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Numerical calculation of the coherence length of ^{24}Mg, ^{39}K, and ^{133}Cs atoms for particles laser

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Abstract

We have used the theory of Bose-Einstein condensate to develop a theoretical model to calculate the coherence length of the interacting particles. Three different types of atoms are treated in this model in order to check its validity. Comparing our results with the available experimental and theoretical data in the literature showed a very good agreement. Since the atom laser is a brand new study and still a basic, according to our knowledge, no other values could traced for atoms like ^{24}Mg, ^{39}K, and ^{133}Cs in the literature. Therefore, we can claim that our calculations for these atoms are made for the first time.

Keywords: Laser cooled atom, BEC atom, Trapping

Introduction

More than 45 years ago, the invention of the first optical lasers marked a scientific breakthrough. With optical lasers the ultimate control over light waves at the quantum limit had reached. Now, a similar spectacular degree of control over matter waves has been achieved with the realization of Bose-Einstein condensation in dilute atomic gases, see Anderson et al [1]; Davis et. al. [2]; and Bradley et. al. [3].

The highly phase coherent ensemble of atoms in a Bose-Einstein condensate is usually created in a trapped state and different techniques have been developed to convert the trapped atoms into freely propagating coherent matter waves, the so called atom lasers Mewes et. al. [4]; Bradley et. al. [5]; Anderson and Kasevich [6]; Hagley et. Al; [7] and Bloch et. al. [8]. These novel atomic beam sources promise to be important constituents of future atom interferometers and atomic clocks and are also believed to be important tools for the field of atom holography Morinaga et. al.[9]. The unique properties of atom lasers will make it possible to enter an experimental regime in atom optics that is not accessible for thermal atom sources. In light optics, the availability of coherent sources has substantially increased the range of photonic applications. Similarly, it expected that coherent matter wave sources will have a profound impact on applications such as atom interferometer Berman [10], or the manipulation of atomic beams on a nanometer scale. Focusing of the atom laser beam is a central step towards an atom laser microscope Balykin and Letokhov, [11], Doak et. al. [12].

To achieve a highly mono energetic atom laser beam of short de Broglie wavelength our novel atom optical element could be used to accelerate the beam. It should then be possible to focus the beam to spot sizes much smaller than achievable in focal light microscopy and comparable to high-energy electron microscopy. To fulfill these expectations it is crucial to invent atom optical elements that are adapted to the demands of the new highly coherent atom sources. For thermal atom, sources varieties of atom optical elements have been investigated Adams et. al. [13]; Holst and Allison [14]; Hinds and Hughes [15], which redirect, split, or shape atomic beams using position- or time-dependent potentials. To preserve the coherence properties of atom lasers a very high "surface quality" of atom optical elements is required. The de Broglie wavelength of an atom laser beam can be well below ten nanometers. The effective surface roughness of atom optical elements should therefore be even smaller.

The simplest approach to realize a coherent matter wave source is to suddenly releasing a Bose-Einstein condensate from the magnetic trap. The mechanical manipulation of released condensates has been demonstrated using optical standing wave fields Kozuma et. al. [16]; Stenger et. al. [17]. Suitably shaped off-resonant laser fields Bongs et. al. [18], and pulsed magnetic fields Arnold [19]. Because of the sudden switch-off of the confining potential, the energy of the repulsive interaction between the atoms is transforming into kinetic energy. The velocity distribution of the released atoms is therefore much broader than the Heisenberg limit associated with the spatial size of the trapped condensate. The velocity spread is drastically reducing for atom lasers employing continuous output coupling, where a Fourier limited output can be approached Band et al. [20], and interaction effects are minimizing. The resulting mono energetic atom laser beam is not susceptible to dispersive effects in the manipulation of coherent matter waves Bongs et. al. [18], however these are unwanted in most atoms optical applications. The atom laser output is extracting from ^{87}Rb Bose-Einstein condensate using continuous output coupling was investigating by Bloch, et al. [8]. Immanuel Block, et. al. [21] reported the experimental results on the continuous output coupling of atoms from magnetically trapped Bose-Einstein condensates.

The idea for an atom laser predates the deconstruction of the exotic quantum phenomenon of Bose-Einstein Condensation (BEC) in dilute atomic gases. However, it was only after the first such condensate was produced in (1995) by Anderson et. al.[1], that the pursuits to create a laser-like source of atomic de Broglie waves become intense. In a Bose condensate, all the atoms occupy the same quantum state and can be describe by the same wave function. The condensate therefore has many unusual properties not found in other states of matter. So, why can we think of Bose condensate as a coherent source of matter wave? To address this crucial point we have to remind ourselves of some of the physics behind the properties of laser light. In laser, all the photons share the same wave function. This is possible because photons have an intrinsic angular momentum, or “spin”, of Planck’s constant ħ. Particles that have a spin that is an integer multiple of ħ obey Bose-Einstein statistic. This means that more than one so-called boson can occupy the same spin such as electrons, neutron and protons, which all have spin ħ/2 and obey Fermi-Dirac statistics, and only one fermion can occupy a given quantum state. A composite particle, such as an atom, is a boson if the sum of its protons, neutrons, and electrons is an even number; the composite particle is fermions if this sum is an odd number. Sodium^{23} atoms, for example, are bosons so a large number of them can force to occupy the same quantum state and therefore have the same wave function. To achieve this, a large number of atoms must confined within a tiny trap and cooled to sub-mille Kelvin temperatures using a combination of optical and magnetic techniques (See Bose condensate make quantum leaps and bound sand, Townsend et. al. [22] in further reading).

In this work, we will concentrate on the properties of the condensates rather than on their creation. We have been able to create a numerical model to calculate the coherence length of highly collimated and mono energetic beams of atoms such as **^{24}Mg, ^{39}K, and ^{133}Cs** for the first time.

Theory

A) Mathematical background

The general vector state to describe a highly collimated non-interacting beam of neutral atoms propagating in non-conducting medium, Castellanos and Lopez [23], is given by:

$$\psi(\vec{r},t)=\int_{-\infty}^{+\infty}A(\vec{k})e^{i(\vec{k}.\vec{r}-\omega t)}d\vec{k} \tag{1}$$

Equation (1) indicates a vector sum, where $\vec{k}$ corresponds to the vector associated to the plane wave solution for a free particle. On the other hand, the integral stems from the possibility of the particle beam of taking any (continuous) value of momentum. In case of discrete changes in momentum, it is customary to write down:

$$\psi(\vec{r},t)=\sum_{k}A_k(\vec{k})e^{i(\vec{k}.\vec{r}-\omega t)} \tag{2}$$

Going from equation (1) to equation (2) is straightforward, but the former it is not so easy to deal with. For instance, the current density,

$$\vec{J} = \frac{\hbar}{2mi}\left(\psi^{*}\vec{\nabla}\psi - \psi\vec{\nabla}\psi^{*}\right) \tag{3}$$

can be written using equation (2) as (notice; time dependence is dropped):

$$\vec{J} = \frac{\hbar}{2mi}\left[2i\sum_{k}\vec{k}\left|A_k(\vec{k})\right|^2 + \sum_{k'\neq k}\sum_{k} i\vec{k}(A_{k'}A_k^{*}e^{i(\vec{k}\ -\vec{k}).\vec{r}*} + A_{k'}A_k e^{-i(\vec{k}\ -\vec{k}).\vec{r}})\right] \tag{4}$$

In the integral form it reads:

$$\vec{J} = \frac{\hbar}{2mi}\left\{2\int\vec{k}\left|A(\vec{k})\right|^2 d\vec{k} + \int_{\vec{k}'}\Gamma(\vec{k}')d\vec{k}'\right\} \tag{5}$$

where,

$$\Gamma(\vec{k}') = \int_{k\neq k'}\vec{k}\left\{A(\vec{k}')A^{*}(\vec{k})e^{-i(\vec{k}-\vec{k}').\vec{r}} + A^{*}(\vec{k}')A(\vec{k})e^{i(k-k').\vec{r}}\right\}d\vec{k} \tag{6}$$

The physical meaning of equation (5) is made visible when one writes:

$$\vec{\tau} = \vec{k} - \vec{k}' \Rightarrow d\vec{\tau} = d\vec{k} \tag{7}$$

for a fixed $\vec{k}'$. Then the first term at the right-hand side of equation (6) (the remaining is a complex conjugate) will be:

$$\gamma = \int\vec{k}A(\vec{k}')A^{*}(\vec{k})e^{-i(\vec{k}-\vec{k}').\vec{r}}d\vec{k} \Rightarrow \gamma = \int\left[A(\vec{k}-\vec{\tau})e^{i(k-\vec{\tau}).\vec{r}}\right]\left[\vec{k}A^{*}(\vec{k})e^{-i\vec{k}.\vec{r}}\right]d\vec{k} \tag{8}$$

Equation (8) dose not quite satisfy the correlation (classical) definition for a pair of functions $g(\xi)$ and $h(\xi)$:

$$\gamma = g(\xi)\otimes h(\xi) \Rightarrow \gamma = \int g(\xi)h^{*}(\xi-\tau)d\xi \tag{9}$$

Because in this expression $\vec{\tau}$ is a fixed correlation parameter and in this treatment (see equation (7)), $\vec{\tau}$ is changing continuously together with $\vec{k}$. However, one still can say that equation (8) has the meaning of a correlation, since the integrand can be understood as a correlation between particles of momentum $\vec{k}$ and $(\vec{k}-\vec{\tau})$ for a given $\vec{k}'$. Keeping this in mind equation (6) makes account of all these correlations for different values of $\vec{k}'$. Then one writes:

$$\Gamma(\vec{k}') = \int\left[\gamma(\vec{k}) + \gamma^{*}(\vec{k})\right]d\vec{k} \tag{10}$$

Now, a particle beam can be experimentally prepared in such a way that $A(\vec{k})$ may be a real number. This only means that for initial condition (t = 0, r = 0) $A(\vec{k})$ has already some definite value (including zero). With this argument, equation (5) now reads:

$$\vec{J} = \frac{\hbar}{2m}\left\{2\int\vec{k}\left|A(\vec{k})\right|^2 d\vec{k} + 2\int d\vec{k}'\int \mathrm{Re}\left[A(\vec{k}-\vec{\tau})e^{i(\vec{k}-\vec{\tau}).\vec{r}}\right]\left[\vec{k}A^{*}(\vec{k})e^{-i\vec{k}.\vec{r}}\right]d\vec{k}\right\} \tag{11}$$

$$\vec{J} = \frac{\hbar}{2m}\left\{2\int\vec{k}\left|A(\vec{k})\right|^2 d\vec{k} + 2\iint A(\vec{k}')A^{*}(\vec{k})\vec{k}\cos\left[(\vec{k}'-\vec{k}).\vec{r}\right]d\vec{k}d\vec{k}'\right\} \tag{12}$$

This equation represents the more general expression for describing non- interacting particle beam propagation. The second term (associated with coherence throughout correlation) describes focusing phenomena both directional (i.e., certain directions contain a bigger amount of particles than other directions) and longitudinal (along a particular direction, where some focus will exist).

From the second term on the right hand side of equation (12) one note that for a pair of atoms traveling in the same direction and with a slight difference in $\vec{k}$, the corresponding focusing along z-axis will correspond to those points where:

$$(\vec{k}'-\vec{k})\bullet\vec{z} = 2n\pi \tag{13}$$

where *n* = 1,2, … ; For de Broglie particles with $\vec{p} = \hbar\vec{k} = m\vec{v}$ we have,

$$z \cong \frac{2n\pi\hbar}{\Delta p} = \frac{2n\pi\hbar}{m\Delta v} \tag{14}$$

where $\Delta v = v' - v$ is the atomic velocity difference between atoms, m the atomic mass and $\hbar$ is the Planck constant. If the coordinate z is measured from the beam origin, eq. (14) can be put in the form

$$\Delta z \Delta p \cong 2n\pi\hbar \qquad (15)$$

To understand eq. (15), it is straightforward from the quantum mechanical point of view. In fact, this expression is identified as the uncertainty principle since $\Delta p = 0$ means that we cannot localize any focus on z–axis. We are in the presence of a perfect monochromatic plane wave (i.e. every particle in the beam has exactly the same energy).

On the other hand, if we choose any two particles into the beam with a momentum difference Δp, and we track them in time, there is a certain possibility, different from zero, that they will focus in some point z given by equation (15). This probability will be smaller as Δp increases. It comes out from this argument that, in geometrical term, the existence of any focus will mean some degree of coherence, and therefore focus at infinity means perfect or total degree of coherence (for a given n). On the contrary, focusing near the origin without any farther focus will mean a lower degree of coherence since the beam will spread along the propagation axes. When we deal with light, we better put equation (15) as:

$$(k' - k)z = 2n\pi \Rightarrow \frac{1}{c}(\Delta\omega) = 2n\pi \Rightarrow \frac{2\pi\Delta\nu\Delta z}{c} = 2n\pi \Rightarrow \Delta z = \frac{n}{\Delta\nu}c \qquad (16)$$

For n=1 we obtain the so-called coherence length in optics $\Delta z = \frac{c}{\Delta\nu}$, Born and Wolf [24]. If we know the bandwidth of a laser, we will know the distance at which the field will oscillate in rigorous phase. Therefore, it is interesting that requiring focusing in equation (13) as a coherence condition, we arrive at the well-known formula of coherence length.

Since the first Bose-Einstein condensate (BEC) was obtained, the laser of atoms became visible, Anderson et. al. [1]. This object is defined as a device producing an intense and well-collimated coherent beam of atoms, Ketterle [25] and Wisemen [26], involving a process of coherent matter-wave amplification, Miesner et. al. [27]. Since atoms have masses and they interact while traveling, the coherence of the beam presents an additional spreading, which has to consider as variant of the Photon Laser. One can do this by replacing equation (1) with the outgoing wave function of the condensate Ψ_0 and then perform the same prosecutor as before. In the following, we describe briefly how to obtain Ψ_0. A more detailed and rigorous deduction can be found in the paper by Gerbier et. al. [28].

These authors consider a (BEC) of ^{87}Rb in the hyperfine level with F=1. This condensate could be in any of the sublevels m=–1,0,1. Atoms in a state with m=–1 is confined in a magnetic potential of the form:

$$V_{trap} = \frac{1}{2}M(w_x^2 x^2 + w_\perp^2 y^2 + w_\perp^2 z^2),$$

Atoms in a state with m=0 are the un trapped ones, and those with m=1 are rejected out of the trap. These are the three components of the spinorial wave function of the condensate $\Psi = [\psi_m]_{m=-1,0,1}$ and they obey a set of Schrödinger coupled equations, Ballagh et.al. [29]. When the limit of weak coupling is considered, Steck et. al. [30], the population N_m satisfy $N_1 \le N_0 \le N_{-1}$ and only the states with m=–1 and m=0 are considered. The condensate atoms are transferred from the state m =–1 (trapped) to the state m=0 (un trapped) by using a rf. pulse:

$$\vec{B}_{rf} = B_{rf}\cos(\omega_{rf}t)\hat{e}_x \qquad (17)$$

Thus the components of (BEC) $\psi_m = \psi'_m e^{-im\omega_{rf}t}$ satisfy the two coupled equations Gerbier et. al. [28]:

$$i\hbar\frac{\partial\psi_{-1}}{\partial t} = \left[\hbar\delta_{rf} + \vec{P}^2/2M + V_{trap} + U|\psi_{-1}|^2\right]\psi_{-1} + \frac{\hbar\Omega_{rf}}{2}\psi_0 \qquad (18)$$

$$i\hbar\frac{\partial\psi_0}{\partial t}=\left[\vec{P}^2/2M-Mgz+U\left|\psi_{-1}\right|^2\right]\psi_0+\frac{\hbar\Omega_{rf}}{2}\psi_{-1} \qquad (19)$$

The intensity of the interaction is given by $U=4\pi\hbar^2 aN/M$, where N is the initial number of trapped atoms, M is the atomic mass, and is the diffusion length for inter atomic collision process which, for the ^{87}Rb is 5nm.

The uncoupling intensity between states m =−1 and m =0 is given by the Rabi flopping frequency:

$$\hbar\Omega_{rf}=\mu_B B_{rf}/2\sqrt{2} \qquad (20)$$

The detuning δ_{rf} is

$$\hbar\delta_{rf}=V_{off}-\hbar\omega_{rf} \qquad (21)$$

and

$$V_{off}=\mu_B B_0/2+Kz^2/2 \qquad (22)$$

B_0 is the background magnetic field due to the coils of the trap. Equations (18) and (19) are uncoupled in the framework of the mean field theory and the weak coupling limit, Gerbier et. al. [28], obtained for Ψ_0,

$$\psi_0(\vec{r},t)\approx A(\Omega_{rf},F)\frac{e^{i\frac{2}{3}\left|\xi_r\right|^{2/3}-i\frac{E_{-1}t}{\hbar}}}{\sqrt{\left|\xi_r\right|^{1/2}}} \qquad (23)$$

where,

$$A(\Omega_{rf},F)=-\sqrt{\pi}\,\frac{\hbar\Omega_{rf}}{Mgl}\phi_{-1}(x,y,z_r)F \qquad (24)$$

Here, F describes the finite extension of an atom laser beam due to the finite coupling time (e.g. the time of rf irradiation) and is constant for each particular laser. The dimensional parameter $\xi_r=(z-z_r)/l$ provides a scale to the size of the trap, $z_r=\eta z_0/2$ is the extraction point from the trap, and

$$\phi_{-1}(x,y,z_r)=(\frac{\mu}{U})^{1/2}\left[1-(x/x_0)^2-(y/y_0)^2-(z/z_0)^2\right]^{1/2} \qquad (25)$$

The quantity $\left|\phi_{-1}(x,y,z_r)\right|^2$ corresponds to the trapped atomic population in the output point z_r. In what follows a definitions of some important figures such as l,z_0,x_0,y_0 and η:

$$l=\left(\frac{\hbar^2}{2M^2g}\right)^{1/3};x_0^2=\frac{2\mu}{Mw_x^2};y_0^2=\frac{2\mu}{Mw_\perp^2};z_0^2=\frac{2\mu}{Mw_\perp^2};\eta=(2\hbar\delta_{rf}+4\mu/7)\frac{1}{2mgz_0} \qquad (26)$$

Where μ is the chemical potential, and is understood as the necessary energy to either add or remove a condensed atom in the trap ensemble. The chemical potential μ is defined as, Dalfovo et. al. [31]:

$$\mu=(\hbar\omega/2)(\frac{15aN_{-1}}{\sigma})^{2/5} \qquad (27)$$

with $\omega=(\omega_x\omega_\perp^2)^{1/3}$ and the harmonic oscillator length defined as $\sigma=(\hbar/M\omega)^{1/2}$. Note that $l<<x_0,y_0,z_0$.

B) Numerical model for Coherence Length for an Atom Laser

We now use Eq. (13) in order to find the coherence length. To do this we need to know the propagation vector $\vec{k}$, which is calculated using Eq. (23) and following the standard procedures, Flugge [32]. We then calculate $\vec{J}$, and find $v = \vec{J}/P$ with $\rho = |\psi_0|^2$.

$$\vec{J} = \frac{\hbar}{2mi}\left(\psi^* \vec{\nabla}\psi - \psi\vec{\nabla}\psi^*\right), v = \frac{J}{P} \Rightarrow v = \frac{\hbar}{ml}\sqrt{\xi_r} \Rightarrow v = \frac{\hbar}{m}\frac{\sqrt{z+z_r}}{l^{3/2}} \tag{28}$$

where $\xi_r = \frac{z}{l} + \frac{z_r}{l}$. We now use the de Broglie relation $\vec{P} = \hbar k = m\vec{v}$ to obtain the propagation vector $\vec{k}$, we obtain for its magnitude:

$$k = \frac{\sqrt{z+z_r}}{l^{3/2}} \tag{29}$$

We then replace equation (29) into equation (13):

$$\left[(z+z_r)^{1/2} - (z'+z_r)^{1/2}\right]z = 2n\pi l^{3/2} \tag{30}$$

It is clear from section A and Eq.(13) that $z - z'$ is the correlation length for the interacting atom laser beam between two different points of the beam. Since we are interested in the coherence length measured from the extraction point z_r, we make $z' = z_r$. Therefore:

$$\left[(z+z_r)^{1/2} - (z_r)^{1/2}\right]z = 2n\pi l^{3/2} \tag{31}$$

By solving this equation for n=1, we obtain the coherence length z for the atom laser.

Results and Conclusion

The coherence of a condensate or output beam can characterized using the theory of "partially" coherent matter-wave fields. The theory incorporates various so-called coherence functions that provide quantitative information about the coherence. The simplest of these, first-order coherence, tells us whether we can see interference fringes formed by two overlapping fields. Higher order coherence functions can represent intensity correlations between fields, for example.

According to the theory, the matter waves output by an atom laser are coherent in two respects: they contain a narrow range of wavelengths and they are much more stable in intensity than thermal beam. An important challenge in building a continuous atom laser is to design an output-coupling scheme that keeps the phase of the matter waves "in step" over a long period. Suppose that we describe the matter wave by *I(x,t)=A(x,t)cos(ωt+f(x,t))* and we can keep the intensity, A, constant. The overall coherence of the output is then limited by the stability of the phase, f, over time and in space. If the phase wanders over time then it will also limit the line width of the atom laser via the uncertainty principle. An atom laser is analogous to an optical laser, but it emits matter waves instead of electromagnetic waves. Its output is a coherent matter wave, a beam of atoms which can be focused to a pinpoint or can be collimated to travel large distances without spreading. The beam is coherent, which means, for instance, that atom laser beams can interfere with each other. Compared to an ordinary beam of atoms, the beam of an atom laser is extremely bright. One can describe laser-like atoms as atoms "marching in lockstep". Although there is no rigorous definition for the atom laser (or, for that matter, an optical laser), all people agree that brightness and coherence are the essential features. A laser requires a cavity (resonator), an active medium, and an output coupler. In atom laser, the "resonator" is a magnetic trap in which the atoms confined by "magnetic mirrors". The active medium is a thermal cloud of ultra cold atoms, and the output coupler is an rf pulse which controls the "reflectivity" of the magnetic mirrors.The analogy to spontaneous emission in the optical laser is elastic scattering of atoms (collisions similar to those between billiard balls). In a laser, stimulated emission of photons causes the radiation field to build up in a single mode. In an atom laser, the presence of a Bose-Einstein condensate (atoms that occupy a "single

mode" of the system, the lowest energy state) causes stimulated scattering by atoms into that mode. More precisely, the presence of a condensate with N atoms enhances the probability that an atom will be scattered into the condensate by $N+1$.
In a normal gas, atoms scatter among the many modes of the system. But when the critical temperature for Bose-Einstein condensation is reached, they scatter predominantly into the lowest energy state of the system, a single one of the myriad of possible quantum states. This abrupt process is closely analogous to the threshold for operating a laser, when the laser suddenly switches on as the supply of radiating atoms is increased.In an atom laser, evaporative cooling does the “excitation” of the “active medium” - the evaporation process creates a cloud, which is not in thermal equilibrium and relaxes towards colder temperatures. This results in growth of the condensate. After equilibration, the net "gain" of the atom laser is zero, i.e., the condensate fraction remains constant until further cooling applied.Un-like optical lasers, which sometimes radiate in several modes (i.e. at several nearby frequencies), the matter wave laser always operates in a single mode. The formation of the Bose condensate actually involves "mode competition": the first excited state cannot be macroscopically populated because the ground state "eats up all the pie".
The output of an optical laser is a collimated beam of light. For an atom laser, it is a beam of atoms. Either laser can be continuous or pulsed - but so far, the atom laser has only realized in the pulsed mode. Both light and atoms propagate according to a wave equation. Light is governing by Maxwell's equations, but matter is describing by the Schrödinger equation. The diffraction limit in optics corresponds to the Heisenberg uncertainty limit for atoms. In an ideal case, the atom laser emits a Heisenberg uncertainty limited beam.Now we have a sufficient background to calculate the coherence length for some atoms. In this calculation, we use the Mathematical program to solve numerically equation (31). In the table shown below we list the results of all possible atom lasers with alkaline species together with a comparison with the available theoretical and experimental data:

Atom	Coherence length in micrometer		
	Our results	Other theoretical results	Experimental results
^{23}Na	2.4814	2.4622*	2.0 to 5.0**
^{87}Rb	1.0246	1.0299*	---
^{7}Li	5.4803	5.4461*	---
^{24}Mg	2.3929	-----	---
^{39}K	1.7428	-----	---
^{133}Cs	0.7727	----	----

Table: Comparison between our results for the coherence length and the results of Castellanos* et. al. [23] and Mark Trippenbach et al. [34]**

In particular, our treatment leads to a quantitative agreement with the experimental results of Trippenbach et al. [34] for the atom of ^{23}Na. In their experimental data, they obtained results as different rf for the coherence length ranging between 2.0 µm to 5.0 µm. Moreover, our results for atoms ^{23}Na, ^{87}Rb, ^{7}Li are in excellent agreement with the theoretical values obtains by Castellanos et al. [23].
In conclusion, we have proposed a very simple form for the calculation of the coherence length for an atom laser, and have shown the validity of the method by comparison with the experimental results of Trippenbach et al. [34]. This agreement encouraged us to calculate the coherence length of atoms like ^{24}Mg, ^{39}K, ^{133}Cs.

REFRENCES

[1] M. H. Anderson, J. R. Ensher, M. R. Matthews, C. E. Wieman, and E. A. Cornell, Science 269, 198 (1995).
[2] K. B. Davis, M. O. Mewes, and W. Ketterle, Phys. Rev. Lett. 75, 3969, (1995).

[3] C. C. Bradley, C. A. Sackett, J. J. Tollett, and R. G. Hulet, Phys. Rev. Lett. 75, 1687, (1995).
[4] M. O. Mewes, M. R. Anderws, N. J. Van Druten, D. M. Stamper-Kurn, D. S. Durfee, C. G. Townsend, and W. Ketterle, Phys. Rev. Lett. 78, 582, (1997).
[5] C. C. Bradley, C. A. Sackett, J. J. Tollett, and R. G. Hulet, Phys. Rev. Lett. 78, 985.(1997).
[6] B. P. Anderson, and M. A. Kasevich, Science 282, 1686, (1998).
[7] E. W. Hagley, L. Deng, M. Kozuma, J. Wen, K. Helmerson, S. Rolston, and W. Phillips, Science 283, 1706, (1999).
[8] I. Bloch, T. W. Hänsch, and T. Esslinger, Phys. Rev. Lett. 82, 3008, (1999).
[9] M. Morinaga, M. Yasuda, T. Kishimoto, and F. Shimiza, Phys. Rev. Lett. 77, 802, (1996).
[10] P. Berman, Atom Interferometry, edited by Academic Press, New York, (1997).
[11] V. I. Balykin, and V. S. Letokhov, Opt. Commun. 64, 151, (1987).
[12] R. B. Doak, R. E. Grisent, S. Rehbein, G. Schmahi, J. P. Toennies, and Ch. Well, Phys. Rev. Lett. 83, 4229, (1999).
[13] C. S. Adams, M. Sigel, and J. Mlynek, Phys. Rep. 240, 143, (1994).
[14] B. Holst, and W. Allison, Nature (London) 390, 244, (1997).
[15] E. A. Hinds, and I. G. Hughes, J. Phys. D 32, R119, (1999).
[16] M. Kozuma, L. Deng, E. W. Hagley, J. Wen, R. Lutwak, K. Helmerson, S. L. Rolston, and W. D. Phillips Phys. Rev. Lett. 82, 871, (1999).
[17] J. Stenger, S. Inouye, A. P. Chikkatur, D. M. Stamper-Kum, D. E. Prithard, and W. Ketterle, Phys. Rev. Lett. 82, 4569, (1999).
[18] K. Bongs, S. Burger, G. Birkl, K. Sengstock, W. Ertmer, and K. Raza, Phys. Rev. Lett. 83, 3577, (1999).
[19] A. S. Arnold, Ph.D. thesis, University of Sussex, (1999).
[20] Y. B. Band, P. S. Julienne, and M. Trippenbach, Phys. Rev. A 59, 3823,(1999)
[21] I. Block, W. H. Theodor, and T. Esslinger, Focus on Coherent Science, RIKEN Review, March No. 33, pp. 6-9, (2001)
[22] C. Townsen, W. Ketterle, And S. Stringari, Physis world March, (1997)
[23] L. M. Castellanos[a], and F. E. Lopez[b], Instuto de Fisica Universal de Antioquia, A. A. 1226, Medellin Colombia, (2003).
[24] M. Born, and E. Wolf, Principles of optics, (Cambridge University Press (2002).
[25] W. Ketterle, Yearbook of Science and Technology (MeGraw Hill, NewYork), (1999).
[26] H. J. Wisemen, Phys. Rev. A 58, 2068, (1997).
[27] H. J. Miesner D. M. Stamper Kurn, M. R. Andrews, D. S. Durfee, S. Inouye and W. Steck H., Narachewski M. and Wallis H., Phys. Rev. Lett. 80, 1 (1998).
[28] F. Gerbier, P. Bouyer, and A. Aspect, Phys. Rev. Lett. 86, 4729 (2001).
[29] R. J. Ballagh, K. Burnet, and T. F. Scott, Phys Rev. Lett. 78, 1607 (1997).
[30] H. Steck, M. Narachewski, and H. Wallis, Phys. Rev. Lett. 80, 1, (1998)
[31] F. Dalfovo, S. Giorgini, L. Pitaevskii, and S. Stringari, Rev. Mod. Phys. 71, 463 (1998).
[32] S. Flugge, Particle Quantum Mechanics, (1994).
[34] M. Trippenbach, Y. B. Band, M. Edwards, M. Doery, P. S. Julienne, E. W. Hagley E., L. Deng, M. Kozuma, K. Helmerson, S. L. Rolston, and W. D. Phillips, J. Phys. B: At. Mol. Opt. Phys. 33, 47 (2000).
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حسابات عددية لطول التناغم في ذرات ^{24}Mg، ^{39}K و ^{133}Cs لليزر الجسيمات

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الخلاصة:

استخدمنا نظرية بوز–اينشتين للتكثيف لتشكيل نموذج نظري لحساب طول التناغم بين الجسيمات المتفاعلة. ثلاثة أنواع مختلفة من الذرات عولجت في هذا النموذج لبيان صحته. قارنا نتائجنا لبعض الذرات التي سبق دراستها مع البيانات العملية والنظرية المتوفرة في المصادر وقد أظهرت المقارنة مطابقة جيدة معها. بالاعتماد على معرفتنا فأن دراسة الليزر الذري تعتبر من الدراسات الحديثة ولا تزال في بدايتها لذلك لا يمكن أن نجد دراسة شملت الذرات المغنيسيوم(24)، البوتاسيوم(39), والسيزيوم(139) في المصادر. لذلك يمكننا الادعاء إن هذه الدراسة قد أنجزت لأول مرة.