\mathrm{s}}\left(\mathrm{t}+\frac{\mathrm{R}}{\mathrm{c}}\right)-\mathrm{P}_{\mathrm{o}}\right]+\frac{\mathrm{R}}{\rho_{\infty} \mathrm{c}} \frac{\mathrm{dP}_{B}(\mathrm{t})}{\mathrm{dt}} \tag{1}
\end{equation*}
$$

Where $\ddot{R}=\frac{d^{2} R}{d t^{2}}$, $c$ is the speed of sound in the liquid, $\rho_{L}$ is the liquid density, $P_{B}(t)$ is the liquid pressure on the external side of the bubble wall, $\mathrm{P}_{\mathrm{s}}(\mathrm{t})$ is a non-constant ambient pressure component such as a sound field, and $\mathrm{P}_{\mathrm{o}}$ is the undisturbed pressure. .

When a bubble is irradiated by an acoustic wave of which wave length is much larger than the bubble radius [16],

$$
\begin{equation*}
P_{s}(t)=-P_{m} \sin \omega t \tag{2}
\end{equation*}
$$

where $\mathrm{P}_{\mathrm{m}}$ is the pressure amplitude of the acoustic wave and $\omega$ is the angular frequency of it. $P_{B}(t)$ is related to the internal bubble pressure $\mathrm{P}_{\mathrm{g}}(\mathrm{t})$ by [20]

$$
\begin{equation*}
P_{B}(t)=P_{g}(t)-\frac{2 \sigma}{R}-\frac{4 \mu \dot{R}}{R} \tag{3}
\end{equation*}
$$

where $\sigma$ is the surface tension and $\mu$ is the liquid viscosity.
In this model, pressure and temperature are assumed to be spatially uniform in a bubble. The liquid temperature on the external side of the bubble wall is assumed to be constant (To) during bubble oscillations. In order to calculate $\mathrm{P}_{\mathrm{g}}(\mathrm{t})$, van der Waals equation of state is
employed [21].

$$
\begin{equation*}
\left[\mathrm{P}_{\mathrm{g}}(\mathrm{t})+\frac{\mathrm{a}}{v^{2}}\right](v-\mathrm{b})=\mathrm{R}_{\mathrm{g}} \mathrm{~T} \tag{4}
\end{equation*}
$$

Where a and b are the van der Waals constants, $v$ is the molar volume, $\mathrm{R}_{\mathrm{g}}$ in the gas constant, and T is the temperature inside the bubble. Formula of the molar volume ( $v$ ) is given in ref. [17]. In this model, the van der Waals constants (a and b) change with time due to the change of $\mathrm{n}_{\mathrm{H} 2 \mathrm{O}}$ and $\mathrm{n}_{\text {air }}$. Equations of the van der Waals constants are described in ref. [17] as a function of $\mathrm{n}_{\mathrm{H} 2 \mathrm{O}}$ and $\mathrm{n}_{\text {air }}$.

Thus temperature discontinuity exists at the bubble wall ( $\Delta \mathrm{T}=\mathrm{T}-\mathrm{T}_{\mathrm{O}}$, where T is temperature inside the bubble). In the adiabatic model, no heat exchange is taken into account between a bubble and the surrounding liquid. Thus the change of the internal energy of a bubble ( $\Delta \mathrm{E}$ ) is brought only by PV work [17].

$$
\begin{equation*}
\Delta \mathrm{E}(\mathrm{t})=-\mathrm{P}_{\mathrm{g}}(\mathrm{t}) \cdot \Delta \mathrm{V}(\mathrm{t}) \tag{5}
\end{equation*}
$$

The volume of the bubble, $\mathrm{V}(\mathrm{t})$, is related to the bubble radius $\mathrm{R}(\mathrm{t})$ by:

$$
\begin{equation*}
\mathrm{V}=\frac{4}{3} \pi \mathrm{R}^{3} \tag{6}
\end{equation*}
$$

The temperature inside the bubble ( T ) is calculated by solving the following equation [17].

$$
\begin{equation*}
\mathrm{E}=\frac{\mathrm{n}_{\text {air }}}{\mathrm{N}_{\mathrm{A}}} \int_{0}^{\mathrm{T}} \mathrm{C}_{\mathrm{V}, \text { air }}\left(\mathrm{T}^{\prime}\right) \mathrm{dT} \mathrm{~T}^{\prime}+\frac{\mathrm{n}_{\mathrm{H} 2 \mathrm{O}}}{\mathrm{~N}_{\mathrm{A}}} \int_{0}^{\mathrm{T}} \mathrm{C}_{\mathrm{V}, \mathrm{H} 2 \mathrm{O}}\left(\mathrm{~T}^{\prime}\right) \mathrm{dT} \mathrm{~T}^{\prime}-\left(\frac{\mathrm{nt}}{\mathrm{~N}_{\mathrm{A}}}\right)^{2} \frac{\mathrm{a}}{\mathrm{~V}} \tag{7}
\end{equation*}
$$

Where $E$ is the internal energy of the bubble, $n_{t}$ is the total number of air and vapour molecules in the bubble ( $\mathrm{n}_{\mathrm{t}}=\mathrm{n}_{\text {air }}+\mathrm{n}_{\mathrm{H} 2 \mathrm{O}}$ ) and $\mathrm{C}_{\mathrm{V}, \text { air }}(\mathrm{T})$ [ $\mathrm{C}_{\mathrm{V}, \mathrm{H} 2 \mathrm{O}}(\mathrm{T})$ ] is the molar heat of air [vapour] at constant volume at temperature T. For the temperature dependence of the heat capacity at constant volume, empirical formulas are employed, which are derived by experiments [22] (see Appendix in ref. [17]).

The molar volume $v$ is calculated by:

$$
\begin{equation*}
v=\frac{N_{A} V}{n_{t}} \tag{8}
\end{equation*}
$$

The density inside the bubble is calculated by:

$$
\begin{equation*}
\rho_{\mathrm{g}}=\frac{10^{-3}}{v}\left(\mathrm{M}_{\mathrm{H} 2 \mathrm{O}} \frac{\mathrm{n}_{\mathrm{H} 2 \mathrm{O}}}{\mathrm{n}_{\mathrm{t}}}+\mathrm{M}_{\text {air }} \frac{\mathrm{n}_{\text {air }}}{\mathrm{nt}}\right) \tag{9}
\end{equation*}
$$

## 3-Model with Evaporation and Condensation

There is a spherical bubble of initial radius Ro containing both vapour and noncondensable gas in a viscous compressible liquid in acoustic field. At time zero, the ambient pressure is Po and then the bubble begins to oscillate accompanied with phase change through the bubble wall. The problem is to investigate this physical effect on the bubble oscillations.

In writing the basic equations, the following assumptions are made:
(a) The bubble is spherically symmetric.
(b) The effect of gravity and diffusion are negligible.
(c) The pressure inside the bubble is uniform.
(d) The vapour and non-condensable gas are obeying van der Waals equation.
(e) The temperature of the vapour and non-condensable gas are equal.
(f) The physical properties of liquid are constant.

We begin this section with a general discussion of the motion of the compressible liquid during the bubble oscillation. The formulation takes in to account of the effect of evaporation and condensation. Fujikawa and Akamats [23] adopt the PLK method to account for liquid compressibility because this procedure is capable of an indefinitely high degree of accuracy. Benjamin [24] first applied this method to solve the problem of the collapsing bubble and later Jahsman [25] and Tomita and Shima [26] further developed it.

Let $\phi(\mathrm{r}, \mathrm{t})$ be the velocity potential for the liquid. Then the continuity equation and momentum equation takes the following forms:

$$
\begin{equation*}
\frac{\partial \rho_{\mathrm{L}}}{\partial \mathrm{t}}+\rho_{\mathrm{L}}\left(\frac{\partial^{2} \phi}{\partial \mathrm{r}^{2}}+\frac{2}{\mathrm{r}} \frac{\partial \phi}{\partial \mathrm{r}}\right)+\left(\frac{\partial \phi}{\partial \mathrm{r}}\right)\left(\frac{\partial \rho_{\mathrm{L}}}{\partial \mathrm{r}}\right)=0 \tag{10}
\end{equation*}
$$

and

$$
\begin{equation*}
\frac{\partial \phi}{\partial \mathrm{t}}+\frac{1}{2}\left(\frac{\partial \phi}{\partial \mathrm{r}}\right)^{2}+\int \frac{\mathrm{dP}_{\mathrm{L}}}{\rho_{\mathrm{L}}}=\text { constant } \tag{11}
\end{equation*}
$$

The sound speed in the liquid is

$$
\begin{equation*}
\tilde{c}^{2}=c^{2}-(n-1)\left[\frac{\partial \phi}{\partial t}+\frac{1}{2}\left(\frac{\partial \phi}{\partial t}\right)^{2}\right] \tag{12}
\end{equation*}
$$

From the equations (10), (11), and (12), Fujikawa and Akamats [23] obtained a partial differential equation concerning the velocity potential $\phi$,

$$
\begin{align*}
\frac{\partial^{2} \phi}{\partial \mathrm{r}^{2}}+\frac{2}{\mathrm{r}} \frac{\partial \phi}{\partial \mathrm{r}}-\frac{1}{\mathrm{c}^{2}} \frac{\partial^{2} \phi}{\partial \mathrm{t}^{2}}=\frac{1}{\mathrm{c}^{2}}[ & 2 \frac{\partial \phi}{\partial \mathrm{r}} \frac{\partial^{2} \phi}{\partial \mathrm{r} \partial \mathrm{t}}+2 \frac{\mathrm{n}-1}{\mathrm{r}} \frac{\partial \phi}{\partial \mathrm{r}} \frac{\partial \phi}{\partial \mathrm{t}}+(\mathrm{n}-1) \frac{\partial^{2} \phi}{\partial \mathrm{r}^{2}} \frac{\partial \phi}{\partial \mathrm{t}}+ \\
& \left.\frac{\mathrm{n}+1}{2}\left(\frac{\partial \phi}{\partial \mathrm{r}}\right)^{2} \frac{\partial^{2} \phi}{\partial \mathrm{r}^{2}}+\frac{\mathrm{n}-1}{\mathrm{r}}\left(\frac{\partial \phi}{\partial \mathrm{r}}\right)^{3}\right] \tag{13}
\end{align*}
$$

where n is a constant in equation of state.
The boundary conditions are:
(i) Continuity at the phase interface, $\quad\left(\frac{\partial \phi}{\partial \mathrm{r}}\right)_{\mathrm{r}=\mathrm{R}}=\dot{\mathrm{R}}-\frac{\dot{\mathrm{m}}}{\rho_{\infty}}$
(ii) In the liquid at the interface the pressure equation, $\left[\frac{\partial \phi}{\partial t}+\frac{1}{2}\left(\frac{\partial \phi^{2}}{\rho_{\infty}}\right)^{2}\right]_{\mathrm{r}=\mathrm{R}}=\mathrm{H}$
(iii) At infinity

$$
\begin{equation*}
\phi=0 \text { as } \mathrm{r} \rightarrow \infty \tag{16}
\end{equation*}
$$

The initial conditions $(t=0)$ are:

$$
\begin{equation*}
\mathrm{R}=\mathrm{R}_{\mathrm{o}} \text { and } \dot{\mathrm{R}}=0 \tag{17}
\end{equation*}
$$

According to the PLK method, Fujikawa and Akamats [23] obtained the equation of motion of the bubble with the second-order correction of the liquid compressibility and the effect of evaporation and condensation:

$$
\begin{aligned}
& \mathrm{R}\left(\ddot{\mathrm{R}}-\frac{\ddot{\mathrm{m}}}{\rho_{\infty}}\right)\left[1-\frac{1}{\mathrm{c}}\left(2 \dot{\mathrm{R}}-\frac{\dot{\mathrm{m}}}{\rho_{\infty}}\right)+\frac{1}{\mathrm{c}^{2}}\left(\frac{23}{10} \dot{\mathrm{R}}^{2}-\frac{31}{10} \frac{\dot{\mathrm{~m}} \mathrm{R}}{\rho_{\infty}}-\frac{1}{5} \frac{\dot{\mathrm{~m}}^{2}}{\rho_{\infty}{ }^{2}}\right)\right]+\frac{3}{2}\left(\dot{\mathrm{R}}-\frac{\dot{\mathrm{m}}}{\rho_{\infty}}\right) \\
& {\left[\left(\dot{\mathrm{R}}+\frac{1}{3} \frac{\dot{\mathrm{~m}}}{\rho_{\infty}}\right)-\frac{4}{3} \frac{\dot{\mathrm{R}}^{2}}{\mathrm{c}}+\frac{1}{\mathrm{c}^{2}}\left(\frac{7}{5} \dot{\mathrm{R}}^{3}-\frac{49}{30} \frac{\dot{\mathrm{~m}} \dot{\mathrm{R}}^{2}}{\rho_{\infty}}-\frac{14}{15} \frac{\dot{\mathrm{~m}}^{2} \dot{\mathrm{R}}}{\rho_{\infty}^{2}}-\frac{1}{6} \frac{\dot{\mathrm{~m}}^{3}}{\rho_{\infty}^{3}}\right)\right]+\frac{1}{\rho_{\infty}}\left[\left(\mathrm{P}_{\mathrm{L} \infty}-\mathrm{P}_{\mathrm{L} 2, \mathrm{R}}\right)-\frac{\mathrm{R}}{\mathrm{c}} \frac{\mathrm{dP}_{\mathrm{L} 1, \mathrm{R}}}{\mathrm{dt}}\right.} \\
& \left.\quad+\frac{1}{\mathrm{c}^{2}}\left\{\left(2 \dot{\mathrm{R}}-\frac{\dot{\mathrm{m}}}{\rho_{\infty}}\right) \mathrm{R} \frac{\mathrm{dP}_{\mathrm{L} 1, \mathrm{R}}}{\mathrm{dt}}+\left(\mathrm{P}_{\mathrm{L} \infty}-\mathrm{P}_{\mathrm{L} 1, \mathrm{R}}\right)\left[\frac{1}{2} \dot{\mathrm{R}}^{2}-\frac{3}{2} \frac{\dot{\mathrm{~m}} \dot{\mathrm{R}}}{\rho_{\infty}}-\frac{\dot{\mathrm{m}}^{2}}{\rho_{\infty}}-\frac{\dot{\mathrm{m}}^{2}}{\rho_{\infty}{ }^{2}}+\frac{3}{2} \frac{\mathrm{P}_{\mathrm{L} \infty}-\mathrm{P}_{\mathrm{Ll}, \mathrm{R}}}{\rho_{\infty}}\right]\right\}\right]=0
\end{aligned}
$$

where

$$
\begin{gather*}
P_{\mathrm{L} 1, \mathrm{R}}=\mathrm{P}_{\mathrm{g}}(\mathrm{t})-\frac{2 \sigma}{\mathrm{R}}-\frac{4 \mu}{\mathrm{R}}\left(\dot{\mathrm{R}}-\frac{\dot{\mathrm{m}}}{\rho_{\infty}}\right)-\dot{\mathrm{m}}^{2}\left(\frac{1}{\rho_{\infty}}-\frac{1}{\rho_{\mathrm{g}}}\right)  \tag{19}\\
\mathrm{P}_{\mathrm{L} 2, \mathrm{R}}=\mathrm{P}_{\mathrm{L} 1, \mathrm{R}}+\frac{4 \mu}{3 \mathrm{c}^{2}}\left[\frac{3 \dot{\mathrm{~m}}}{2 \rho_{\infty} \mathrm{R}}\left(\dot{\mathrm{R}}-\frac{\dot{\mathrm{m}}}{\rho_{\infty}}\right)^{2}-\frac{1}{\rho_{\infty}} \frac{\mathrm{dP}_{\mathrm{L} 1, \mathrm{R}}}{\mathrm{dt}}+\frac{\dot{\mathrm{m}}\left(\mathrm{P}_{\mathrm{L} \infty}-\mathrm{P}_{\mathrm{L}, \mathrm{R}}\right)}{\rho_{\infty}^{2} \mathrm{R}}\right]
\end{gather*}
$$

In this model, the pressure and temperature are assumed to be spatially uniform in a bubble as in the adiabatic model. The liquid temperature on the external side of the bubble wall is assumed to be constant ( $\mathrm{T}_{\mathrm{o}}$ ) during bubble oscillations. Thus at the bubble wall, temperature discontinuity ( $\Delta \mathrm{T}=\mathrm{T}-\mathrm{T}_{\mathrm{o}}$ ) exists as in the adiabatic model.

The number of water vapour ( $\mathrm{n}_{\mathrm{H} 2 \mathrm{O}}$ ) changes with time by evaporation and condensation at bubble wall. Mixture of the vapour and air in the bubble is called gas.

$$
\begin{equation*}
\mathrm{n}_{\mathrm{H} 2 \mathrm{O}}(\mathrm{t}+\Delta \mathrm{t})=\mathrm{n}_{\mathrm{H} 2 \mathrm{O}}(\mathrm{t})+4 \pi \mathrm{R}^{2} \dot{\mathrm{~m}} \Delta \mathrm{t} \tag{21}
\end{equation*}
$$

Where $\dot{\mathrm{m}}$ is the net rate of evaporation per unit area and unit time (when, $\dot{\mathrm{m}}\langle 0$, condensation takes place $) . \quad \dot{\mathrm{m}}=\dot{\mathrm{m}}_{\text {eva }}-\dot{\mathrm{m}}_{\text {con }}$

Where

$$
\begin{equation*}
\dot{\mathrm{m}}_{\mathrm{eva}}=\frac{10^{3} \mathrm{~N}_{\mathrm{A}} \alpha_{\mathrm{M}} \mathrm{P}_{\mathrm{v}}^{*}}{\mathrm{M}_{\mathrm{H} 2 \mathrm{O}} \sqrt{2 \pi \mathrm{RvT}_{\mathrm{O}}}} \tag{22}
\end{equation*}
$$

$$
\begin{equation*}
\dot{\mathrm{m}}_{\mathrm{con}}=\frac{10^{3} \mathrm{~N}_{\mathrm{A}} \alpha_{\mathrm{M}} \Gamma \mathrm{P}_{\mathrm{v}}}{\mathrm{M}_{\mathrm{H} 2 \mathrm{O}} \sqrt{2 \pi \mathrm{RvT}}} \tag{24}
\end{equation*}
$$

Eq. (22) means that the net rate of evaporation (the difference between the actual rate of evaporation and that of condensation), $\alpha \mathrm{m}$ is the accommodation coefficient for evaporation or condensation (assumed constant), $\mathrm{P}_{\mathrm{v}}^{*}$ is the saturated vapour pressure at temperature $\mathrm{T}_{\mathrm{o}}$, and $\mathrm{P}_{\mathrm{v}}$ is the actual vapour pressure.

$$
\begin{equation*}
\mathrm{P}_{\mathrm{V}}=\frac{\mathrm{n}_{\mathrm{H} 2 \mathrm{O}}}{\mathrm{nt}} \mathrm{P}_{\mathrm{g}} \tag{25}
\end{equation*}
$$

The correction factor ( $\Gamma$ ) in eq. (24) is expressed as [23]

$$
\begin{equation*}
\Gamma=\exp \left(-\Omega^{2}\right)-\Omega \sqrt{\pi}\left(1-\frac{2}{\sqrt{\pi}} \int_{0}^{\Omega} \exp \left(-x^{2}\right) d x\right) \tag{26}
\end{equation*}
$$

in which

$$
\begin{equation*}
\Omega=\frac{\dot{\mathrm{m}}}{\mathrm{P}_{\mathrm{v}}} \sqrt{\frac{\mathrm{R}_{\mathrm{v}} \mathrm{~T}}{2}} \tag{27}
\end{equation*}
$$

Evaporating vapour molecules carry their own energy into the bubble from the surrounding liquid. Condensing vapour molecules carry their own energy into the surrounding liquid from the bubble. Thus the change of the internal energy of a bubble ( $\Delta \mathrm{E}$ ) is expressed by [24].

$$
\begin{equation*}
\Delta \mathrm{E}(\mathrm{t})=-\mathrm{P}_{\mathrm{g}}(\mathrm{t}) \Delta \mathrm{V}(\mathrm{t})+4 \pi \mathrm{R}^{2} \Delta \mathrm{t}\left(\dot{\mathrm{~m}}_{\mathrm{eva}} \mathrm{e}_{\mathrm{eva}}-\dot{\mathrm{m}}_{\mathrm{con}} \mathrm{e}_{\mathrm{con}}\right) \tag{28}
\end{equation*}
$$

The first term in the right hand side of eq. (28) is the work by pressure $\mathrm{P}_{\mathrm{g}}(\mathrm{t})$. The second term is the energy by evaporating (condensing) vapour molecules.

The energy carried by a condensing vapour molecule $\left(\mathrm{e}_{\mathrm{con}}\right)$ is calculated by

$$
\begin{equation*}
\mathrm{e}_{\mathrm{con}}=\frac{1}{\mathrm{~N}_{\mathrm{A}}} \int_{0}^{\mathrm{T}} \mathrm{C}_{\mathrm{V}, \mathrm{H} 2 \mathrm{O}}\left(\mathrm{~T}^{\prime}\right) \mathrm{dT}^{\prime} \tag{29}
\end{equation*}
$$

and

$$
\begin{equation*}
\mathrm{e}_{\mathrm{eva}}=\frac{1}{\mathrm{~N}_{\mathrm{A}}} \int_{0}^{\mathrm{T}} \mathrm{C}_{\mathrm{V}, \mathrm{H} 2 \mathrm{O}}\left(\mathrm{~T}^{\prime}\right) \mathrm{dT}^{\prime} \tag{30}
\end{equation*}
$$

As an equation of state, van der Waals equation (eq. (4)) is employed as in the adiabatic model. In this model, the van der Waals constant (a and b) varies with time. The temperature inside the bubble ( T ) is calculated by solving eq. (7), the density inside the bubble ( $\rho_{\mathrm{g}}$ ) is calculated by eq. (9) as in the adiabatic model.

## 4- Results and Discussions

Calculations are performed under a condition of $\mathrm{T}_{\mathrm{o}}=20^{\circ} \mathrm{C}$ and $\mathrm{R}_{0}=4.5 \mathrm{~mm}$, where $R_{0}$ is the initial bubble radius. Physical quantities employed in the calculations are listed in Table $1[24,25]$. The frequency and the amplitude of the acoustic field are chosen to be 26.5 kHz and 1.325 bar, respectively. The undisturbed pressure is taken to be $\mathrm{P}_{0}=1$ bar. $\alpha \mathrm{m}$ is chosen to be 0.04 [17]. Calculations start from the time $t=0$ with the initial conditions that:

$$
\mathrm{R}=\mathrm{R}_{\mathrm{o}}, \dot{\mathrm{R}}=0, \frac{\mathrm{dP}_{\mathrm{g}}}{\mathrm{dt}}=0, \dot{\mathrm{~m}}=\ddot{\mathrm{m}}=0, \text { and } \mathrm{P}_{\mathrm{s}}(0)=0
$$

Table 1: Physical quantities employee in calculations

| Bubble | Liquid (water) |
| :--- | :--- |
| $\mathrm{R}_{\mathrm{o}}=4.5(\mu \mathrm{~m})$ | $\mathrm{P}_{\mathrm{v}}=2.3381 \times 10^{3}(\mathrm{~Pa})$ |
| $\mathrm{n}_{\text {air }, \mathrm{o}}=1.23822 \times 10^{10}$ | $\mathrm{c}=1481(\mathrm{~m} / \mathrm{s})$ |
| $\mathrm{n}_{\mathrm{H} 2 \mathrm{O}, \mathrm{o}}=2.20415 \times 10^{8}$ | $\sigma=7.275 \times 10^{-2}(\mathrm{~N} / \mathrm{m})$ |
|  | $\mu=1.002 \times 10^{-3}(\mathrm{~Pa} \cdot \mathrm{~s})$ |
|  | $\rho=998.2\left(\mathrm{~kg} / \mathrm{m}^{3}\right)$ |

### 4.1 Case (1): Adiabatic Model

Under the physical conditions employed in the calculations described above, a periodic solution is obtained by numerical calculations by the adiabatic model. The bubble radius ( R ) is shown in Fig. 1. The pressure inside the bubble $\left(\mathrm{P}_{\mathrm{g}}\right)$ is shown in Fig. 2. Both the radius and pressure change with time periodically with the frequency of the acoustic field. Other physical quantity of the bubble such as temperature $\left(\mathrm{T}_{\mathrm{g}}\right)$ changes with time periodically with the same frequency as shown in Fig. 3. High pressure and temperature occur when the bubble radius becomes minimum.


Fig. 1. The bubble radius $(\mathbb{R})$ as a function of time.


Fig. 2. The pressure inside the bubble $\left(\mathrm{P}_{\mathrm{g}}\right)$ as a function of time with logarithmic vertical axis.


Fig. 3. The temperature inside the bubble ( $\mathrm{T}_{\mathrm{g}}$ ) as a function of time.

### 4.2 Case (2): Model with Evaporation and Condensation

As in the adiabatic model, a periodic solution is obtained by numerical calculations as shown in Figs. (4~11).

In Fig.4, the bubble radius (R) is shown as a function of time. In Fig. 5 (Fig. 6), the pressure (temperature) inside the bubble is shown as a function of time. Other physical quantities of the bubble such as velocity ( R ), partial pressure of water vapour ( $\mathrm{P}_{\mathrm{v}}$ ), and internal energy ( E ) all change with time periodically with the same frequency as shown in Figs. (7~9), respectively. In Fig. 10, the total number of molecules in the bubble ( $n_{t}$ ) and the number of vapour and air molecules ( $\mathrm{n}_{\mathrm{H} 2 \mathrm{O}}$ and $\mathrm{n}_{\text {air }}$ ) are shown with logarithmic vertical axis. The number of molecules ( n ) all change with time periodically with the frequency of the acoustic field (strictly, $\mathrm{n}_{\text {air }}$ is almost constant due to the negligence of the gas diffusion).

From Fig. 10, it is seen that evaporation of water vapour takes place in the expansion phase of bubble oscillations. It is because the partial pressure of water vapour in the bubble $\left(\mathrm{P}_{\mathrm{v}}\right)$ decreases in the expansion phase. On the other hand, at collapses, condensation of water vapour takes place. It is because $P_{v}$ increases at all the collapses.

In Fig. 11, the density of the gas (vapour and air) is shown as a function of time. It becomes high when the bubble radius becomes minimum. It is concluded from the numerical calculations that the partial ${ }_{*}$ pressure of water vapour ( $\mathrm{P}_{\mathrm{v}}$ ) differs considerably from the saturated vapour pressure ( $\mathrm{P}_{\mathrm{v}}^{*}$ ) at collapses of a bubble. On the other hand, in the expansion phase, $\mathrm{P}_{\mathrm{v}} \sim \mathrm{P}_{\mathrm{v}}$ holds well.


Fig. 4. The bubble radius $(\mathbf{R})$ as a function of time.


Fig. 5. The pressure inside the bubble $\left(\mathrm{Pg}_{\mathrm{g}}\right)$ as a function of time with logarithmic vertical axis.


Fig. 6. The temperature inside the bubble ( $\mathrm{Tg}_{\mathrm{g}}$ ) as a function of time.


Fig. 7. The bubble wall velocity $(\dot{\mathbf{R}})$ as a function of time.


Fig. 8. The partial pressure of water vapour $\left(\mathrm{P}_{\mathbf{v}}\right)$ as a function of time with logarithmic vertical axis.


Fig. 9. The internal energy of the bubble (E) as a function of time.


Fig. 10. The number of molecules in the bubble as a function of time with logarithmic vertical axis.


Fig. 11. The density of the gas inside the bubble ( $\rho_{g}$ ) as a function of time.


Fig. 12. Comparison between the calculated result and the experimental data [26] of radius-time curve for acoustic cycle.

### 4.3 Comparison between the adiabatic model and the model with evaporation and condensation

In Figs. (13, 14), the comparisons are shown between the calculated results by the adiabatic model and those by the model with evaporation and condensation (hereafter it is called "the model with EC") for one acoustic cycle. In the model with EC, pressure and temperature are lower from the adiabatic model. The dash line shows the calculated by the adiabatic model, while the line shows those by the model with EC, and the circles show those by the experimental data [26]. In Fig. 12, the bubble radius (R) is shown as a function of time. The comparison is given between the calculated result by the model with EC and the experimental data [26] of radius-time curve for one acoustic cycle. The calculated result by EC fits well with the experimental data.

It is concluded from the figures that the effect of evaporation and condensation for water vapour is considerable on bubble dynamics in acoustic field.


Fig. 13. The pressure inside the bubble $\left(\mathrm{P}_{\mathrm{g}}\right)$ as a function of time with logarithmic vertical axis for one acoustic cycle.


Fig. 14. The temperature inside the bubble ( $\mathrm{T}_{\mathrm{g}}$ ) as a function of time for one acoustic cycle.

## 5- Conclusions

An equation of bubble radius is derived including the effect of evaporation and condensation of water vapour at bubble wall. A new model of bubble dynamics in acoustic field is constructed including this effect. Numerical calculations are performed both by the adiabatic model and the present model.

It is concluded that the effect of evaporation and condensation is considerable on bubble oscillations in acoustic field. It is also concluded that the partial pressure of water vapour ( $\mathrm{P}_{\mathrm{v}}$ ) differs considerably from the saturated vapour pressure ( $\mathrm{P}^{*}{ }_{\mathrm{v}}$ ) at collapses of a bubble. The calculated result of EC fits well with the experimental data by others.

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تأثير التبخير و التكثيف على الحركة اللاخطية للفقاعة
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## لالخلاصتة

تم تشكيل علاقات رياضبـة للنصـرف الفِزبـاوي لفقاعـة في وسط صـوتي. تم إنشـاء نموذج جديد لحركـة الفقاعـة اللاخطية يتضمن تأنيرات انضغاطية السائل و تأثنيراتُ التبخير والنتكيِّف لبخار ألّماء عند سطح الفقاعة. المعادلة الحركية
 العددية لهذا النموذ ج مع النموذ ج الأدبيانيكي (المعزول).
 لبخـار المـاء يماثل ضـغط بخأر النتـبع عدا عند انهيار الفقاعة. عند الانهيارات الشُديدة يكون الضـغط الجزئي للبخـار



