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A Numerical Method for Accelerating the Convergence of the Power Method

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الخلاصة

هذا البحث يصف وسيلة جديدة تربط بين طريقة Wilkinson مع طريقة Aitken من أجل الحصول على أفضل تقريب لأكبر قيمة ذاتية . المصفوفات المتناظرة وغير المتناظرة تم حلها . وثم ملاحظة أن الطريقة المقترحة تقترب بسرعة وأنها غير حساسة نوعاً ما إلى خواص المصفوفات المستخدمة . وتمت ا لمقارنة باستخدام خمس مسائل عددية وبالاعتماد على عدد التكرارات وعلى الزمن المضبوط المستغرق للحاسبة (CPU time). النتائج العملية أثبتت أن الوسيلة العددية الجديدة أكثر كفاءة من طرق Power، Wilkinson وAitken .

ABSTRACT

This paper describes a procedure which combines between the Wilkinson and Aitken methods in order to obtain a best approximation of the greatest eigenvalue. Both the symmetric and the nonsymmetric matrices are solved. It shows that our suggested method converges quickly and it is quit insensitive to the properties of the matrices used. A comparison between these approximations for five numerical examples is given, depending on the number of iterations and running computer time. Experimental results indicate that the new numerical procedure is more efficient than Power, Wilkinson and Aitken methods.

Introduction

Eigenvalues arise in many physical problems. Some of the physical situations where eigenvalues are required can be listed: flexural oscillations of a tapered rod, vibration problems, stability problems, buckling of columns, stability of electrical circuits etc. While trying to analyse the solution to these type of problems, we come across eigenvalue problem. These physical problems require for their solution the values of λ which satisfy the characteristic polynomial $P(\lambda) = \det(A - \lambda I) = 0$ [10].

The problem we are considering is this: Given an n×n real matrix A, find numerical approximations to the eigenvalues and eigenvectors of A. This numerical eigenproblem is difficult to solve in general. In many applications, A may be symmetric, or tridiagonal or have some other special form or property. Consequently, most numerical methods are designed for special matices [3,9]. If λ_1 is an eigenvalue of A that is larger in absolute value than any other eigenvalue, it is called the dominant eigenvalue. An eigenvector V_1 corresponding to λ_1 is called a dominant eigenvector [6].

Jennings [5] showed from theoretical considerations of Aitkin's process for accelerating the convergence of iterative processes involving matrices and with test examples to overcome the basic unreliability of Aitkin's method for a significant range of applications. Wilkinson [12] described a technique for calculating the eigenvectors of a matrix automatically by means of the Lanczos transformation. This method can be made to give accurate results for both of the Lanczos transformations. Wood et al. [13] described and contrasted the structure and implementation of a new general iterative method for diagonalising large matrices with other more commonly used iterative techniques. The method requires the direct diagonalisation of only a small submatrix, does not require the storage of the large matrix and provides eigensolutions to within a prescribed precision in a rapidly convergent iterative procedure. This technique makes possible efficient solution of a variety of quantum mechanical matrix problems where large basis set expansions are required. Canright et al. [3] introduced a fully distributed Power method for finding the principal eigenvalue of arbitrary matrices that are stored in weighted links of an overlay network. We addressed the problem of synchronization and normalization. Through the example of link analysis we demonstrated both theoretically and experimentally that the proposed method is competitive with the Power method. We have also demonstrated extreme fault tolerance. Al-Bayti and Ahmed [1] produced new technique to improve and calculate the eigenvalues and eigenvectors of the linear system. The procedure is compared with the well-known Power and Aitken methods with promising numerical results. Sun and

Wei [11] presented some notes of the PageRank algorithm, including its L_1 condition number and some observation of the numerical tests of two variant algorithms (standard Power method and Aitken extrapolation) which are based on the extrapolation method.

In this paper, the convergence of the Power method is improved to calculate the greatest eigenvalue of any matrix by using Wilkinson's method and Aitkin's method, and also suggested a new algorithm which combines between the Wilkinson with the Aitken methods in order to obtain the best approximation.

1. The Power Method

Assume that the n×n matrix A has n distinct eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_n$ and that they are ordered in decreasing magnitude: that is,

$$|\lambda_1| > |\lambda_2| \geq |\lambda_3| \geq \ldots \geq |\lambda_n| \qquad \text{.................. (1)}$$

If x_0 is chosen appropriately, then the sequences $\left\{X_k = [x_1^{(k)} \; x_2^{(k)} \; \ldots \; x_n^{(k)}]^t\right\}$ and $\{c_k\}$ generated recursively by

$$Y_k = \mathrm{A}X_k \qquad \text{.................. (2)}$$

and

$$X_{k+1} = \frac{1}{c_{k+1}} Y_k \qquad \text{.................. (3)}$$

where $c_{k+1} = x_j^{(k)}$ and $x_j^{(k)} = \max_{1 \le i \le n} \left\{ |x_i^{(k)}| \right\}$ will converge to the dominant eigenvector V_1 and eigenvalue λ_1, respectively [7]. That is $\lim_{k \to \infty} X_k = V_1$ *and* $\lim_{k \to \infty} c_k = \lambda_1$

The rate of convergence of the Power method depends upon how fast the ratios $\left(\lambda_i / \lambda_1 \right)^k$ go to zero. Therefore the Power method may converge slowly when the roots are very much closer and hence we would like to accelerate the convergence by some other techniques [4,11].

Algorithm 1 (Power Method): [1]

To approximate the dominant eigenvalues and an associated eigenvector of the nxn matrix A given a non-zero vector X .

INPUT dimension n, matrix A, vector X, tolerance TOL. Maximum number of iterations N.

OUTPUT approximate eigenpair (λ, X) with $\|X\|_\infty = 1$ or a message that the maximum number of iterations is exceeded

Step 1: Set k=1

Step 2: Find an integer p with $1 \le p \le n$ and $|x_p| = \|X\|_\infty$

Step 3: Set $X = \frac{1}{x_p} X$

Step 4: If $(k > N)$ goto step 12

Step 5: Set Y=AX

Step 6: Set $\mu = y_p$

Step 7: Find an integer p with $1 \le p \le n$ and $|y_p| = \|Y\|_\infty$

Step 8: If $y_p = 0$ stop and start with a new vector X .

Step 9: Set $ERR = \|X - (1/y_p)Y\|_\infty$ and X= $(1/y_p)$ Y

Step 10: If ERR < TOL then output (μ,X)(procedure completed successfully) Stop.

Step 11: Else set k=k+1 goto step 5.

Step 12: Output (Maximum number of iterations exceeded); (procedure completed unsuccessfully) stop.

2. The Wilkinson's Method

Suppose that all the eigenvalues are real but that the Power method is converging slowly because there are two eigenvalues nearly equal in magnitude. By the theory of eigenvalues, we know that A and A-qI have the same eigenvectors. If λ is the eigenvalue of A, then $(\lambda - q)$ is the eigenvalue of (A-qI) and the eigenvector in both the cases being same. The optimal choice of q if we wish to converge to λ_1 is that value which minimizes $\max_{2 \le i \le n} \left| \frac{\lambda_i - q}{\lambda_1 - q} \right|$. Hence we have

$$AX = \lambda X$$

and $$(A - qI)X = (\lambda - q)X \qquad \ldots\ldots\ldots\ldots\ldots (4)$$

now, we proceed to find the largest eigenvalue of (A-qI) (given in eq.(4)) using the Power method. By a judicious choice of q, it may be possible to speed up the convergence markedly. The Power method applied on the new system (A-qI) would give the largest eigenvalue of (A-qI) which is the smallest eigenvalue of A. It is found that, if all the eigenvalues are positive, the optimum value of q is $\frac{\lambda_2 + \lambda_n}{2}$ where λ_n is the smallest eigenvalue of A. If we choose $q = \lambda_n$, there is also an improved rate of convergence. The iteration technique with $q = \frac{\lambda_2 + \lambda_n}{2}$ will have maximum rate of convergence to $\lambda_1 - q$ and alternatively with

$q=\frac{\lambda_1+\lambda_{n-1}}{2}$ we have maximum rate of convergence to λ_n-q. Hence the choice of q depends on the eigenvalues $\lambda_1,\lambda_2,...,\lambda_n$ of the matrix A [10,13].

Algorithm 2 (Wilkinson Method):

INPUT dimension n, matrix A, vector X, tolerance TOL. Maximum number of iterations N .

OUTPUT approximate eigenpair (λ, X) with $\|X\|_\infty=1$ or a message that the maximum number of iterations is exceeded

Step 1: Find $q=\frac{\lambda_2+\lambda_n}{2}$

Step 2: Evaluate A-qI

Step 3: Set k=1

Step 4: Find an integer p with $1\le p\le n$ and $|x_p|=\|X\|_\infty$

Step 5: Set $X=\frac{1}{x_p}X$

Step 6: If $(k>N)$ goto step 14

Step 7: Set Y=(A-qI)X

Step 8: Set $\mu=y_p$

Step 9: Find an integer p with $1\le p\le n$ and $|y_p|=\|Y\|_\infty$

Step 10: If $y_p=0$ stop and start with a new vector X .

Step 11: Set $ERR=\|X-(1/y_p)Y\|_\infty$ and X= $(1/y_p)$ Y

Step 12: If ERR < TOL then output (μ,X) (procedure completed successfully) Stop.

Step 13: Else set k=k+1 goto step 7.

Step 14: Output (Maximum number of iterations exceeded); (procedure completed unsuccessfully) stop.

3. The Aitken's Method

This process uses the form

$$\dot{\mu}=\mu_0-\frac{(\mu_1-\mu_0)^2}{\mu-2\mu_1+\mu_0} \qquad \text{...............} \quad (5)$$

which is approximation to the greatest eigenvalue λ_1, where $\mu_0,\mu_1, and\ \mu$ are the eigenvalues in the first step, second step and third step respectively, which are obtained from the Power method. This method is applicable to any linearly convergence sequence, the rate of convergence of this method for symmetric matrices is $O((\lambda_2/\lambda_1)^{2m})$ see [3].

Algorithm 3 (Aitken Method): [1]

It is modification of Power algorithm

INPUT dimension n, matrix A, vector X, tolerance TOL. Maximum number of iterations N.

OUTPUT approximate eigenpair (λ, X) with $\|X\|_\infty = 1$ or a message that the maximum number of iterations is exceeded.

Step 1: k=1 set $\mu_0=0$, $\mu_1=0$

Step 2: If (k >N) goto step 10

Step 3: Set Y=AX

Step 4: Set $\mu = y_p$; $\dot{\mu} = \mu_0 - \dfrac{(\mu_1 - \mu_0)^2}{\mu - 2\mu_1 + \mu_0}$

Step 5: Find an integer p with 1≤p≤n and $|y_p| = \|Y\|_\infty$

Step 6: If $y_p=0$ stop and start with a new vector X .

Step 7: Set ERR=$\|X - (1/y_p)Y\|_\infty$ and X=$(1/y_p)$Y

Step 8: If ERR <TOL then output (μ,X) (procedure completed successfully) stop.

Step 9: Else set k=k+1

$$\mu_0 = \mu_1$$
$$\mu_1 = \mu$$

Goto step 3

Step 10: Output (Maximum number of iterations exceeded and procedure completed unsuccessfully) stop.

4. The Suggested Method

This method combines between the Wilkinson method and Aitken method in order to generate the best approximation. Firstly in this method we compute the matrix (A-qI) where the optimal choice of q is that value which minimizes

$$\max_{2 \le i \le n} \left| \frac{\lambda_i - q}{\lambda_1 - q} \right|$$

that is the optimum value of q is $\frac{\lambda_2 + \lambda_n}{2}$, where λ_n is the smallest eigenvalue of A, and then we use the Power method to find the dominant eigenvalue. Secondly we apply the Aitken formula to results from the first step. This iteration technique with $q = \frac{\lambda_2 + \lambda_n}{2}$ will have maximum rate of convergence to $\lambda_1 - q$, and if the matrix A has rank 2×2 then we choose $q = \lambda_n = \lambda_2$, there is also an improved rate of convergence. We will show that the suggested method converges fastly.

Theorem(1): Let a real sequence $\mu_n\,(n=1,2,...)$ converge to λ so that $d_n \neq 0\, where\; d_n = \mu_n - \lambda$ for large enough n and let

$$\frac{d_{n+1}}{d_n} \to \alpha\ (n \to +\infty), |\alpha| < 1$$

If we define $\dot{\mu}_n$ by

$$\dot{\mu}_n = \mu_n - \frac{(\mu_{n+1} - \mu_n)^2}{\mu_{n+2} - 2\mu_{n+1} + \mu_n}$$

then $$\frac{\dot{\mu}_n - \lambda}{\mu_n - \lambda} \to 0,\ \ (n \to \infty)$$

Therefore $\dot{\mu}_n$ converges to λ faster than μ_n .

Proof: See [8].

Theorem(2): Let $\mu_n\,(n=1,2,...)$ satisfy the conditions of Theorem (1) then

$$\frac{\mu_n^* - \lambda}{\mu_n - \lambda} \to 0,\ \ (n \to +\infty)$$

Therefore μ_n^* converges to λ faster than μ_n .

Proof: Using Theorem (1), we have

$d_n = \mu_n - \lambda - q + q = (\mu_n - q) - (\lambda - q)$, for large enough n and

$$\frac{d_{n+1}}{d_n} \to \hat{A}\,(n \to +\infty), |\hat{A}| < 1$$

Taking the ratio of the eigenvalues of the proposed algorithm, we have:

$$\frac{\mu_n^* - \lambda}{\mu_n - \lambda} = 1 - \frac{(\Delta\mu_n)^2}{(\mu_n - \lambda)\Delta^2\mu_n}$$

$$= 1 - \frac{\left(\sum_{t=0}^{1} (-1)^t \binom{1}{t} \hat{A}^{1-t}\right)^2}{\sum_{t=0}^{2} (-1)^t \binom{2}{t} \hat{A}^{2-t}}$$

$$= 1 - \frac{(\hat{A} - 1)^2}{(\hat{A} - 1)^2} = 0\ \ , where\ \ n \to +\infty$$

This completes the proof.

Algorithm 4 (New):

INPUT dimension n, matrix A, vector X, tolerance TOL. Maximum number of iterations N .

OUTPUT approximate eigenpair (λ,X) with $\|X\|_\infty = 1$ or a message that the maximum number of iterations is exceeded.

Step 1: Find $q = \frac{\lambda_2 + \lambda_n}{2}$

Step 2: Evaluate A-qI

Step 3: Set k=1 and μ_0=0 , μ_1=0

Step 4: If (k > N) goto step 12

Step 5: Set Y=(A-qI)X

Step 6: Set $\mu = y_p$

Step 7: Find an integer p with 1≤p≤n , $|y_p| = \|Y\|_\infty$

and compute $\mu^* = \mu_0 - \frac{(\mu_1 - \mu_0)^2}{\mu - 2\mu_1 + \mu_0}$

Step 8: If y_p =0 stop and start with a new vector X .

Step 9: Set ERR=$\|X - (1/y_p)Y\|_\infty$ and X=(1/y_p)Y

Step 10: If ERR <TOL then output (μ,X) stop.

Step 11: Else set k=k+1

$\mu_0 = \mu_1$

$\mu_1 = \mu$

Goto step 5

Step 12: Output (Maximum number of iterations exceeded) stop.

5. Numerical Results

In this section we present five numerical examples to test the effectiveness of these methods and compare the suggested algorithm with the usual Power, Wilkinson and Aitken methods, where identical Matlab7 programs are used and R.T. is running time (CPU time).

Example (1): [6] consider the matrix

$$A = \begin{bmatrix} 1 & 6 \\ 9 & 2 \end{bmatrix}$$

The exact eigenvalues of A are $\lambda_1 = 8.8655$, $\lambda_2 = -5.8655$

we compute $q = \lambda_2 = -5.8655$

The numerical values of this solution are presented in table (1).

Table (1): Results of the numerical methods to find the dominant eigenpair of the matrix in ex.(1)

k	Approximate values of the dominant eigenvalue by the methods			
	Power	Wilkinson	Aitken	Proposed
1	11	11		
2	7.7273	8.8655		
3	9.7294	8.8655	8.9695	8.8655
4	8.3446		8.9108	
5	9.2316		8.8853	
6	8.6329		8.8742	
7	9.0235		8.8693	
8	8.7627		8.8671	
9	8.9342		8.8662	
10	8.8203		8.8658	
11	8.8955		8.8656	
12	8.8457		8.8655	
13	8.8786		8.8655	
14	8.8568			
15	8.8712			
16	8.8617			
17	8.8680			
18	8.8638			
19	8.8666			
20	8.8647			
21	8.8659			
22	8.8651			
23	8.8657			
24	8.8653			
25	8.8656			
26	8.8654			
27	8.8655			
R.T.	00:00:015	00:00:01	00:00:015	00:00:01
Eigenvector		1.0000 0.7628		

Example (2):[2] consider the matrix

$$A = \begin{bmatrix} 4 & -1 & 1 \\ -1 & 3 & -2 \\ 1 & -2 & 3 \end{bmatrix}$$

The exact eigenvalues of A are $\lambda_1 = 6, \lambda_2 = 3, \lambda_3 = 1$

we compute $q = \frac{\lambda_2 + \lambda_3}{2} = \frac{3+1}{2} = 2$

The numerical values of this solution are presented in table (2).

Table (2): Results of the numerical methods to find the dominant eigenpair of the matrix in ex.(2)

k	Approximate values of the dominant eigenvalue by the methods			
	Power	Wilkinson	Aitken	Proposed
1	4	4		
2	4.5000	5		
3	5	5.6667	-Inf	7.0003
4	5.4000	5.9091	7.0000	6.0476
5	5.6667	5.9767	6.2003	6.0028
6	5.8235	5.9942	6.0472	6.0003
7	5.9091	5.9985	6.0120	5.9999
8	5.9538	5.9996	6.0027	6.0000
9	5.9767	5.9999	6.0008	6.0000
10	5.9883	6.0000	6.0002	
11	5.9942	6.0000	6.0003	
12	5.9971		5.9999	
13	5.9985		5.9998	
14	5.9993		6.0004	
15	5.9996		5.9998	
16	5.9998		6.0002	
17	5.9999		6.0000	
18	6.0000		5.9997	
19	6.0000		6.0000	
R.T.	00:00:016	00:00:016	00:00:016	00:00:015
Eigenvector		1.0000 -1.0000 1.0000		

Example (3):[7] Let

$$A = \begin{bmatrix} 2 & 3 & 1 \\ 5 & 4 & 8 \\ 0 & 8 & 1 \end{bmatrix}$$

The exact eigenvalues of A are $\lambda_1 = 11.8194,\ \lambda_2 = 1.3111, \lambda_3 = -6.1305$

we compute $q = \frac{1.3111 - 6.1305}{2} = \text{-}2.4097$

The numerical values of this solution are presented in table (3).

Table (3): Results of the numerical methods to find the dominant eigenpair of the matrix in ex.(3)

k	Approximate values of the dominant eigenvalue by the methods			
	Power	Wilkinson	Aitken	Proposed
1	17	17		
2	10	10.8690		
3	12.9412	12.0977	12.0710	11.8926
4	11.2886	11.7511	11.8831	11.8274
5	12.1077	11.8381	11.8363	11.8206
6	11.6734	11.8147	11.8239	11.8197
7	11.8961	11.8207	11.8206	11.8195
8	11.7799	11.8191	11.8197	11.8194
9	11.8400	11.8195	11.8195	11.8194
10	11.8088	11.8194	11.8194	
11	11.8249	11.8194	11.8194	
12	11.8165			
13	11.8209			
14	11.8186			
15	11.8198			
16	11.8192			
17	11.8195			
18	11.8193			
19	11.8194			
20	11.8194			
R.T.	00:00:015	00:00:016	00:00:016	00:00:015
Eigenvector		0.3808 1.0000 0.7394		

Example (4):[10] Let

$$A = \begin{bmatrix} 1 & 4 & 6 \\ -3 & 4 & 3 \\ 2 & -1 & 5 \end{bmatrix}$$

The exact eigenvalues of A are

$\lambda_1 = 7$, $\lambda_2 = 1.5000 + 2.9580i$, $\lambda_3 = 1.5000 - 2.9580i$

and $q = 1.5$

The numerical values of this solution are presented in table (4).

Table (4): Results of the numerical methods to find the dominant eigenpair of the matrix in ex.(4)

k	Approximate values of the dominant eigenvalue by the methods			
	Power	Wilkinson	Aitken	Proposed
1	11	11		
2	5.7273	4.8947		
3	5.7937	7.2713	5.7929	6.6054
4	7.0575	6.6474	5.7236	6.7771
5	6.9006	7.3197	6.9179	6.9710
6	6.9843	6.7254	6.9552	7.0043
7	7.0174	7.0230	7.0391	6.9237
8	6.9690	6.9688	6.9977	6.9772
9	6.9891	7.0266	6.9832	6.9968
10	6.9996	6.9759	7.0111	6.9996
11	6.9986	7.0019	6.9987	6.9931
12	7.0004	6.9974	6.9992	6.9981
13	6.9999	7.0022	7.0000	6.9997
14	6.9996	6.9980		7.0000
15	7.0000	7.0002		
16		6.9998		
17		7.0000		
R.T.	00:00:016			
Eigenvector		1.0000 0.0000 1.0000		

Example (5):[6] consider the matrix

$$A = \begin{bmatrix} 2.5 & -2.0 & 2.5 & 0.5 \\ 0.5 & 5.0 & -2.5 & -0.5 \\ -1.5 & 1.0 & 3.5 & -2.5 \\ 2.0 & 3.0 & -5.0 & 3.0 \end{bmatrix}$$

The exact eigenvalues of A are $\lambda_1 = 6, \lambda_2 = 4, \lambda_3 = 3, \lambda_4 = 1$

we compute $q = \dfrac{\lambda_2 + \lambda_4}{2} = 2.5$

The numerical values of this solution are presented in table (5).

Table (5): Results of the numerical methods to find the dominant eigenpair of the matrix in ex.(5)

k	Approximate values of the dominant eigenvalue by the methods			
	Power	Wilkinson	Aitken	Proposed
1	3.5000	4.5000		
2	6.0000	8.6250		
3	7.2857	5.4490	8.6470	6.8306
4	6.8039	6.6540	6.9352	6.3226
5	6.4870	5.8988	5.8780	6.1898
6	6.3025	6.1042	6.0454	6.0603
7	6.1924	5.9814	6.0295	6.0273
8	6.1243	6.0187	6.0139	6.0100
9	6.0812	5.9966	6.0069	6.0048
10	6.0534	6.0034	6.0029	6.0018
11	6.0353	5.9994	6.0015	6.0009
12	6.0234	6.0006	6.0006	6.0003
13	6.0155	5.9999	5.9999	6.0002
14	6.0103	6.0001	6.0003	6.0001
15	6.0069	6.0000	6.0005	6.0000
16	6.0046	6.0000	5.9998	6.0000
17	6.0030		5.9993	
18	6.0020		6.0003	
19	6.0014		6.0005	
20	6.0009		5.9984	
21	6.0006		6.0002	
22	6.0004		6.0000	
23	6.0003		6.0002	
24	6.0002		6.0003	
25	6.0001		6.0001	
26	6.0001		6.0000	
27	6.0000		6.0000	
28	6.0000			
R.T.	00:00:016	00:00:016	00:00:016	00:00:015
Eigenvector		-0.5000 0.5000 -0.5000 1.0000		

7. Discusions and Concluions

The convergence of Power method of first degree and the speed of convergence is slow so that more iteration steps are needed, it is the simplest iterative method. The Aitken and Wilkinson methods converge faster than the Power method. The Wilkinson method requires that all the eigenvalues are real and the Aitken method requires more computations per iteration compard with the Power method and Wilkinson method. The hybrid algorithm converges faster than the Power, Wilkinson and Aitken methods and requires more computations per iteration. It is quit insensitive to the properities of the matrices used and it evalutes the greatest eigenpair of any matrix.

All the algorithms are used the initial unit vector and the tolerance is 10^{-4}, our numerical results, in tables (1,2,3,4 and 5), indicate that the proposed algorithm performs better than the Power method and the results are given in Table (6). Specifically quoting the number of iteration (NOI) and running computer time (R.T.) as a basis of the comparison.; namely taking the Power method as 100% NOI the proposed method needs 35.8% NOI while Aitken method needs 64.2% NOI and Wilkinson method needs 50.9% NOI. Note that a clear looking to test example no. (4) it is obvious that the new method is stable for finding the dominant eigenvalue while the other methods are alternating to reach the solution with the same running time, and the Wilkinson method is very slow because some of eigenvalues of this example are complex.

Finally, we have noticed that the proposed method seems to be accurate, fast, and gives very encouraging numerical results for the considered matrices.

Table (6): Comparison between the suggested method and the Power, Wilkinson and Aitken methods.

No. of examples	NOI of Power	NOI of Wilkinson	NOI of Aitken	NOI of Proposed
1	27	2	10	1
2	18	10	15	6
3	19	10	8	6
4	15	17	11	12
5	27	15	24	13
Total	106	54	68	38

(a)

Power method	Wilkinson method	Aitken method	Proposed method
100%	50.9%	64.2%	35.8%

(b)

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