Collective C2 and C3 Transitions in ^{17}O

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Abstract:

The longitudinal form factors for C2 and C3 transitions to some positive and negative parity excited states in ^{17}**O** nucleus are investigated in the framework of Particle-Vibration Coupling Model (PVCM). Configuration mixing is allowed for the PVCM states that include up to two octupole phonons and three quadrupole phonons of the collective states, and $1d_{5/2}$, $2s_{1/2}$ and $1d_{3/2}$ harmonic oscillator shells of the single-particle states. The total Hamiltonian is diagonalized in this space. Admixture of higher configurations is considered for the ground state wave function, and the core polarization effect is introduced by giving the odd nucleon an effective charge to obtain the best description for the data.

Keywords: (e,e') longitudinal C2 and C3 form factors, Particle-Vibration Coupling Model (PVCM).

1. Introduction:

Inelastic electron scattering gives valuable information about the nuclear structure of the excited states due to the firm correlation of the nuclear form factors with the local charge and current density operators [1]. Those density operators are constructed, in the framework of PVCM, from two parts: the single-particle and the vibrating core, and are given by Brown *et al.* [2] and Walecka [3], respectively. In spite of the widespread application of this semi-microscopic collective model to study the nuclei in the rare-earth region and neighboring to the ^{208}**Pb** [4], it is applied in the present work to study the structure of some states that are excited strongly by C2 and C3 transitions and are mainly collective in nature.

The experimental values of the form factors for electron scattering from ^{17}**O** are measured by Manley *et al.* [5] and interpreted in their work within the framework of weak-coupling model of particles and holes to facilitate a simple description of the observed spectrum. According to the PVCM, in the present work, states of ^{17}**O** are constructed from a single neutron that moves in a harmonic oscillator potential well and being in coupling with the collective vibrational motion of ^{16}**O** core nucleus that described macroscopically.

2. Theory:

The total Hamiltonian of PVCM is given by [6]

$$H = H_{coll} + H_{s.p.} + H_{\text{int}} , \qquad \text{….(1)}$$

with the collective vibration Hamiltonian of the core,

$$H_{coll} = \sum_{\lambda\mu} \hbar\omega_\lambda [b^\dagger_{\lambda\mu} b_{\lambda\mu} + \tfrac{1}{2}], \qquad \text{….(2)}$$

the single-particle shell model Hamiltonian,

$$H_{s.p.} = \sum_{jm} \varepsilon_j a^\dagger_{jm} a_{jm} , \qquad \text{…(3)}$$

and the particle-core interaction Hamiltonian,

$$H_{\text{int}} = -\sum_{\lambda\mu} \sqrt{\frac{\pi}{2\lambda+1}} \xi_\lambda \hbar\omega_\lambda [b_{\lambda\mu} + (-1)^\mu b^\dagger_{\lambda-\mu}] Y_{\lambda\mu}(\hat{r}) \; \text{..(4)}$$

The operators $b^{\dagger}_{\lambda\mu}$ and $a^{\dagger}_{jm}$ are, respectively, boson (phonon) and particle (neutron) creation operators and, $b_{\lambda\mu}$ and a_{jm} are the corresponding annihilation operators. $\hbar\omega_{\lambda}$ is the energy of the phonon of multipolarity λ, which can be taken from the excitation energy corresponding to the transition $0 \rightarrow \lambda$ in the adjacent even-even nucleus. The single-particle energy ε_j, may be determined experimentally.

The strength parameter ξ_{λ}, for the coupling of the single nucleon, moves in the harmonic oscillator potential, with the vibrating core can be estimated to be:

$$\xi_{\lambda} = 41\, A^{-1/3}\,(N + \tfrac{3}{2})\sqrt{\frac{2\lambda + 1}{2\pi\,\hbar\omega_{\lambda} C_{\lambda}}}\ , \quad \ldots(5)$$

where $N = 2(n-1) + \ell$, with n and ℓ are the principal and orbital quantum numbers, and the stiffness parameter of the surface of the nuclear core C_{λ}, can be given in terms of the reduced transition probability $B(E\lambda\uparrow)$, by [7]:

$$C_{\lambda} = \left(\frac{3}{4\pi} ZeR_0^{\lambda}\right)^2 \frac{(2\lambda+1)\hbar\omega_{\lambda}}{2B(E\lambda\uparrow)}, \quad \ldots(6)$$

A and Z are the mass and atomic numbers of the core nucleus of equilibrium radius $R_0\,(fm) = 1.2A^{1/3}$.

Diagonalization of the total Hamiltonian matrix elements between the mixed configurations of PVCM states $\left|(N_3R_3N_2R_2)R, j; I\right\rangle$, yields eigenvalues ε, and eigenvectors $c_{\alpha}(R, j; I)$, which are used to calculate the electron scattering form factor of a given multipolarity J, and momentum transfer q, as follow:

$$\begin{aligned}
&|F_J(q)|^2 \\
&= \frac{4\pi}{Z^2}(2I'+1)\Bigg|\sum_{\substack{N_3'R_3'N_2'\\ R_2'R'j'}}\ \sum_{\substack{N_3R_3N_2\\ R_2Rj}} c_{\beta}((N_3'R_3'N_2'R_2')R', j'; I')\, c_{\alpha}((N_3R_3N_2R_2)R, j; I) \\
&\times\Bigg[(-1)^{R+j+I'+J}\begin{Bmatrix} j' & j & J \\ I & I' & R' \end{Bmatrix}\left\langle j'\left\| T_J^{(1)}(q)\right\| j\right\rangle F_{f.s.}(q)F_{c.m.}(q)\,\delta_{N_3N_3'}\delta_{R_3R_3'}\delta_{N_2N_2'} \qquad \ldots(7) \\
&\times\delta_{R_2R_2'}\delta_{RR'} + (-1)^{R'+j'+I+J}\begin{Bmatrix} R' & R & J \\ I & I' & j \end{Bmatrix}\left\langle R'\left\| T_J^{(2)}(q)\right\| R\right\rangle F_d(q)\delta_{\ell\ell'}\delta_{jj'}\Bigg]\Bigg|^2
\end{aligned}$$

where N_3 and N_2 are the number of octupole and quadrupole phonons that coupled to angular momentum R_3 and R_2, respectively. R and j are, respectively the angular momentum of the core and the single-particle that coupled to the total angular momentum of the nucleus I. α and β are the remaining quantum numbers for the initial (unprimed) and final (primed) states, respectively.

The reduced single-particle matrix elements of the longitudinal electron scattering operator $T_J^{(1)}(q)$, used in this work are those of ref. [2]. While the reduced collective matrix elements of the longitudinal collective operator, linear in the deformation parameter, will take the following forms according to the value of the multipolarity J,

$$\left\langle R' \left\| T^{(2)}_{J=2}(q) \right\| R \right\rangle = \sqrt{\frac{\hbar\omega_2}{2C_2}} \frac{3Z}{4\pi} j_2(qR_0)\,(-1)^{R'+R'_3+2} \sqrt{(2R+1)(2R'+1)} \begin{Bmatrix} R'_2 & R_2 & 2 \\ R & R' & R'_3 \end{Bmatrix}, (8)$$
$$\times\left[(-1)^{R_2}\left\langle N'_2 R'_2 \left\| b^\dagger_2 \right\| N_2 R_2 \right\rangle + (-1)^{R'_2}\left\langle N_2 R_2 \left\| b^\dagger_2 \right\| N'_2 R'_2 \right\rangle\right]\delta_{N_3 N'_3}\delta_{R_3 R'_3}$$

and

$$\left\langle R' \left\| T^{(2)}_{J=3}(q) \right\| R \right\rangle = \sqrt{\frac{\hbar\omega_3}{2C_3}} \frac{3Z}{4\pi} j_3(qR_0)\,(-1)^{R+R_2+3} \sqrt{(2R+1)(2R'+1)} \begin{Bmatrix} R'_3 & R_3 & 3 \\ R & R' & R'_2 \end{Bmatrix}, (9)$$
$$\times\left[(-1)^{R'_3}\left\langle N'_3 R'_3 \left\| b^\dagger_3 \right\| N_3 R_3 \right\rangle + (-1)^{R_3}\left\langle N_3 R_3 \left\| b^\dagger_3 \right\| N'_3 R'_3 \right\rangle\right]\delta_{N_2 N'_2}\delta_{R_2 R'_2}$$

where the reduced matrix elements of the quadrupole and octupole boson creation operators can be regarded as boson coefficient of fractional parentage (BCFP) given in ref.[7]. $F_{f.s.}(q) = \exp\left(-0.43q^2/2\right)$ [8] is the single-nucleon form factor that corrects the finite size of the nucleon, $F_{c.m.}(q) = \exp\left(q^2b^2/4A\right)$ [8] is the center-of-mass form factor that corrects the lack of translational invariance in shell model wave functions, and $F_d(q) = \exp\left(-0.693q^2/2\right)$ [9] is the diffuseness form factor that corrects the diffuseness of the nuclear surface of the core nucleus. The parameter b is the harmonic oscillator size parameter.

3. Results and discussion:

In the application of PVCM on ^{17}**O**, the odd neutron is taken to occupy the orbits within the 2s-1d harmonic oscillator shells, i.e. $1d_{5/2}$, $2s_{1/2}$ and $1d_{3/2}$ which has the following unperturbed energies: 0, 0.87 and 5.08 MeV [10], respectively. The core parameters are estimated from the experimental data of ^{16}**O**, thus, $\hbar\omega_\lambda, C_\lambda$ and ξ_λ are respectively, 6.917 MeV [11], 229 MeV and 1.25 for $\lambda = 2$, and 6.13 MeV [11], 40 MeV and 3.747 for $\lambda = 3$. $B(E2\uparrow) = 23\,e^2\,fm^4$ [12] and $B(E3\uparrow) = 1484.7\,e^2\,fm^6$ [13] are used. A harmonic oscillator single-particle wave function with size parameter b=1.8 *fm* [5] is employed.

The form factors are calculated with ground state wave function which is modified to include higher configurations up to the orbits of 3s-2d shells using mixing parameter γ that mixes the state $\left| R, n\ell j; I \right\rangle$ with the state $\left| R, n+1\ell j; I \right\rangle$.

3.1. C2 transitions

The electro-excitation of ^{17}**O** from its ground state at $\frac{5}{2}^+_1$ to the low lying $T = \frac{1}{2}$ excited states at $I^\pi_n(E_x, MeV) = \frac{3}{2}^+_2(5.87), \frac{1}{2}^+_2(6.36), \frac{3}{2}^+_3(7.2), \frac{7}{2}^+_1(7.58),$ $\frac{5}{2}^+_4(8.4)$, and $\frac{9}{2}^+_1(8.47)$ has a predominant longitudinal C2 component. Our PVCM calculation found these states at 7.3, 7.97, 8.27, 6.92, 11.2, and 6.3 MeV, respectively. According to the weak-coupling model calculation, the collective positive parity states at 5.57, 7.58, and 8.47 MeV are expected to be a predominantly 5p-4h levels and they found at 5.81, 7.56, and 8.68 MeV, respectively [5]. Our calculation for the longitudinal C2 form factors are displayed in figs. 1 to 7.

3.1.1. The 5.87 MeV, $\frac{3}{2}^{+}_{2}$ state

The C2 form factor for the transition to $\frac{3}{2}^{+}_{2}$ state at 5.87 MeV is shown in fig. 1. The PVCM calculations that are based on the use of the bare charge of the neutron with **γ=1.0,** are displayed as a dashed curve. Giving the odd neutron an effective charge $\mathbf{e_n=0.5e}$ enhances the calculated form factor, as shown in the dash-dot curve. An overall agreement with the experimental data is obtained by giving the odd neutron an effective charge $\mathbf{e_n=0.5e}$ and extending the ground state wave function of the PVCM with **γ=0.95**.

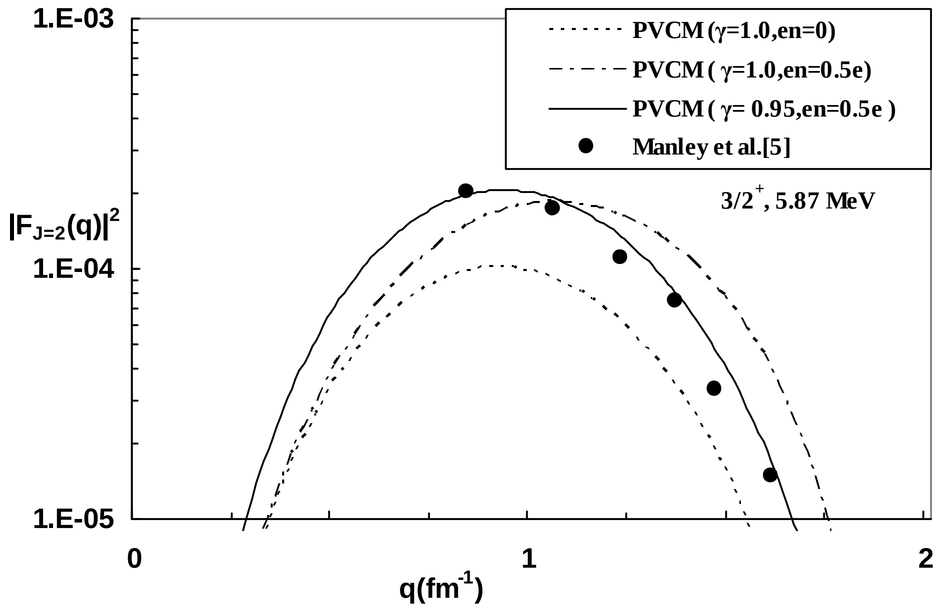

Fig. 1. The longitudinal C2 form factor for $\frac{3}{2}^{+}$, $E_x = 5.87\,\mathrm{MeV}$ **state in ^{17}O nucleus.**

3.1.2. The 6.36 MeV, $\frac{1}{2}^{+}_{2}$ and 7.2 MeV, $\frac{3}{2}^{+}_{3}$ states

Figs. 2 and 3 show the longitudinal C2 form factors for the transitions to $\frac{1}{2}^{+}_{2}$ and $\frac{3}{2}^{+}_{3}$ states at E_x=6.36 and 7.2 MeV, respectively. These states are not easily visible in the measured spectra [5]. Furthermore, the measured form factors for these levels are subjected to rather large errors.

However, our calculations of these C2 form factors which are performed by using the value of the free neutron charge in the framework of PVCM describe them very well as compared with the experimental data.

3.1.3. The 7.58 MeV, $\frac{7}{2}^{+}_{1}$ state

Fig. 4 shows the longitudinal C2 form factor for the transition to the first $\frac{7}{2}^{+}$ state at E_x=7.58 MeV in ^{17}**O** nucleus. The PVCM calculations with $\mathbf{e_n=0.35e}$ are displayed by the dashed curve. Extension of the ground state wave function with $\gamma = -0.15$ shifted the maximum of the calculated C2 form factor to the correct position and reproduces the

data excellently as shown in the solid curve.
in ^{17}**O** nucleus.

However, extending the ground state wave function with such as extreme value of **γ,** decreases the contribution of the original orbits (γ=**1),** and gives the higher orbits a predominant contribution.

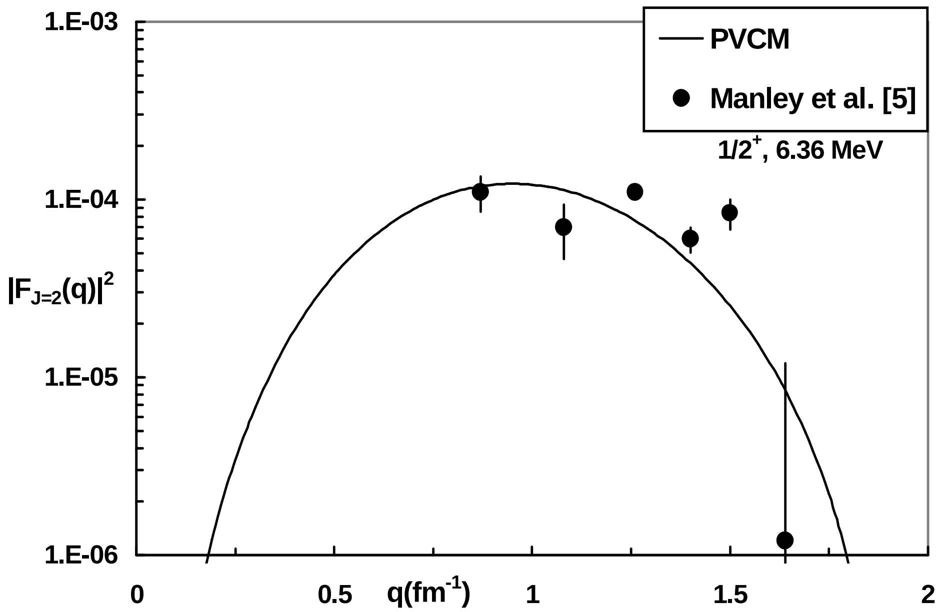

Fig. 2.The longitudinal C2 form factor for $\frac{1}{2}^{+}$, $E_X = 6.36\,\text{MeV}$ **state in** ^{17}**O nucleus.**

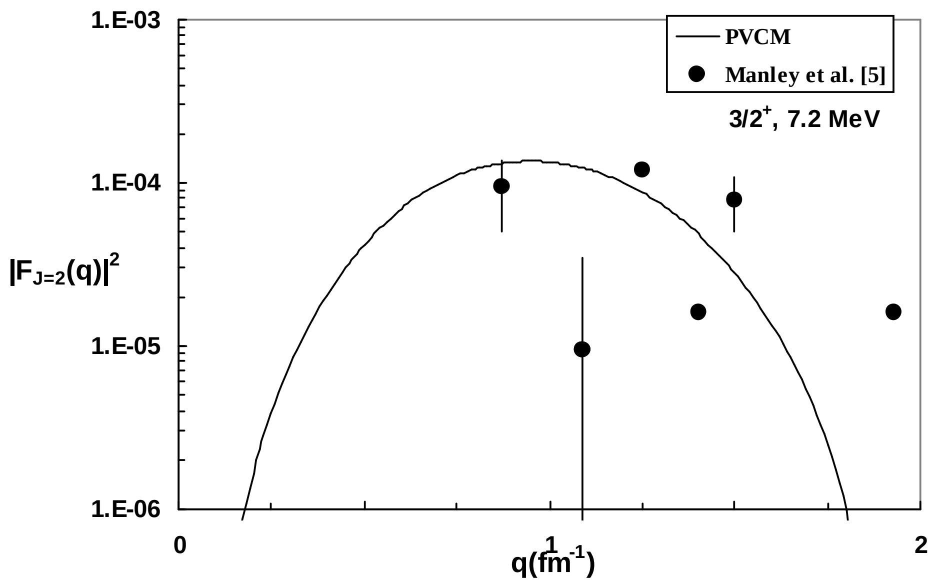

Fig. 3.The longitudinal C2 form factor for $\frac{3}{2}^{+}$, $E_X = 7.2\,\text{MeV}$ **state in** ^{17}**O nucleus.**

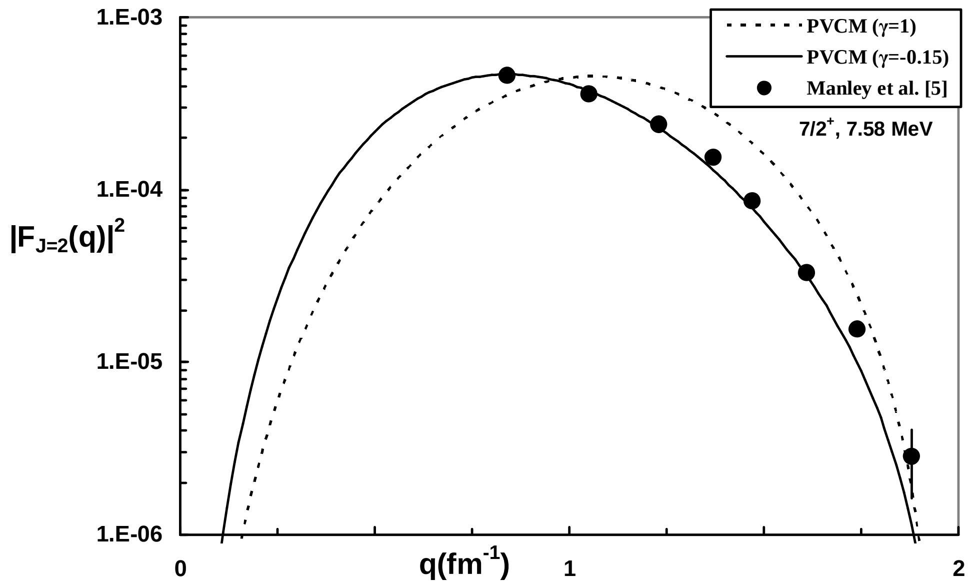

Fig. 4.The longitudinal C2 form factor for $\frac{7}{2}^+$, $E_x = 7.58\,\mathrm{MeV}$ state

3.1.4. The 8.4 MeV, $\frac{5}{2}^+_4$ state

Fig. 5 shows the longitudinal C2 form factor for the transition to $\frac{5}{2}^+_4$ state at E_x=8.4 MeV in ^{17}**O** nucleus. The dashed curve shows the PVCM calculation, giving the odd neutron an effective charge **e_n=0.91 e.** The extension of the ground state wave function with **γ=0.1** reproduces the position of the maximum with a magnitude lowered by a factor of ≈ 10.

Such an effective charge was used by Kim *et al.* [14] to obtain the best fit to their measured values of the Coulomb form factor of $\frac{3}{2}^+_1$ state at 5.08 MeV in ^{17}**O.**

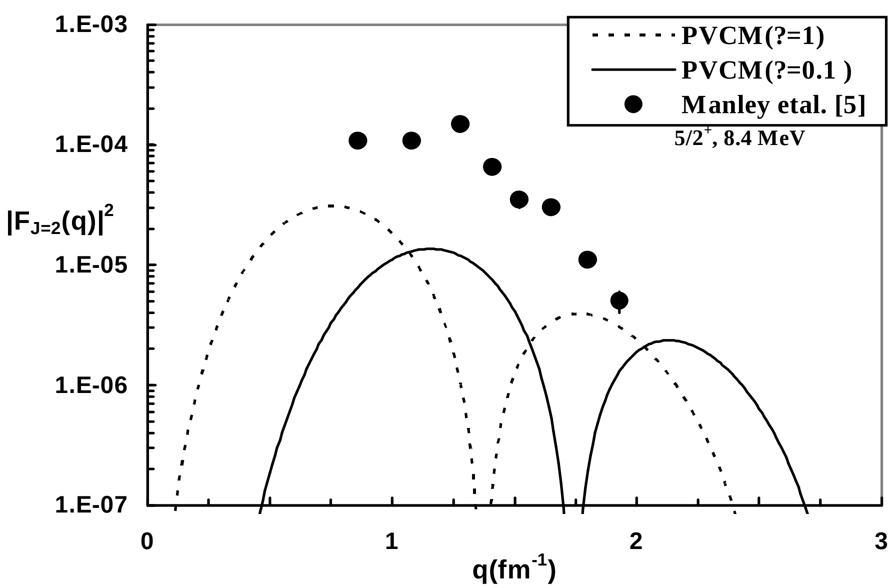

Fig. 5.The longitudinal C2 form factor for $\frac{5}{2}^+_4$, $E_x = 8.4\,\mathrm{MeV}$ state in ^{17}O nucleus.

3.1.5. The 8.47 MeV, $\frac{9}{2}_1^+$ state

Fig. 6 shows the longitudinal C2 form factor for the transition to $\frac{9}{2}_1^+$ state at E_x=8.47 MeV in ^{17}**O** nucleus. The experimental data are explained very well by using the PVCM calculation with neutron effective charge **e_n=0.65 e**.

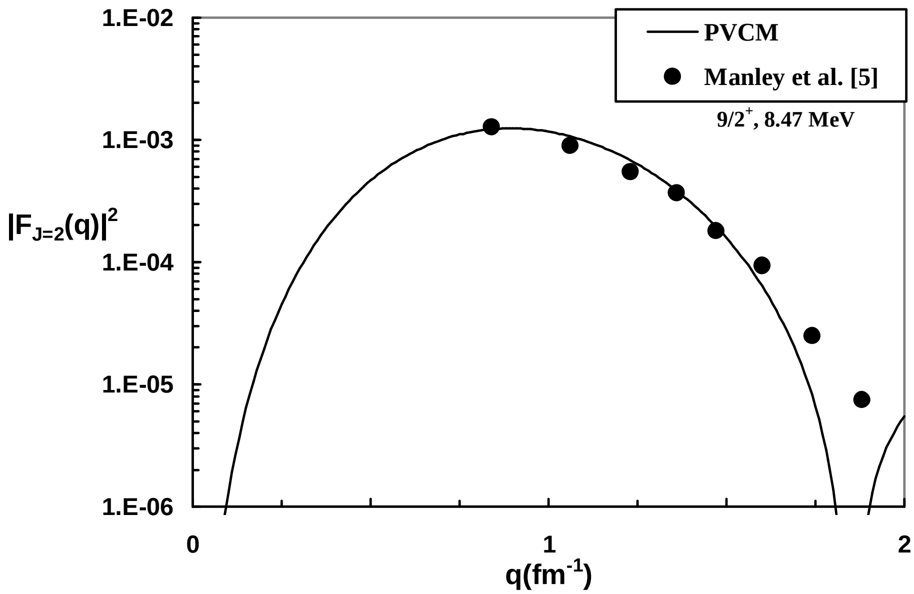

Fig. 6.The longitudinal C2 form factor for $\frac{9}{2}_1^+$, $E_x = 8.47\ \text{MeV}$ **state in ^{17}O nucleus.**

3.2. C3 Transitions

The electro-excitation of ^{17}**O** nucleus to the $T=\frac{1}{2}$ states at $\frac{1}{2}_1^-$,3.06 MeV, $\frac{9}{2}_1^-$, 5.22 MeV and $\frac{11}{2}_1^-$, 7.76 MeV have a C3 component which contributes appreciably to their longitudinal form factors. The energy eigenvalues of these states are not discriminated in our PVCM calculations where they found at 6.29 MeV, respectively. However, weak-coupling model calculation found $\frac{1}{2}_1^-$ and $\frac{11}{2}_1^-$ states at 4.39 and 8.1 MeV which are expected to be 4p-3h and 2p-1h configurations, respectively [5].

3.2.1. The 3.06 MeV, $\frac{1}{2}_1^-$ state

Fig. 7 shows the longitudinal C3 form factor for the transition to $\frac{1}{2}_1^-$state at E_x=3.06 MeV in ^{17}**O** nucleus. The dashed curve, which is calculated according to the PVCM, reproduces the shape of the measured form factor as a function of the momentum transfer, *q*. A very well agreement with the data is obtained by raising the stiffness parameter of the octupole vibrations of the core **C_3** by a factor of **4**. The result is displayed by the solid curve of fig. 7.

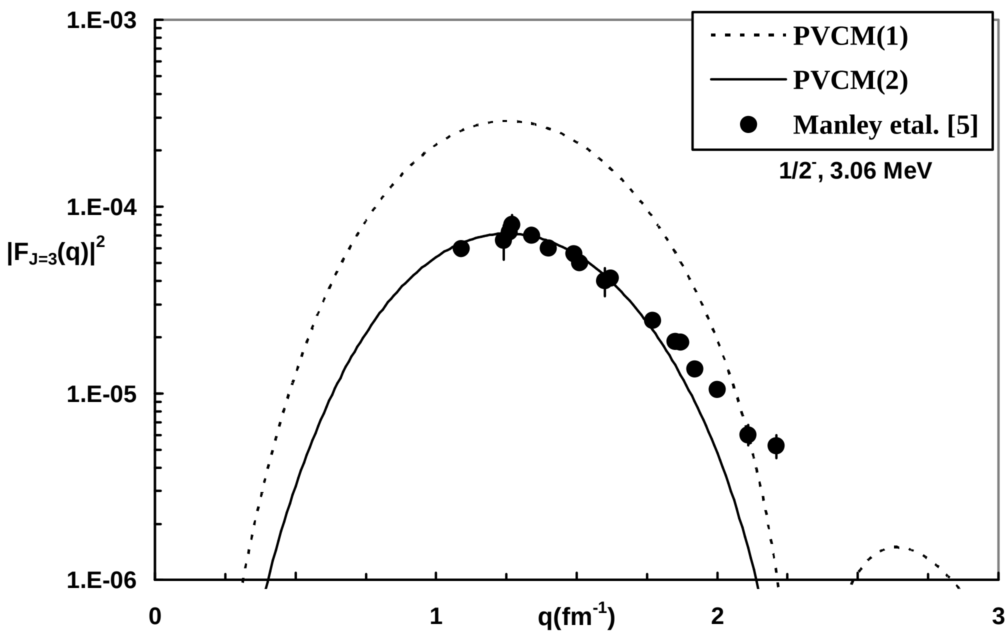

Fig. 7.The longitudinal C3 form factor for $\frac{1}{2}{}_{1}^{-}$, $E_x = 3.06\,\mathrm{MeV}$ **state in** ^{17}**O nucleus.**

3.2.2. The 5.22 MeV, $\frac{9}{2}{}_{1}^{-}$ state

Fig. 8 shows the longitudinal C3 form factor for the transition to $\frac{9}{2}{}_{1}^{-}$ state at E_x=5.22 MeV in ^{17}**O** nucleus. The PVCM based calculation shows an agreement with the experimental data up to $q \approx 1.5\ fm^{-1}$. The data are underestimated beyond $q \approx 1.5\ fm^{-1}$.

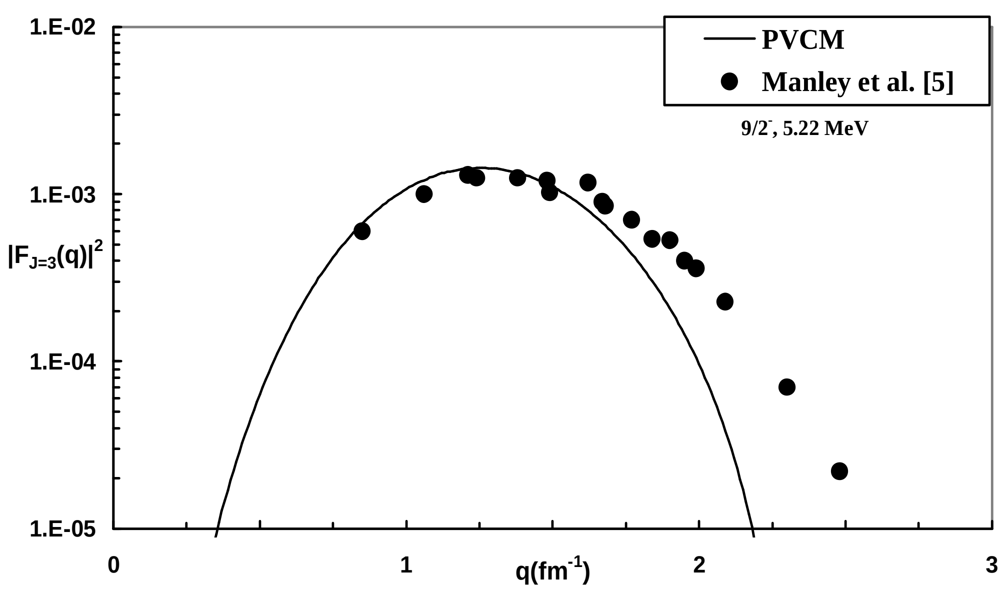

Fig. 8.The longitudinal C3 form factor for $\frac{9}{2}{}_{1}^{-}$, $E_x = 5.22\,\mathrm{MeV}$ **state in** ^{17}**O nucleus.**

3.2.3. The 7.76 MeV, $\frac{11}{2}^-_1$ state

Fig. 9 shows the longitudinal C3 form factor for the transition to $\frac{11}{2}^-_1$ state at E_x=7.76 MeV in ^{17}**O** nucleus. Our PVCM calculations are shown by the dashed curve. Raising the $\mathbf{C_3}$ parameter by a factor of **1.375** explains the measured form factors up to $q \approx 1.3\ fm^{-1}$ and underestimates the higher q data.

Calculations using 2p-1h and 4p-3h wave functions are in a reasonable agreement with the C3 form factors of those states which measured for momentum transfer up to 1.2 fm^{-1} [14].

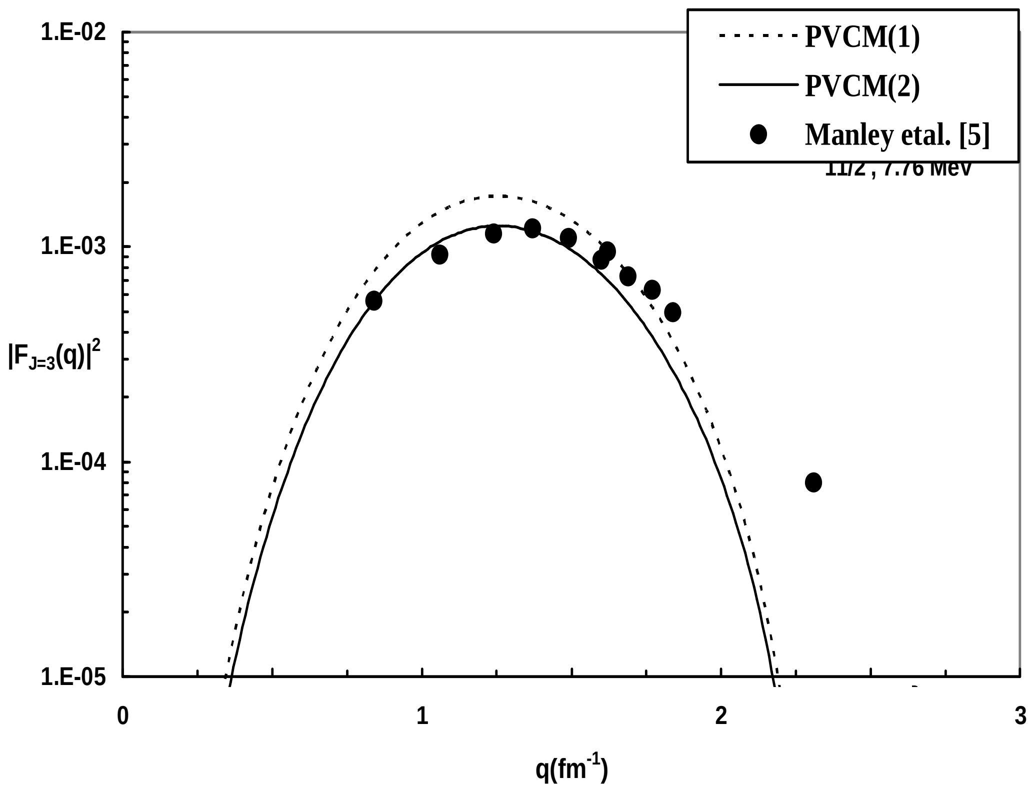

Fig. 9.The longitudinal C3 form factor for $\frac{11}{2}^-_1$ **,** $E_x = 7.76\,\text{MeV}$ **state in** ^{17}**O nucleus.**

Conclusions:

One can conclude, from the above results, the reliability of the PVCM to describe the C2 and C3 form factors of some low lying $T = \frac{1}{2}$ excited states in ^{17}**O**. It is found that PVCM must be extended to give a very well description of the data. Thus, core polarization effects are introduced by giving the odd neutron an effective charge. Also, the model space of PVCM ground state is extended to include configurations from higher orbits. Those extensions are, in fact, add more degrees of collectivity to the studied states. On the other hand, the stiffness parameter of the vibrating surface of the core may take values that deviate from that calculated at the photon point to give the best fit to the measured data.

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الانتقالات C2 و C3 التجمعية في ^{17}O

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الخلاصة:

فحصت عوامل التشكل الطولية في اطار نموذج اقتران جسيم–ذبذبة للانتقالات C2 و C3 الى بعض الحالات المتهيجة ذات التماثل الموجب و السالب لنواة **^{17}O** . سمح بالتشكيلات الممتزجة لحالات PVCM التي تتضمن الحالات التجمعية لغاية اثنين من الفونونات ثمانية القطب و ثلاثة من الفونونات رباعية القطب, و حالات الجسيم المفرد التي تضمن قشرات المتذبذب التوافقي $1d_{5/2}$, $2s_{1/2}$ و $1d_{3/2}$. حول الهاملتوني الكلي الى مصفوفة قطرية في هذا الفضاء. و للحصول على الوصف الافضل للبيانات العملية, فأن مزيجا من التشكيلات العليا أعتبرت في الدالة الموجية للحالة الارضية, كما أدخل تأثير أستقطاب اللب بأعطاء النيوكليون المفرد شحنة مؤثرة.