A relations among some kinds of continuous and open functions in topological , bitopological and tritopological spaces

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ABSTRACT :-

In this work , I introduce a relations among four kinds of continuous functions (i.e. continuous , α^*-continuous , weakly - δ-continuous , weakly - $\delta*$-continuous) and relations among four open functions (i.e. open , α^{**}-open , λ-open , λ^*-open) in Topological , Bitopological and Tritopological spaces .

Keywords :- α^*-continuous function , weakly - δ-continuous function , weakly - $\delta*$-continuous function , α^{**}-open function, λ-open function , λ^*-open function .

1-INTRODUCTION :-

Throughout this paper I adopt the notations and terminology of [4] , [5] , [3] and [2] , X and Y are finite sets and the following conventions :(X,T), (X,T,Ω), (X,T,Ω,ρ) will always denot to Topological space , Bitopological space and Tritopological space respectively .

Let (X,T) be a topological space , and let A be asubset of X, then A is said to be a α-open set iff $A \subseteq T-\text{int}(T-cl(T-\text{int}(A)))$ [4] , and the family of all α-open sets is denoted by $\alpha.O(X)$. The complement of α-open set is called α-closed set .

Let (X,T,Ω) be a Bitopological space , and let A be asubset of X, then A is said to be a δ-open set iff $A \subseteq T-\text{int}(\Omega-cl(T-\text{int}(A)))$ [3] , and the family of all δ-open sets is denoted by $\delta.O(X)$. The complement of δ-open set is called δ-closed set .

And let (X,T,Ω,ρ) be a Tritopological space , a subset A of X is said to be $\delta*$-open set iff $A \subseteq T-\text{int}(\Omega-cl(\rho-\text{int}(A)))$ [2] , and the family of all $\delta*$-open sets is denoted by $\delta^*.O(X)$. The complement of $\delta*$-open set is called a $\delta*$-closed set .

The definitions of the four continuous functions are :-

1. The function f : $(X,T) \longrightarrow (Y,T')$ is said to be **continuous** at x∈ X iff for every T'- open set V in Y containing f(x) there exists T- open set U in X containing x such that f(U) ⊂ V . We say f is continuous on X iff f is continuous at each x∈ X [6]
2. The function f : $(X,T) \longrightarrow (Y,T')$ is said to be α^***- continuous** at x∈ X iff for every T'- open set V in Y containing f(x) there exists α- open set U in X containing x such that f(U) ⊂V . We say f is α^*- continuous on X iff f is α^*- continuous at each x∈ X . [4]
3. The function f : $(X,T,\Omega) \longrightarrow (Y,T')$ is said to be **weakly** - δ**-continuous** at x∈ X iff for every T'- open set V in Y containing f(x) there exists δ-open set U in X containing x such that f(U) ⊂V . We say f is weakly -δ-continuous on X iff f is weakly -δ-continuous at each x∈ X .[3]
4. The function f : $(X,T,\Omega,\rho) \longrightarrow (Y,T')$ is said to be **weakly** -$\delta*$-

continuous at x∈ X iff for every T'- open set V in Y containing f(x) there exists $\delta*$-open set U in X containing x such that f(U) ⊂ V . We say f is weakly - $\delta*$-continuous on X iff f is weakly - $\delta*$-continuous at each x∈ X . [2]

I make relations among these four continuos functions by using theorems [every T-open set can be α -open set] , [every T-open set can be δ -open set] and [every T-open set can be $\delta*$- open set] and the converse is not hold [1] .

And The definitions of the four open functions are :

1. The function f : $(X,T) \longrightarrow (Y,T')$ is said to be **open** iff f(G) is a T'- open set in Y for every T - open set G in X . [6]
2. The function f : $(X,T) \longrightarrow (Y,T')$ is said to be α^{**} - **open** iff f(G) is α -open set in Y for every T - open set G in X . [4]
3. The function f : $(X,T) \longrightarrow (Y,T',\Omega')$ is said to be λ - **open** iff f(G) is δ- open set in Y for every T - open set G in X . [3]
4. The function f : $(X,T) \longrightarrow (Y,T',\Omega',\rho')$ is said to be λ^*- **open** iff f(G) is $\delta*$-open set in Y for every T - open set G in X . [2]

Also i make relations among these four open functions by using theorems [every T-open set can be α -open set] , [every T-open set can be δ -open set] and [every T-open set can be $\delta*$- open set] and the converse is not hold [1] .

Moreover I introduce the following theorems :

2- Some important theorems :

2.1 Theorem :

Continuous function leads to a α^* - continuous function .

Proof:

Let f be a continuous function , to be f a α^* - continuous function ; let V be a T'- open set in Y , then f^{-1}(V) is T - open set in X because f is continuos function , and because [every T-open set can be a α -open set] therefore f^{-1}(V) is α - open set .

Hence f a α^* - continuous function .

2.2 Remark :

The converse of the above theorem is not held , because α - open set not necessary to be a T - open set [4] .

2.3 Theorem :

Continuous function leads to a weakly - δ - continuous function .

Proof:

Let f be a continuous function , to be f a weakly - δ - continuous function ; let V be a T'- open set in Y , then f^{-1}(V) is T - open set in X because f is continuous function , and because [every T-open set can be a δ - open set] therefore f^{-1}(V) is δ -open set .

Hence f a weakly - δ - continuous function .

2.4 Remark :

The converse of the above theorem is not held , because the bitopological space does not always represent a topology [3] . (i.e. δ -open set not necessary to be a T - open set) .

2.5 Theorem :

Continuous function leads to a weakly - $\delta*$- continuous function .

Proof:

Let f be a continuous function , to be f a weakly - $\delta*$- continuous function ; let V be a

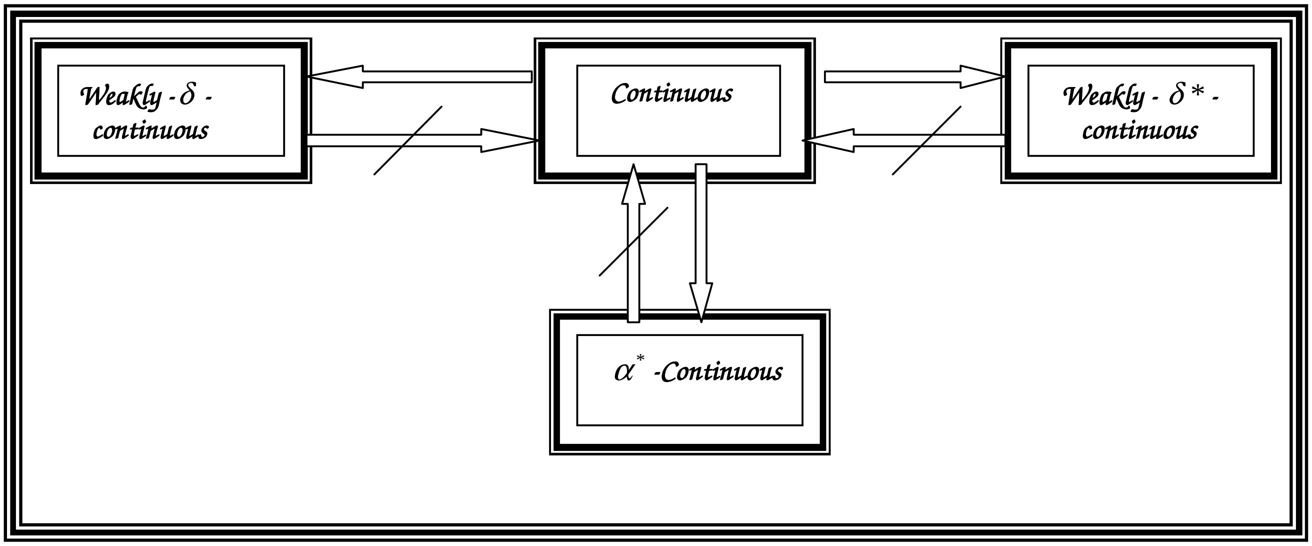
T' - open set in Y , then f^{-1}(V) is T - open set in X because f is continuos function , and because [every T-open set can be a $\delta*$- open set] therefore f^{-1}(V) is $\delta*$- open set .

Hence f a weakly -$\delta*$- continuous function .

2.6 Remark :

The converse of the above theorem is not held , because the tritopological space does not always represent a topology [2] . (i.e. $\delta*$-open set not necessary to be a T - open set) .

The following diagram illustrates the relations among that four continuos functions in topological , bitopological and tritopological spaces .



2.7 Theorem :

Open function leads to a α^{**} - open function .

Proof:

Let f be an open function , to be f a α^{**}- open function ; let G be a T - open set in X , then f (G) is T'- open set in Y because f is an open function ,
and because [every T-open set can be a α -open set] therefore f (G) is α -open set .

Hence f a α^{**} - open function .

So that every Open function can be a α^{**} - open function .

2.8 Remark :

The converse of the above theorem is not held , because α - open set not necessary to be a T - open set [4] .

2.9 Theorem :

Open function leads to a λ - open function .

Proof:

Let f be an open function , to be f a λ - open function ; let G be a T - open set in X , then f (G) is T'- open set in Y because f is an open function , and
because [every T-open set can be a δ -open set] therefore f (G) is δ -open set .

Hence f a λ - open function .

So that every Open function can be a λ - open function .

2.10 Remark :

The converse of the above theorem is not held , because the bitopological space does not always represent a topology [3] . (i.e. δ -open set not necessary to be a T - open set) .

2.11 Theorem :

Open function leads to a λ^* - open function .

Proof:

Let f be an open function , to be f a λ^* - open function ; let G be a T - open set in X , then f (G) is T' - open set in Y because f is an open function , and because [every T-open set can be a $\delta*$-open set] therefore f (G) is $\delta*$-open set.

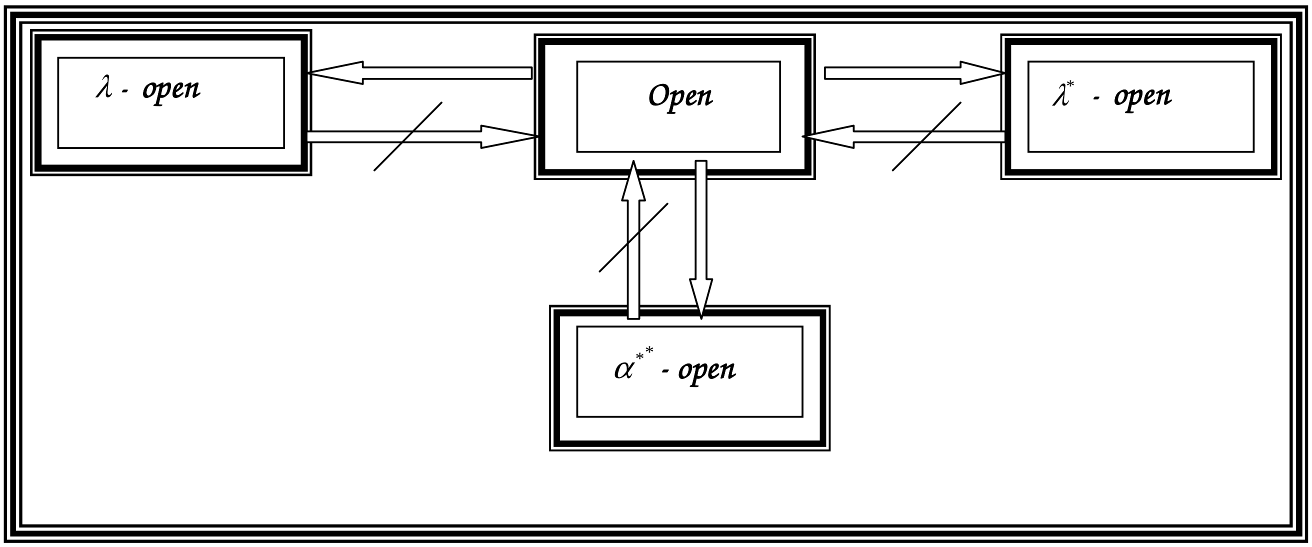
Hence f a λ^* - open function .

So that every Open function can be a λ^* - open function .

2.12 Remark :

The converse of the above theorem is not held , because the tritopological space not always represent a topology [2] . (i.e. $\delta*$-open set not necessary to be a T - open set)

The following diagram illustrates the relations among that four open functions in topological , bitopological and tritopological spaces .



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الخلاصة :

في هذا العمل قدمت علاقات بين أربعة أنواع من الدوال المستمرة ؛ وهي (المستمرة و α^* - مستمرة و δ - مستمرة الضعيفة و $\delta*$ -مستمرة الضعيفة) ، وكذلك قدمت علاقات بين أربعة أنواع من الدوال المفتوحة ؛ وهي (المفتوحة و λ - المفتوحة و λ^* - المفتوحة و α^{**} - المفتوحة) في الفضاء التبولوجي و الفضاء الثنائي التبولوجي و الفضاء الثلاثي التبولوجي .