# Study of an Acoustic Bubble Oscillation <br> <br> in Oil Using Linear Wave Equation 

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#### Abstract

This paper presents a mainly theoretical study of the behavior of a bubble in oil in a sound field. An equation for the radial motion, including the effect of the liquid compressibility is presented. The diffusion of gas, and evaporation and condensation at bubble wall are neglected. By using values of the initial bubble radius, the ratio of oscillating pressure amplitude to the static pressure in the liquid, and the angular frequency of the pressure variations, the effects of the viscosity of the liquid on the bubble oscillation are clarified.From the results, the effect of viscosity, a fairly large difference in the waveform of the oscillation is found. The calculations for the bubble oscillation in water are carried out and they are compared with that in oil. It has been clarified that the oscillating bubbles in oil are hard to collapse in comparison with that in water.


Keywords: Acoustic Bubble, Cavitation, Bubble dynamics

## 1- Nomenclature

A: Ratio of oscillating pressure amplitude to static pressure in liquid.
$\mathrm{c}_{\infty}$ : Sound speed in the liquid at infinity ( $\mathrm{m} / \mathrm{s}$ ).
h: Enthalpy (J/kg)
H: Enthalpy at bubble wall (J/kg)
k: Thermal Conductivity (W/m.K)
O: Central point of a bubble.
$\mathrm{P}_{\mathrm{o}}$ : Static pressure in liquid ( Pa ).
$\mathrm{P}(\mathrm{R})$ : Pressure at bubble wall $(\mathrm{Pa})$
$\mathrm{P}_{\mathrm{g}}$ : Gas pressure in a bubble ( Pa )
$\mathrm{P}_{\infty}$ : Pressure of Liquid at infinity ( Pa )
r: Radial distance from point $\mathrm{O}(\mathrm{m})$
Ro: Initial bubble radius ( m )
R : Bubble radius ( m )
$\dot{\mathrm{R}}$ : Bubble wall velocity ( $\mathrm{m} / \mathrm{s}$ )
$\mathrm{R}_{0}$ : Initial bubble radius (m)
t : Time ( s )
T : Temperature inside a bubble (K)
u : Velocity in liquid ( $\mathrm{m} / \mathrm{s}$ )
$\gamma$ : Ratio of specific heats of gas
$\mu$ : Liquid viscosity ( $\mathrm{N} . \mathrm{s} / \mathrm{m}$ )
$\rho$ : Liquid density $\left(\mathrm{kg} / \mathrm{m}^{3}\right)$
$\sigma$ : Surface tension of liquid ( $\mathrm{N} / \mathrm{m}$ )
$\phi$ : Velocity potential in liquid ( $\mathrm{m} / \mathrm{s}$ )
$v$ : Velocity of gas in a bubble ( $\mathrm{m} / \mathrm{s}$ )
$\omega$ : Angular frequency of pressure variation ( $\mathrm{s}^{-1}$ )
$\infty$ : Refers to the condition at great distance from a bubble.

## 2-Introduction

Recently, with the increase in the use of the hydraulic torque converters and oil hydraulic components, the requirements to the structure and performance of this machinery have become more and more strict in the direction for small size, light weight, and high efficiency. So, the study on cavitation in oil is one of the important subjects.

In the present work, therefore, a problem, of bubble oscillation in oil has been talked.
As the theoretical studies of the problem related to the oscillation of cavitation bubbles, there has been a paper of Nolting and Neppiras [1], which numerically obtained the variation of the bubble radius with time in the pulsating pressure field. In 1964, Borotnikova and Soloukhin [2] numerically clarified the effects of the ratio of oscillating pressure amplitude to the static pressure and the surface tension on the bubble oscillation. In these studies, both their analyses assume that the bubble oscillations are adiabatic. While, under the consideration that the gas in bubble changes isothermally, Solomon and Plesset [3] have theoretically analyzed about the bubble oscillation.

The difference between the case for an adiabatic change of the gas in a bubble and the case for an isothermal change was numerically given by Flynn [4], however, frequencies used for the calculations of his analysis were considerable high. In these studies [1-4], none take into consideration the viscosity of liquid. In 1990, Arrigo et al. [5] studied the motion of a planar consisting of an adiabatic gas surrounded by a liquid. Effects of viscosity of the liquid and different state equation were considered. In 1991, Prosperetti [6] discussed several aspects of the oscillation of a gas bubble in a slightly compressible liquid by means of a simplified model based on the assumption of a spatially uniform internal pressure.

In 1998, Niederdrank et al. [7] used Keller-Miksis model with heat transfer only to study the effect of the ambient liquid temperature and the initial bubble radius on radial oscillation of a bubble under acoustic field. In 2001, Yasui [8] studied oscillation of an argon bubble in water. He has used the modified Keller-Miksis model in ref. [9]. In this study, the effect of non-equilibrium evaporation-condensation, thermal conduction both inside and outside the bubble and liquid compressibility are taken into account. The calculations showed that the maximum bubble radius increases with increasing the ambient liquid temperature.

At least, when one treats the bubble oscillation in oil, it will be necessary to take into consideration the compressibility, the viscosity, and the surface tension of the liquid.

In the present work, therefore, the theoretical analysis has been made for the motion of the bubble in oil when harmonic pressure oscillations are imposed, by considering the compressibility, the viscosity, and the surface tension of the liquid. With further assumption that the temperature at the bubble wall is equal to the liquid temperature and that air in the bubble behaves according to the perfect gas laws. Here, the mass transfer (diffusion of gas, and evaporation and condensation) at the bubble wall is neglected. And then using a computer has carried out the numerical calculations for the change in time of the bubble radius. The effect of the angular frequency and amplitude of the pressure variations, the initial bubble radius, and the viscosity of liquid on the change in time of the bubble radius were found. As it was clarified in the calculated range, when the initial radius is small the oscillations become more violent, that no account of the viscosity of liquid the waveform of oscillation become smooth and the bubble collapse was hard to yield.

The calculations for the bubble oscillation in water are carried out and they are compared with that in oil.

## 3- Mathematical Description

### 3.1 The Radial Equation of Motion

As it is shown in Fig. A, we assume that a spherical gas bubble of the initial radius Ro with uniform interior at a point O in oil. Let a distance from point O to an arbitrary point denotes as r. Moreover, we shall introduce the following assumptions:
(i) The liquid is compressible.
(ii) The effect of gravity is neglected.
(iii) The gas density in a bubble is very small as compared with it in liquid.
(iv) The diffusion of gas, and evaporation-condensation at bubble wall is negligible.
(v) The shape of bubble keeps to the spherical symmetry.


Fig. A. A spherical bubble in oil
Under these assumptions, the equation of bubble motion is derived by the following equations. We begin by discussing the basis of these equations, which will serve to derive an equation of motion of fundamental importance in the following.

We consider a single spherical bubble which is isolated in a liquid that extends to infinity. The starting points are the equations of conservation of mass and momentum of the liquid, which we write as [10]

$$
\begin{align*}
& \frac{1}{\rho}\left(\frac{\partial \rho}{\partial t}+\frac{\partial \rho}{\partial r}\right)+\nabla \cdot u=0  \tag{1}\\
& \frac{\partial u}{\partial t}+u \frac{\partial u}{\partial r}=-\frac{1}{\rho} \frac{\partial \rho}{\partial r} \tag{2}
\end{align*}
$$

Here u is the velocity directed along the radial direction, $\rho$ and P denote density and pressure, respectively, and r is the distance measured from the center of the spherical bubble. With the assumption of isentropic motion in the liquid we may write [11].

$$
\begin{align*}
\mathrm{d} \rho & =\mathrm{c}^{-2} \mathrm{dP}  \tag{3}\\
\mathrm{dh} & =\rho^{-1} \mathrm{dP} \tag{4}
\end{align*}
$$

We may also introduce a velocity potential $\phi$ such that $u=\nabla \phi$. In terms of these quantities the preceding equations may be written,

$$
\begin{align*}
& \nabla^{2} \phi+\frac{1}{\mathrm{c}^{2}}\left(\frac{\partial \mathrm{~h}}{\partial \mathrm{t}}+\frac{\partial \phi}{\partial \mathrm{r}} \frac{\partial \mathrm{~h}}{\partial \mathrm{r}}\right)=0  \tag{5}\\
& \frac{\partial \phi}{\partial \mathrm{t}}+\frac{1}{2}(\nabla \phi)^{2}+\mathrm{h}=0 \tag{6}
\end{align*}
$$

The second equation was obtained by integrating eq. (2) and the constant of integration set to zero, thus implicitly assuming that $\phi$ tends to zero at large distances from the bubble and that the enthalpy is referred to its value at infinity. These equations can be simplified if the speed of sound in the liquid is assumed to be large. Indeed we note that, by a Taylor series expansion [11].

$$
\begin{equation*}
\mathrm{h}=\int_{\mathrm{P}_{\infty}}^{\mathrm{P}} \rho^{-1} \mathrm{dP}=\int_{\mathrm{P}_{\infty}}^{\mathrm{P}}\left(\frac{1}{\rho_{\infty}}-\frac{\mathrm{P}^{\prime}-\mathrm{P}_{\infty}}{\rho_{\infty}{ }^{2} \mathrm{c}_{\infty}{ }^{2}}+\ldots .\right) \mathrm{dP}^{\prime} \tag{7}
\end{equation*}
$$

Then

$$
\begin{equation*}
\mathrm{h}=\frac{\mathrm{P}-\mathrm{P}_{\infty}}{\rho_{\infty}}-\frac{1}{2} \mathrm{c}_{\infty}{ }^{-2}\left(\frac{\mathrm{P}-\mathrm{P}_{\infty}}{\rho_{\infty}}\right)^{2}+\mathrm{O}\left(\mathrm{c}_{\infty}{ }^{-4}\right) \tag{8}
\end{equation*}
$$

and similarly,

$$
\begin{align*}
\mathrm{c}^{-2} & =\mathrm{c}_{\infty}^{-2}+\left(\mathrm{P}-\mathrm{P}_{\infty}\right) \frac{\mathrm{dc}^{-2}}{\mathrm{dP}}+\ldots \\
& =\mathrm{c}_{\infty}^{-2}-\frac{\mathrm{P}-\mathrm{P}_{\infty}}{\mathrm{c}_{\infty}^{4}} \frac{\mathrm{dc}^{2}}{\mathrm{dp}}+\ldots . \tag{9}
\end{align*}
$$

Where $c_{\infty}, P_{\infty}, \rho_{\infty}$ are the static values of the speed of sound, pressure, and density at large distances from the bubble. Correct to order $\mathrm{c}_{\infty}$ the preceding equations can therefore be
written as:

$$
\begin{gather*}
\nabla^{2} \phi+\frac{1}{\rho_{\infty} \mathrm{c}_{\infty}}\left(\frac{\partial \mathrm{P}}{\partial \mathrm{t}}+\frac{\partial \phi}{\partial \mathrm{r}} \frac{\partial \mathrm{P}}{\partial \mathrm{r}}\right)=0  \tag{10}\\
\frac{\partial \phi}{\partial \mathrm{t}}+\frac{1}{2}(\nabla \phi)^{2}+\frac{\mathrm{P}-\mathrm{P}_{\infty}}{\rho_{\infty}}\left(1-\frac{\mathrm{P}-\mathrm{P}_{\infty}}{2 \rho_{\infty} \mathrm{c}_{\infty}^{2}}\right)=0 \tag{11}
\end{gather*}
$$

We can now consider the limiting form of these equations near and far from the bubble. In the former limit, the finite speed of propagation of the signals are unimportant. Thus the appropriate form for the near field is
and

$$
\begin{equation*}
\frac{\partial \phi}{\phi \mathrm{t}}+\frac{1}{2}(\nabla \phi)^{2}+\frac{\mathrm{P}-\mathrm{P}_{\infty}}{\rho_{\infty}}=0 \tag{12}
\end{equation*}
$$

that is, the customary incompressible formulation. Far from the bubble, on the other hand, one may expect that all perturbations are small so that a linear model is adequate. Thus, for the far field, we are led to:

$$
\begin{align*}
& \nabla^{2} \phi+\frac{1}{\rho_{\infty} c_{\infty}^{2}} \frac{\partial \mathrm{P}}{\partial \mathrm{t}}=0  \tag{14}\\
& \frac{\partial \phi}{\partial \mathrm{t}}+\frac{\mathrm{P}-\mathrm{P}_{\infty}}{\mathrm{P}_{\infty}}=0 \tag{15}
\end{align*}
$$

From which one can obtain $\quad \nabla^{2} \phi-\frac{1}{\mathrm{c}_{\infty}^{2}} \frac{\partial^{2} \phi}{\partial \mathrm{t}^{2}}=0$
This is called linear Wave Equation. An approximate model, valid simultaneously in the near and far-field, can be obtained by using the linear wave equation eq. (16) together with the incompressible Bernoulli integral (eq. (11)). Indeed, eq. (16) is correct in the far field, and its difference from eq. (12) in the near field is negligible. Similarly, eq. (11) differs from eq. (15) by the presence of the terms $(\nabla \phi)^{2}$, which is small in the far field. This mathematical formulation was first proposed by Keller and Kolodner [12], and subsequently resumed and generalized by Keller and Miksis [13]. Using then eq. (11) and eq. (16) and following the same procedure used by Keller in the papers referred to, the following equation of motion for the radial oscillations of a bubble of instantaneous radius $\mathrm{R}(\mathrm{t})$ is obtained

$$
\begin{equation*}
R \ddot{R}\left(1-\frac{\dot{R}}{c}\right)+\frac{3}{2} \dot{R}^{2}\left(1-\frac{\dot{R}}{3 c}\right)=\left(1+\frac{\dot{R}}{c}\right) \frac{P(R, t)-P_{s}\left(t+\frac{R}{c}\right)}{\rho}+\frac{R}{\rho c} \frac{d P(R, t)}{d t} \tag{17}
\end{equation*}
$$

where

$$
\begin{equation*}
\mathrm{P}_{\mathrm{s}}(\mathrm{t})=\mathrm{P}_{\mathrm{O}}(1+\mathrm{A} \sin \omega \mathrm{t}) \tag{18}
\end{equation*}
$$

Here $\mathrm{P}_{\mathrm{s}}(\mathrm{t})$ is the static pressure plus the pressure of the sound field driving the oscillations of the bubble. Dots denote differentiation with respect to time, and the subscript $(\infty)$ has been dropped from $\rho$ and $c$. The liquid pressure just outside the bubble, $P(R, t)$ is connected to the pressure $\mathrm{P}_{\mathrm{g}}(\mathrm{R}, \mathrm{t})$ acting on the inner side of the bubble surface by the condition on the normal stresses. In the incompressible Limit ( $\mathrm{c} \rightarrow \infty$ ) and eq. (17) becomes the well-known equation:

$$
\begin{equation*}
\mathrm{R} \ddot{\mathrm{R}}+\frac{3}{2} \dot{\mathrm{R}}^{2}=\frac{1}{\rho}\left[\mathrm{P}_{\mathrm{g}}(\mathrm{R}, \mathrm{t})-\mathrm{P}_{\mathrm{s}}(\mathrm{t})-\frac{2 \sigma}{\mathrm{R}}-4 \mu \frac{\dot{\mathrm{R}}}{\mathrm{R}}\right] \tag{20}
\end{equation*}
$$

### 3.2 The Bubble Interior

The mathematical model for the bubble interior is described in detail in Prosperetti et al. [14], Kamath and Prosperetti [15] and Prosperetti [16]. The model accounts for the compressibility of the gas and heat transport by convection and by conduction inside the bubble. The main assumptions, discussed below, are those of perfect-gas behavior and of spatial uniformity of the gas pressure. The internal pressure $\mathrm{P}_{\mathrm{g}}$ is found by integrating

$$
\begin{equation*}
\frac{\mathrm{dP}_{\mathrm{g}}}{\mathrm{dt}}=\frac{3}{\mathrm{R}}\left[\left.(\gamma-1) \mathrm{k} \frac{\partial \mathrm{~T}}{\partial \mathrm{r}}\right|_{\mathrm{r}=\mathrm{R}}-\gamma \mathrm{P}_{\mathrm{g}} \dot{\mathrm{R}}\right] \tag{21}
\end{equation*}
$$

where $\gamma$ is the ratio of the specific heats of the gas and k is the gas thermal conductivity. The gas temperature field. $\mathrm{T}(\mathrm{r}, \mathrm{t})$ is obtained from

$$
\begin{equation*}
\frac{\gamma}{\gamma-1} \frac{\mathrm{P}_{\mathrm{g}}}{\mathrm{~T}}\left(\frac{\partial \mathrm{~T}}{\partial \mathrm{t}}+v \frac{\partial \mathrm{~T}}{\partial \mathrm{r}}\right)=\frac{\mathrm{dP}_{\mathrm{g}}}{\mathrm{dt}}+\frac{1}{\mathrm{r}^{2}} \frac{\partial}{\partial \mathrm{r}}\left(\mathrm{kr}^{2} \frac{\partial \mathrm{~T}}{\partial \mathrm{r}}\right) \tag{22}
\end{equation*}
$$

with the following expression for the velocity field [15].

$$
\begin{equation*}
v=\frac{1}{\gamma \mathrm{P}_{\mathrm{g}}}\left[(\gamma-1) \mathrm{k} \frac{\partial \mathrm{~T}}{\partial \mathrm{r}}-\frac{1}{3} \mathrm{r} \frac{\mathrm{dP}_{\mathrm{g}}}{\mathrm{dt}}\right] \tag{23}
\end{equation*}
$$

In order to avoid dealing with the energy equation in the liquid, the gas energy equation (eq. (22)) is solved assuming that the liquid temperature at the surface of the bubble remains undisturbed (constant). In Kamath et al. [15] it was shown that the surface temperature changes had only a minor effect. If liquid temperatures at the bubble wall rise considerable, phase-change effects should be modelled.

In deriving eq. (22) the gas specific heat is assumed to be constant. Since the specific heat of gases in an increasing function of temperature, this approximation will tend to overpredic the gas temperature somewhat. For the temperature dependence of the gas thermal conductivity ( k ), we use the kinetic-theory result [17].

$$
\begin{equation*}
\mathrm{k}=5.39 \times 10^{-5} \mathrm{~T}+0.0108 \tag{24}
\end{equation*}
$$

Where k is in $\mathrm{W} / \mathrm{m} . \mathrm{K}$ and T in K . As for the assumption of spatially uniform pressure, it of course rules out the presence of shock waves. Therefore, eq. (17) is solved numerically using Runge-Kutta method [18] coupling with the energy equation of the gas inside the bubble eq. (22), which is solved using finite difference method [19].

## 4. Results and Discussions

Using silicone oil as a liquid and air as the gas in the bubble, the calculations, which the bubble radius varies with time, have been carried out (where, physical properties of silicone oil show in Table 1 [20]). The static pressure in liquid is $\mathrm{P}_{\mathrm{o}}=1.013$ bar and the liquid temperature is $20^{\circ} \mathrm{C}$.

Table 1. Physical properties of Liquids at $20{ }^{\circ} \mathrm{C}$

| Liquids | $\boldsymbol{\rho}\left(\mathbf{k g} / \mathbf{m}^{\mathbf{3}}\right)$ | $\boldsymbol{\mu}\left(\mathbf{N} . \mathbf{s} / \mathbf{m}^{2}\right)$ | $\boldsymbol{\sigma}(\mathbf{N} / \mathbf{m})$ |
| :---: | :---: | :---: | :---: |
| Oil | 969.97 | $4.8513 \times 10^{-1}$ | $2.1104 \times 10^{-2}$ |
| Water | 982 | $1.01 \times 10^{-3}$ | $7.28 \times 10^{-2}$ |

The initial bubble radius was chosen to be $R_{o}=1,0.1$ and 0.01 mm , the ratio of oscillating pressure amplitude to static pressure in liquid was $\mathrm{A}=0.2,0.5,1$ and 2 , the angular frequency of the pressure variation was $\omega=0.2 \times 10^{5}, 0.5 \times 10^{5}$, and $1 \times 10^{5} \mathrm{~s}^{-1}$, the ratio of specific heat was $\gamma=1.4$, and the speed of sound in the liquids oil and water were 1012.4 and $1481 \mathrm{~m} / \mathrm{s}$, respectively [20]. The effects of $\mathrm{R}_{\mathrm{o}}, \mathrm{A}, \omega$, and $\mu$ on the changes in time of the bubble radius have been numerically clarified. Also, to compare with the case of water, the calculations were similarly carried out for the case where a bubble oscillating is in water.

### 4.1 The Effect of the Ratio of Oscillating Pressure Amplitude to the Static Pressure

The effects of the ratio of oscillating pressure amplitude to the static pressure, A, on the change in time of the bubble radius are shown in Figs. 1-3.

In each figure, $\omega$ is $0.5 \times 10^{5} \mathrm{~s}^{-1}$ and Figs. 1,2, and 3 represent for the cases of $R_{0}=1$, 0.1 and 0.01 mm , respectively. As shown in these figures, the larger the value of $A$ is the larger the amplitude of bubble oscillation, for example for the cases where $A=1$ at $R_{0}=0.1$ mm (Fig. 2) the bubble collapses are found. While, when $\mathrm{R}_{\mathrm{o}}=1 \mathrm{~mm}$, the effects of A are not much larger than that for $\mathrm{R}_{0}=0.1$ and 0.01 mm .


Fig. 1. Effect of A on relation between bubble radius and time.


Fig. 2. Effect of A on relation between bubble radius and time.


Fig. 3. Effect of A on relation between bubble radius and time.

### 4.2 The Effect of the Angular Frequency of Pressure Variations

The effects of the angular frequency $\omega$ on the bubble radius variations with time are shown in Figs. 4~6. In these figures, as $\mathrm{A}=0.5$, Figs. 4, 5 , and 7 are for the cases of $\mathrm{Ro}=1$, 0.1 and 0.01 mm , and $\omega$ correspond to $0.2 \times 10^{5}, 0.5 \times 10^{5}$, and $1 \times 10^{5} \mathrm{~s}^{-1}$.

It is found from these figures that the larger value of $\omega$ becomes, the larger the frequency of bubble motion becomes, and that for the cases of $\mathrm{R}_{0}=0.1$ and 0.01 mm (Figs. 5 and 6) the bubble oscillations have an adequate periodic character. On the other hand, when $\mathrm{R}_{0}=1 \mathrm{~mm}$ (Fig, 4) according as $\omega$ increase, the amplitude of bubble oscillation becomes small.


Fig. 4. Effect of $\boldsymbol{\omega}$ on relation between bubble radius and time.


Fig. 5. Effect of $\boldsymbol{\omega}$ on relation between bubble radius and time.


Fig. 6. Effect of $\boldsymbol{\omega}$ on relation between bubble radius and time.

### 4.3 The Effect of Initial Bubble Radius

The effects of the initial bubble radius $\mathrm{R}_{0}$ on the change in time of the bubble radius are shown in Figs. 7~8. In each figure, as $\omega=0.5 \times 10^{5} \mathrm{~s}^{-1}$, and Fig. 7 is for $\mathrm{A}=0.5$ and Fig. 8 for the case of $A=1$. Also, in these figures indicate for the cases of $R_{0}=1,0.1,0.01 \mathrm{~mm}$. As shown in these figures, when $R_{0}=0.1 \mathrm{~mm}$ the effect of $R_{0}$ on the bubble oscillation strongly works, and the violent change of the bubble radius occurs.

In particular, for the case where $A=1$ at $\mathrm{R}_{0}=0.1 \mathrm{~mm}$ (Fig. 8) the bubble collapses. For the case of the large bubble as $\mathrm{R}_{0}=1 \mathrm{~mm}$ the oscillating waveform becomes smooth. And for the case of the small bubble as $\mathrm{R}_{\mathrm{o}}=0.01 \mathrm{~mm}$, the oscillating amplitude is fairly larger than for the case of $\mathrm{R}_{0}=1 \mathrm{~mm}$, and then there is an indicate of more periodic variation.


Fig. 7. Effect of $\mathrm{R}_{\mathrm{O}}$ on relation between bubble radius and time.


Fig. 8. Effect of $R_{0}$ on relation between bubble radius and time.

### 4.4. The Effect of Viscosity

The effects of viscosity on the change in time of the bubble radius are shown in Figs. $9 \sim 15$. Figs. 9,10 and 11 show for the case of $R_{0}=1 \mathrm{~mm}$, and in these figures are for $A=0.5$, $\omega=1 \times 10^{5} \mathrm{~s}^{-1}$, for $A=0.5, \omega=0.5 \times 10^{5} \mathrm{~s}^{-1}$, and for $A=1, \omega=0.5 \times 10^{5} \mathrm{~s}^{-1}$, respectively.

Figs. 12, 13, 14, and 15 show for the case of $R_{0}=0.1 \mathrm{~mm}$, and in these figure are for $A=$ $0.5, \omega=0.2 \times 10^{5} \mathrm{~s}^{-1}$, for $\mathrm{A}=0.5, \omega=0.5 \times 10^{5} \mathrm{~s}^{-1}$, for $\mathrm{A}=0.5, \omega=1 \times 10^{5} \mathrm{~s}^{-1}$ and for $A=1, \omega=0.5 \times 10^{5} \mathrm{~s}^{-1}$, respectively. In these figures, the solid lines and the dotted lines stand for the cases with the viscosity and without, respectively.

In general, the damping effect appears on the bubble oscillations because of the effect of viscosity, and this effect was fairly notably yield in the cases of $R_{0}=0.1 \mathrm{~mm}$. That is, by dint of the viscosity, the bubble oscillations become smoother.

And yet, for Fig. 14, it can be found that if the effect of viscosity is included, the bubble oscillates periodically and that unless its effect is included, the bubble collapses and due to the effect of viscosity the oscillating waveforms become to quite differ

Also, for Fig. 15, the bubble collapses, but the collapsed time of the bubble without the viscosity becomes fairly short compared with it with the effect.

For the case where the initial radius is large as the case of $R_{0}=1 \mathrm{~mm}$, the effect of viscosity on the bubble oscillations does not almost appears (Fig. 9).


Fig. 9. Effect of $\boldsymbol{\mu}$ on relation between bubble radius and time.



Fig. 11. Effect of $\boldsymbol{\mu}$ on relation between bubble radius and time.


Fig. 12. Effect of $\boldsymbol{\mu}$ on relation between bubble radius and time.


Fig. 13. Effect of $\boldsymbol{\mu}$ on relation between bubble radius and time.


Fig. 14. Effect of $\boldsymbol{\mu}$ on relation between bubble radius and time.


### 4.5 Comparison with Bubble Oscillation in Water

The comparisons with the calculated result of which the bubble oscillates in water are shown in Figs. 16-18.

Figs. 16 and 17 are for $\mathrm{R}_{0}=0.1 \mathrm{~mm}$, Fig. 18 for $\mathrm{Ro}=0.01 \mathrm{~mm}$, and Figs. 16, 17 and 18 represent for $\mathrm{A}=0.5, \omega=0.5 \times 10^{5} \mathrm{~s}^{-1}$, for $\mathrm{A}=1, \omega=0.5 \times 10^{5} \mathrm{~s}^{-1}$, and for $\mathrm{A}=1, \omega=0.5 \times 10^{5}$ $\mathrm{s}^{-1}$, respectively. In the figures the solid lines and the dotted lines denote the cases for oil and water, respectively.

According to these comparisons, it is found that the oscillating amplitude for the case of oil becomes smaller than for the case of water.

Yet, when the bubbles in oil and water collapse as shown in Fig.17, and when the oscillations are either periodic in both cases of oil and water, the changes in time of the bubble radius are similar.

But there is an indication for the notable difference that for the case of oil as in Fig. 18 the bubble oscillations have the periodic character, and for the case of water the bubble collapses.


Fig. 16. Comparison of the calculated result with case of water.


5. Conclusibits ${ }^{18 .}$ Comparison of the calculated result with case of water.

The calculations of the relation between the bubble radius and the time elapsed have been carried out, and the effects of $R_{0}, A, \omega$, and the viscosity of liquid, $\mu$, were clarified, and further, from the comparison with the case of the bubble oscillations in water, the following results were obtained in the calculated range.
a. The larger A becomes, the larger the amplitude of the bubble oscillations becomes, in particular, when $R_{0}$ is small and $A=1$, the bubble collapses.
b. The frequency of the bubble oscillations becomes large as $\omega$ becomes large. Also, when the initial bubble radius is large, for example as $\mathrm{R}_{0}=1 \mathrm{~mm}$, the amplitude of the bubble oscillations becomes small with increasing of $\omega$.
c. When $R_{0}=0.1 \mathrm{~mm}$, the bubble oscillations are most violent and the bubble is easy to collapse.
d. The effect of the viscosity notably appears in the case for $\mathrm{R}_{0}=0.1$ and 0.01 mm , then, the oscillating waveform becomes smooth and the bubble is hard to collapse.
e. For the case of the bubble oscillations in oil, as compared with the case of water, the oscillating waveform becomes smooth and the bubble is hard to collapse.

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قسم الهنسسة الميكانيكية - كلية المنسسة - الأسدي


#### Abstract

الخلاصتة يقدم هذا البحث بشكل رئبسي دراسـة نظربـة لتصبرف فقاعة في زيت في وسط صوتي. تم نقديم معادلة لحركـة  قيم لقطر الفقاعة الابتدائي ونسبة سعة ضـغط الموجة الصوتية إلى ضـنـط النـائلِ المحيط وتردد الموجـة الصونية، تم توضيح تأثير لزوجة السائل على تذبذب الفقاعة. توضـح هذه الدراسة أن اللزوجة لها تأثئبر كبير على تذبذب الفقاعة في  تذبنب الفقاعات في الزيت حاد بالمقارنة مع تذبذبها في الماء.


