# Numerical Calculation of the Temperature Distribution In Hot Burning Plasma 

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ISSN -1817-2695
Received 26/3/2006, Accepted 27/8/2006


#### Abstract

The influence of the laser beam focusing condition on the efficiency of the localized interaction with the gas indicated and the method proposed for increasing this efficiency in the presence of hot burning plasma are studied. The intensity distribution of the laser radiation in the focal plane and in the adjacent region of the laser beam is determined. Our calculations depended on the solution of two-dimensional Navier-Stokes equations together with paraxial ray equation; ionisation equation and the energy equation. A finite differences Algorithm employed to solve these equations. The active area divided into 3321 non-uniform mesh. All calculation depends on the real gas properties including the absorption coefficient. Some results will be present in this paper.


Keyword:L aser produce plasma; Optical discharge; Plasma

## INTRODUCTION

The Interaction between the laser beams with a gas is in the intensity range $10^{16}-10^{17}$ $\mathrm{w} / \mathrm{cm}^{2}$ which has been explored theoretically and experimentally, see for example Roda [1], Jaanimagi [2], Tabak [3], Shargee [4], Al-Kelly [5], Al-Hashmiy [6-8]. Some of these studies show that at laser intensities $3 \times 10^{16} \mathrm{w} / \mathrm{cm}^{2}$ and in the focussing conditions of the experiments, the laser absorption is about $30 \%$ and divided between resonance absorption, enhanced collision absorption or the parametric decay instability and the inverse bremsstrahlung mechanism [9-12]. Based on the previous study, we can assume that the total energy of electron comprises a few percent of absorbed energy. The actual energy distribution of these electrons will depend on the assumption about the mechanisms by which there generate. In addition, the interaction of the intense cw laser radiation with a gas medium may accompanied by the formation of an optical discharge, which is sustained by the power in the laser beam. Even in the first studies in which this phenomenon was observed Locke etal. [13], it notes that the presence of such plasma in the beam could substantially reduce the fraction of the power pulse incident on the gas. Interaction with cw radiation as a rule involves considerably less intense beams and performs under constant pressure conditions. This must strongly affect the parameter of the plasma and consequently the nature of its influence on the interaction. In a cw laser beam, steady-state optical discharge plasma can exist in the form of euro-jet plasma of the ionised gases. It is of great practical interest to investigate the distribution of the temperature in such plasma and to investigate the distribution of the laser power absorbed by the gas in the presence of optical discharge since it is the possibility concentrating the laser power in a small interaction zone. The present paper reports an investigation of the laser beam intensity distribution in the laser beam focus together with temperature distribution in this region where the hot plasma created by the
absorption of laser. This investigation stands as a core element for laser driven space rocket in near future application.

## THEORY

Laser thermal propulsion promises to deliver higher specific impulse than be attained with conventional chemical propellants, Molvik etal. [14]. So one who likes to attack this problem must be aware of different kind of theoretical background such as, the flow through the rocket nozzle, the intensity distribution of laser beam, the factor influence the cascade ionisation as well as the real gas properties. To visualize our discussion we will divide the theory of this problem into three parts as follows:

## a) Part one: theory of the laser beam intensity distribution

In order to compute the intensity distribution of the laser beam throughout the focal point for conditions that conform closely to experiment, the spherical aberrations must be taken into account. Most investigations that use lasers, the high intensities required for breakdown have been attained with simple lenses having relatively focal length of the order of ( 2 to 10 ) cm . Such lenses used in connection with lasers having active elements of about 1 cm in diameter produce distorted wave fronts with values of primary spherical aberration function of several wavelengths. The value of the intensity at any point in the region of the focus of a given lens and laser system may compute from the well-known Haygens-FresenelKirchoff equation. For a single mode laser giving a circular beam of a radius a with a Gaussian electric field intensity profile of the form:
$\mathrm{E}(\mathrm{r})=\mathrm{E}_{0} \exp \left(-\mathrm{r}^{2} / \mathrm{R}^{2}\right)$ for $0 \leq \mathrm{r} \leq \mathrm{a}$
Where R is the lens aperture and a is the beam radius. The spatial variation of intensity modified to include the effect of the Gaussian intensity distribution and Linfoot and Wolf [15] give the lens aberration:
$I(P)=\left[\frac{A_{o} a^{2}}{\lambda f^{2}}\right]^{2}\left|\int_{0}^{2 \pi} \int_{0}^{1} \exp \left(-\frac{\rho^{2} a^{2}}{R^{2}}\right) \exp \left[-i\left[-k \phi(\rho)+v \rho \cos \theta+\frac{u \rho^{2}}{2}\right]\right] \rho d \rho d \theta\right|^{2}$.
where $\rho=\frac{r}{a} ; \ldots . . v=\frac{2 \pi}{\lambda}\left(\frac{a}{f}\right) r ; \ldots \ldots . . k=\frac{2 \pi}{\lambda}$
$A_{0}$ is proportional to the maximum amplitude of the wave on the beam axis, $f$ is the focal length and $\Phi(\rho)$ is the primary spherical aberration function given by:
$\phi(\rho)=-\frac{(a \rho)^{4} P^{3}}{32}\left[\frac{n^{2}}{(n-1)}-\frac{n}{n+2}+\frac{4(n+1)^{2}}{n(n+2)}\right]$
For the purpose of this work equation (2) will be expressed in terms of the real coordinates $(\mathrm{r}, \mathrm{z})$. So, by taking into account that $\rho=\mathrm{r} / \mathrm{a}$, equation (2) become.
$I(r, z)=\left.\left(\frac{6.34 \pi P}{\lambda^{2} f^{2} a^{2}}\right) \int_{0}^{a} \exp \left(-\frac{r^{2}}{2 a^{2}}\right) \exp \left(i\left[-\frac{1}{4} k B r^{4}-\frac{1}{2} \Gamma_{1} r^{2} z\right]\right) J_{0}\left(\Gamma_{2} r^{2}\right) r d r\right|^{2}$
where $\Gamma_{1}=\frac{2 \pi}{f^{2} \lambda} ; \Gamma_{2}=\frac{2 \pi}{f \lambda}$
and $J_{0}$ is the first order Bessel's function

## b) Part two: Theory of the plasma distribution

In optical discharges, ionisation cause by collision of free electrons with the gas atoms. The electrons gain their energies from the laser beam by collisions, which change their ordered oscillatory motion to random motion. However, in contrast, various loss mechanisms contribute to a reduction in electron concentration. A steady state discharge occurs when the production of new charged particles in the volume equals their loss to the volume by any of the loss processes. In order to formulate this work, we shall make the following assumption: 1) A sufficiently strong field is applied which is necessary for the electron to excite the rapidly ionised by the absorption of a few photons. However if the field is not strong enough to
provide rapid ionisation of the excited atom, the energy lost by the electron to the excitation hinders the development of the cascade.
2) The discharge is a quasi-neutral in which $N_{e} \approx N_{i}$ where $N_{e}$ and $N_{i}$ stand for electron and ions densities.
3) The degree of ionisation $N_{e} / \mathrm{N}$ is small compared to unity so that the conservation equations for the neutral species may be uncoupled from those of the charge particles.
4) Diffusion controlled discharge will be considered, while the recombination is assumed to take place at out let of pipe only.
5) Any internal drift velocity may occur, which will incorporate into an effective diffusion coefficient.
According to the above assumptions and taking into account the fact that steady state discharge occurs when the production of new charge particles in the volume equal these losses to the volume by any of the loss processes, and if we consider the only given term is the cascade ionisation, so that the ionisation equation take the form;
$\frac{1}{L^{2}} \frac{\partial^{2} n}{\partial z^{2}}+\frac{1}{R_{0}^{2}} \frac{\partial^{2} n}{\partial r^{2}}+\frac{1}{R_{0}^{2}} \frac{1}{r} \frac{\partial n}{\partial r}-\frac{U_{z}}{L D_{a}} \frac{\partial n}{\partial z}+\frac{\alpha_{c}}{D_{a}} n=0$
Where $L, R_{0}, D_{a}, U_{z}$ and $\alpha$, are the cylinder length, radius, diffusion length, velocity and ionization coefficient.

## c) Part three Theory of the Gas Dynamics

To discuss the interaction between the laser beam and the flowing gas, it is necessary to take into account the hydrodynamic equations of flow. These are continuity equation and Navier-stock momentum equation. In order to study the temperature distribution of the gas due to the laser beam heating, the energy equation must take into account. In the connection with all of these equations, the state equation of the gas will also consider. A collection of these equations will give below;

1) For steady state, the continuity equation in axi-symmetric cylindrical coordinates takes the form:
$\frac{\partial\left(\rho v_{z}\right)}{\partial z}+\frac{1}{r} \frac{\partial\left(\rho v_{r}\right)}{\partial r}=0$
) The momentum equations (Navier-Stocks equations) of motion in cylindrical coordinates, for steady state and axi-symmetrical flow, neglecting the components of body force which look like:
$\frac{\partial}{\partial z}\left(\rho v_{r} v_{z}-\frac{4 \mu}{3} \frac{\partial v_{z}}{\partial z}\right)+\frac{1}{r} \frac{\partial}{\partial r}\left(r \rho v_{r} v_{z}-\mu r \frac{\partial v_{z}}{\partial z}\right)=-\frac{\partial p}{\partial z}+S_{1}$
and
$\frac{\partial}{\partial z}\left(\rho v_{r} v_{z}-\mu \frac{\partial v_{r}}{\partial z}\right)-\frac{1}{r} \frac{\partial}{\partial r}\left(r \rho v_{r} v_{z}-\frac{4 \mu r}{3} \frac{\partial v_{r}}{\partial r}\right)=-\frac{\partial p}{\partial r}+S_{2}$
Where $v_{r}, v_{z}$ are the velocity in r and z directions respectively, $\mu$ is the gas viscosity, $\rho$ is the gas density, p is the gas pressure and $\mathrm{S}_{1}, \mathrm{~S}_{2}$ are the source term in numerical calculation with

$$
\begin{equation*}
S_{1}=\frac{1}{r} \frac{\partial}{\partial r}\left(\mu r \frac{\partial v_{r}}{\partial z}\right)-\frac{\partial}{\partial z}\left(\frac{2 \mu}{3 r} \frac{\partial}{\partial r}\left(r v_{r)}\right) . .\right. \tag{10}
\end{equation*}
$$

and
$S_{2}=\frac{\partial}{\partial z}\left(\mu \frac{\partial v_{z}}{\partial r}\right)-\left(\frac{2 \mu v_{r}}{r^{2}}\right)-\frac{1}{r} \frac{\partial}{\partial r}\left(\frac{2 \mu r}{3}\left(\frac{\partial v_{z}}{\partial z}+\frac{v_{r}}{r}\right)\right)+\frac{2 \mu}{3 r}\left(\frac{\partial v_{z}}{\partial z}+\frac{1}{r} \frac{\partial\left(r v_{r}\right)}{\partial z}\right)$.
3) The energy equation in axi-symmetric cylindrical coordinates is given by:

$$
\begin{equation*}
\frac{\partial}{\partial z}\left(\rho v T-\frac{\kappa}{c_{p}} \frac{\partial T}{\partial z}\right)+\frac{1}{r} \frac{\partial}{\partial r}\left(\rho v_{r} r T-\frac{r \kappa}{c_{p}} \frac{\partial T}{\partial r}\right)=S_{3} . \tag{12}
\end{equation*}
$$

here T is the gas temperature $\kappa$ is the gas thermal conductivity, $\mathrm{c}_{\mathrm{p}}$ is the specific heat at constap $\neq \ddagger$ pressure, and $S_{3}$ the source term given by:

$$
\begin{equation*}
S_{3}=\frac{k t}{} \text {.. } \tag{13}
\end{equation*}
$$

Where $\alpha$ and $I$ stand for the absorption coefficient, and the laser beam intensity respectively.
Treatment of the boundary Conditions To obtain a solution for the equations mentioned above, boundary conditions for all the dependent variables must specified at all points on the boundaries inclosing the field flow as indicated below:
a) Inlet: We shall restrict this treatment to the following boundaries

$$
\begin{aligned}
& v_{z}=v_{0}\left(1-\frac{r^{2}}{R^{2}}\right) ; \ldots \ldots \ldots \ldots \ldots v_{r}=0 \\
& E(r)=E_{0} \exp \left(-\frac{r^{2}}{R^{2}}\right), \text { for } 0 \leq r \leq a \\
& N(r, z)=N(r, \pm \infty)=0 ; \ldots \ldots . . T=300 k^{0}
\end{aligned}
$$

b) Wall: we shall assume that there is no mass flow crossing the boundaries so that the fluid velocity has no components on the wall i.e. $v_{z}=v_{r}=0$; the plasma density at the wall is zero i.e. $\mathrm{N}\left(\mathrm{R}_{0}, \mathrm{z}\right)=0$ and $\mathrm{T}=300 k^{0}$.
c) Axis: At the axis the boundary conditions are:

$$
\frac{\partial v_{z}}{\partial r}=0 . \ldots \ldots \ldots \ldots . . . . ; \frac{\partial T}{\partial r}=0 \ldots \ldots \ldots \ldots . \ldots ; v_{z}=0
$$

d) Outlet: There is no explicit specification of the boundary at the outlet. This is because the boundary conditions prevailing there strongly depend on the upstream conditions in the calculation domain.
e) In addition to these specification of the dependent variables at the boundary, we shall assume that the plasma density $\mathrm{N}(\mathrm{r}, \mathrm{z})$, and $\partial \mathrm{N} / \partial \mathrm{z}$ are continuous at $\mathrm{z}= \pm \mathrm{L}$, where L the active area.

## RESULT AND DISCUSSION

A) The laser beam distributions: The numerical solutions of the integral diffraction equation (4) obtain using the well-known trapezoidal rule for integration. The mesh size chose such that convergence of the integration insured to one part in $10^{-8}$ at successive steps. This occurs by running the program a number of times at a few points using a steadily decreasing step length until two solutions using consecutive lengths converged. For the reason of the rapidly oscillating nature of the integral, it is necessary to use very small step lengths. For the cases of aberration, it is necessary to restrict the number of points of computation in (r,z) coordinates to another coarse mesh to insure the accuracy mention above. The aberration function $\Phi(p)$ is strongly dependent on both focal length of the lens and the diameter of the laser beam as shown in figure (1) and figure (2). The numerical solutions of the integral diffraction equation first checked with the analytical solutions obtained by Linfoot and Wolf [15], and by Morgan [16], for a uniform incident wave front and a perfect lens. In this comparison, the ruby laser is considered and it seems that the results of the numerical solutions of the intensity distributions are in good agreement with those obtained analytically and experimentally. However, the analytical model restricted by a much-tied condition and it seemed difficult to attack verity of cases, for example among other things the continuous laser beam. Figure (3) shows the expected shape and size of the focal region in the case of aberration free lens while figure (4) shows the expected shape and size of the focal region in the case of lenses exhibits spherical aberration. Figure (5) shows the intensity distribution of laser beam for an aberration free lens. We can see that the Gaussian focus and the diffraction focus coincide and the other diffraction maxima lie on the optic axis on either side of the Gaussian focus and are weak. The intensity distribution is symmetrical about the focal plane. Figure (6) shows the intensity distribution by a real lens with spherical aberration in the order
of $0.5 \lambda$. One can see that for small amount of primary spherical aberration, the principle diffraction focus no longer coincide with the Gaussian focus. In addition, the off-axis maxima appear as larger than those of perfect lens and the intensity distribution is no longer symmetrical about the focal plane. Figures $(7,8)$ present the case for the focal lengths $4 \mathrm{~cm}, 3$ cm , which are corresponding to spherical aberration $6 \lambda$ and $10.3 \lambda$ respectively. It is clear from these figures that the distortion of the intensity distribution increasing by decreasing the focal length of the lens or by increasing the radius of the aperture of the lens whose lead to increasing the spherical aberration. In addition, we can see that the symmetrical about the focal plane was decreased and many axial and off-axis maxima appeared. From these figure one can say that it is important to know the distribution of the laser beam throughout the working area in order to know the amount of energy deposit into the gas.
B) The temperature distribution: The main purpose of this section is to give a full discussion of the behaviour of the working gas arising from the numerical solution of the general partial differentials equations $(5,8,9,12)$. To demonstrate the validity of the model presented, we shall consider that the system used is part of cylinder concentric with an open cylinder figure (9).
In order to derive the finite difference equation, we shall assume that a non-uniform grid network covers the finite area of interest filed. The finite difference equation will eventually be expresses in terms of the values of variables of these nodes. This technique will apply to the ionisation and energy equations only. The reason is that the pressure gradient forms part of the source term for momentum equation in numerical calculations. The presence of the pressure causes a real difficulty in the calculation of the velocity components because there is no obvious equation for obtaining it, to know more about such difficulties one should refer to the book of Patenker [17]. In order to resolve this difficulty we adopted a different kind of grid to evaluate the velocity and pressure components. This grids, is called staggered grid, were used by Al-Hashmiy [18], Al-Kelly [5]. There are two advantages of using staggered grid; the first is that the algebraic continuity equation would contain the differences of adjacent velocity components rather than alternate. The second is that the difference between two adjacent grid points of the pressure becomes the natural driving force for the velocity component located between these grid points. Figure (10), shows the axial distribution of the temperature throughout the focus region. It is clear that in the absence of the heat addition outside the hot central zone, the temperature is constant as might be expected, but over the central region where the heat addition zone is located the peak temperature reaches about $32,000^{0} \mathrm{~K}$. This means that although a high temperature at the central region the gas near the wall stays cool. The corresponding radial distribution of the temperature throughout the focus region is present in figure (11). The complete view of the temperature behaviors is shown in figure (12), for aberration free lens and figure (13) for lens exhibit primary spherical aberration. Figure (12), however shows the symmetrical distribution of the temperature in both axial and radial distance, while figure (13) shows how the influence of the spherical aberration has changed the behavior of the temperature distribution. This behavior is very important in order to understand what called the continuous optical discharge and one can strongly expect the dependence of the discharge on the temperature distribution. Figure (14) shows the variation of the plasma density with laser power. The plasma density increases rapidly with laser power. Finally, we can say that the properties of the plasma in flowing gas need to understood, and more work must carried out in this aspect. This will be done in next future.


Fig.(2) The spherical aberration dependenceon the laser beam at constant focal distance ( 5 cm )


Fig.(4) Beam geometry: lower half of this figure shows Non uniform numerical grid for fluid dynamics; upper Half shows rays used for beam calculation for lens exhibits aberration.


Fig.(5) Intensity
distributionaberration free lens


Fig.(7) Intensity distribution in lens exhibits aberration, $f=4 \mathrm{~cm}$.


Fig.(8) Intensity distribution in lens exhibits aberration, $f=3 \mathrm{~cm}$.


Fig(9) Flow geometry used in the analysis


Fig(10) The axial distribution of the temperature in active Discharge plasma for different values of focus radius


Fig.(11) The radial distribution of the temperature in active Discharge plasma



Fig.(14) The plasma number density as a function of laser power

## Reference:

[1] Roda A.V., Gamaly E. G., and Davies B. L.; Phys. Plasma 4(10), pp3676-83. (1997).
[2] Jaanimagi P.A., Ebrahim N. A., Burnet N. H., and Joshi C.; Appl. Phys. Lett.; $\mathbf{3 8}$ pp 734. (1981).
[3] Tabak M., Hammer M., Glinsky M. E., Kruer W. L., Wilks S. C., Woodworth J. G., Campbell E. M., Perry M. D., and Mason R. J.; Phys. Plasmas; $\underline{1}$ pp 1826. (1994).
[4] Shargee S. N.; "Numerical analysis of laser produced COD in flowing gases"; Ph.D. thesis; Basrah Univ.; Iraq. (1997).
[5] Al-Kelly A. A.; "Theoretical study of the continuous optical discharge in the laser beam focus"; Ph.D. Thesis; Basrah Univ.; Iraq. (1999).
[6] Al-Hashmiy N. H., and Al-Kelly A. A.; Basrah J. Science A; Vol 14 No. 1, pp 77. (1996).
[7] Al-Hashmiy N. H.; Basrah J. Science A; Vol 17 No. 2, pp 100. (1999).
[8] Al-Hashmiy N. H., and Al-Kelly A. A.; Faculty of Science Bulletin (FSB); Vol. 14; pp 13. (2001).
[9] Kozlov G. I., and Kuznetsov V. A.; Technical Phys. Lett.; vol. 20 pp 197. (1994).
[10] Surzhikov S. T.; J. High Temperature;Vol. 32 pp 275-81.(1994).
[11] Lebehot A., and Campargue R.; Physics of Plasmas; vol. $\underline{\mathbf{3}}$ pp 2502-10. .; (1996).
[12] Raizer Yu. P., and Sarzhikov S. T.; AIAA; vol. 34, pp 1584. (1996).
[13] Locke E. V., Hoag E. D., and Hella R. A.; IEEE J Quantum Electron. QE-8, 132. (1972).
[14] Molvik G. A., Choi D., and Merkle L. C.; AIAA vol. 25 No.7, pp 1053-1060. (1985).
[15] Linfoot E. H., and Wolf E.Proc. Phys. Soc.; B69 pp 823. (1956).
[16] Morgan C. G.; "Radiative processes in discharge plasmas"; Plenum publishing corporation; UK. (1986).
[17] Patanker S. V.; "Numerical heat transfer and fluid flow"; Hemisphere publishing corporations; USA. (1986).
[18] Al-Hashmiy N. H.; "Theoretical Investigation of Laser Produced Ionization in a Flowing Gas"; Ph.D. Thesis; Swansea univ.; U.K. (1990).

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حساب عددي لتوزيع الحرارة في البلازما المجمرة
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لـخلاصة:
تم توضيح تأثير حالات تركيز اللليزر على كفاءة تفاعل اللليزر مع الغاز وقد تم دراسة الطريقة المقترحة لزيادة هذه
 جميع حساباتتا تعتمد على حل معادلات Navier-Stokes في بعحين، معادلـة الآشعة المحورية، معادلة النأين و معادلة الطاقة. استخدمت طريقة الفروق المتتاهية لحل هذه المعادلات حيث فسمت منطقة التأثنثر إلى 3321 خلية غبر منظمة. الجمقيع الحسابات تتعتد على الخواص الحققية للغاز بما في ذلك معامل الامتصاص وقد تم عرض بعض النتائج في هذه

