# Application of Adomian decomposition method to solve Fisher's equation 

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#### Abstract

In this paper, we apply Adomian decomposition method (ADM) to solve the generalized nonlinear Fisher's equation with give it is convergence proof. The results of numerical test for three types of this equation are show the efficiency and simplicity application of this method.


Key words: Adomian decomposition method, Adomian polynomials, Fisher's equation.

## 1-Introduction

Mathematical modeling on many phenomena in applied sciences such as physics, chemistry, biology and engineering leads to nonlinear partial differential equations for example, in mathematical biology Fisher's equation [11, 2] is defined by
$u_{t}=u_{x x}+u(1-u)(\rho u+\beta),(\mathbf{1 a})$
with the initial condition

$$
\begin{equation*}
u(x, 0)=f(x) \tag{1b}
\end{equation*}
$$

where $\rho$ and $\beta$ are arbitrary constants.
Several methods have been proposed to solve (1) approximately or analytically $[2,8,3,9,10]$. Recently, the Adomian decomposition method has been applied to the study of linear and nonlinear problems. For many problems, the decomposition method has shown reliable results in providing
analytical approximation that converges rapidly. In $[12,13]$ introduced formula to generate Adomian polynomials for all forms of nonlinear operators, some feasible methods for the calculation Adomian polynomials in simple way has been studied [5, 1] without any need for the formulas which introduced by $[12,13]$. We believe that a simple and reliable method can be established to calculate Adomian polynomials less dependable on so many formulas as before.

The main goal of this paper is provide a promising method that can be used to calculate Adomian polynomials for nonlinear operators in easy way. This paper organized as follows: In Section 2, we will introduce the analysis of ADM. In Section 3, we will prove the convergence of this method applied to Fisher's equation. In Section 4, the effectiveness of ADM is shown through three cases for $\rho$ and $\beta$ as examples. Conclusions follow in section 5 .

## 2-Adomian decomposition Method

Let us rewrite (1a) in operator form

$$
\begin{equation*}
L(u)=u_{x x}+F(u) \tag{2}
\end{equation*}
$$

where

$$
L=\frac{\partial}{\partial t}
$$

and
$F(u)=u(1-u)(\rho u+\beta)$ is the nonlinear term.

Applying the inverse operator
$L^{-1}(\cdot)=\int_{0}^{t}(\cdot) d t$ on both sides of (2), we obtain
$u(x, t)=f(x)+L^{-1}\left(u_{x x}\right)+L^{-1}(F(u)),(3)$
and $A_{k}{ }^{\prime} s$ the Adomian polynomials.

In the ADM , the solution $u(x, t)$ is given by the series form [13]

$$
\begin{equation*}
u(x, t)=\sum_{n=0}^{\infty} u_{n}(x, t) \tag{4}
\end{equation*}
$$

and the nonlinear term $F(u)$ is decomposed as

$$
\begin{equation*}
F(u(\lambda))=\sum_{k=0}^{\infty} \lambda^{k} A_{k} \tag{5}
\end{equation*}
$$

where,
$u(\lambda)=\sum_{k=0}^{\infty} \lambda^{k} u_{k}$

For calculation the Adomian polynomials, we use equation (5) to compute $A_{k}$ as follows:

$$
\begin{align*}
& \sum_{k=0}^{\infty} \lambda^{k} A_{k}=F\left(\sum_{k=0}^{\infty} \lambda^{k} u_{k}\right) \\
& =\beta\left(\sum_{k=0}^{\infty} \lambda^{k} u_{k}\right)+(\rho-\beta)\left(\sum_{k=0}^{\infty} \lambda^{k} u_{k}\right)^{2}-\rho\left(\sum_{k=0}^{\infty} \lambda^{k} u_{k}\right)^{3} \\
& = \\
& =u_{0}\left(1-u_{0}\right)\left(\rho u_{0}+\beta\right)+\lambda\left(\beta u_{1}+2(\rho-\beta) u_{0} u_{1}-3 \rho u_{0}^{2} u_{1}\right) \\
& + \\
& +\lambda^{2}\left(\beta u_{2}+(\rho-\beta)\left(u_{1}^{2}+2 u_{0} u_{2}\right)-3 \rho\left(u_{0} u_{1}^{2}+u_{0}^{2} u_{2}\right)\right) \\
& +  \tag{7}\\
& \lambda^{3}\left(\beta u_{3}+2(\rho-\beta)\left(u_{0} u_{3}+u_{1} u_{2}\right)-3 \rho\left(2 u_{0} u_{1} u_{2}+u_{0}^{2} u_{3}\right)\right) \\
& + \\
& \quad \lambda^{4}\left(\beta u_{4}+(\rho-\beta)\left(2 u_{0} u_{4}+2 u_{1} u_{3}+u_{2}^{2}\right)-3 \rho\left(u_{0}^{2} u_{4}+\right.\right. \\
& \\
& \\
& \left.\left.\quad 2 u_{0} u_{1} u_{3}+u_{0} u_{2}^{2}+u_{1}^{2} u_{2}\right)\right)+\cdots
\end{align*}
$$

$A_{k}{ }^{\prime} s$ are obtained by equating the coefficients of the first four expressions for the Adomian the terms with identical power of $\lambda$. For example, polynomials $A_{k}$ of the nonlinear term $F(u)$ are

$$
\begin{align*}
A_{0} & =u_{0}\left(1-u_{0}\right)\left(\rho u_{0}+\beta\right) \\
A_{1} & =\beta u_{1}+2(\rho-\beta) u_{0} u_{1}-3 \rho u_{0}^{2} u_{1} \\
A_{2} & =\beta u_{2}+(\rho-\beta)\left(u_{1}^{2}+2 u_{0} u_{2}\right)-3 \rho\left(u_{0} u_{1}^{2}+u_{0}^{2} u_{2}\right) \\
A_{3} & =\beta u_{3}+2(\rho-\beta)\left(u_{0} u_{3}+u_{1} u_{2}\right)-3 \rho\left(u_{0} u_{1} u_{2}+u_{0}^{2} u_{3}\right)  \tag{8}\\
A_{4}= & \beta u_{4}+(\rho-\beta)\left(2 u_{0} u_{4}+2 u_{1} u_{3}+u_{2}^{2}\right)-3 \rho\left(u_{0}^{2} u_{4}+2 u_{0} u_{1} u_{3}\right. \\
& \left.+u_{0} u_{2}^{2}+u_{1}^{2} u_{2}\right)
\end{align*}
$$

The unknown function $u_{n}(x, t), n \geq 0$ is one can be obtained by the following decomposed into a sum of components (4), each decomposition formula:
$u_{0}(x, t)=f(x)$,
$\left.u_{n+1}(x, t)=\int_{0}^{t}\left[A_{n}\left(u_{0}, u_{1}, \ldots, u_{n}\right)+\left(u_{n}\right)_{x x}\right] d t . \quad n \geq 0\right\}$

## 3-Convergence analysis

To demonstrate the accuracy of the ADM to (1), we will discuss the convergence analysis in
the same manner [6, 7, 4]. Let us consider the Hilbert space H defined as follows:

$$
H=L^{2}((a, b) \times[0, T]):
$$

$$
u:(a, b) \times[0, T] \rightarrow R \text { with } \int_{(a, b) \times[0, T]} u^{2}(x, s) d s d \tau<+\infty
$$

the scalar product
$\langle u, v\rangle_{H}=\int_{(a, b) \times[0, T]} u(x, s) v(x, s) d s d \tau$,
(10)
and the associated norm
$\|u\|_{H}^{2}=\int_{(a, b) \times[0, T]} u^{2}(x, s) d s d \tau$.
The ADM applied to (1) is convergent if the following two hypotheses are satisfied [4]
$\mathrm{H}_{1}:\langle L(u)-L(v), u-v\rangle \geq k\|u-v\|^{2}, k>0, \forall u, v \in H$,
$\mathrm{H}_{2}:$ For any $M>0, \exists$ a constant $C(M)>0$, such that for $u, v \in H$ with $\|u\| \leq M,\|v\| \leq M$, We have $\langle L(u)-L(v), w\rangle \leq C(M)\|u-v\|\|w\| \quad$ for every $w \in H$.

Theorem:[4] (Sufficient condition of convergence). The ADM applied to the Fisher's equation (1) converges towards an exact solution.
Proof: To prove this theorem, firstly, we will verify the convergence hypothesis $\mathrm{H}_{1}$ for the operator $L(u)$. From (2) we have

$$
\begin{align*}
L(u)-L(v)= & {\left[\frac{\partial^{2} u}{\partial x^{2}}-\frac{\partial^{2} v}{\partial x^{2}}\right]+(\rho-\beta)\left(u^{2}-v^{2}\right)-\rho\left(u^{3}-v^{3}\right)+\beta(u-v) } \\
= & {\left[\frac{\partial^{2} u}{\partial x^{2}}-\frac{\partial^{2} v}{\partial x^{2}}\right]+(\rho-\beta)(u-v) \sum_{i=1}^{2} u^{2-i} v^{i-1}-\rho(u-v) } \\
& \sum_{i=1}^{3} u^{3-i} v^{i-1}+\beta(u-v) \tag{12}
\end{align*}
$$

Therefore,

$$
\begin{align*}
\langle L(u)-L(v), u-v\rangle & =\left\langle\frac{\partial^{2}}{\partial x^{2}}(u-v), u-v\right\rangle-(\rho-\beta)\left\langle-(u-v) \sum_{i=1}^{2} u^{2-i} v^{i-1}, u-v\right\rangle \\
& +\rho\left\langle-(u-v) \sum_{i=1}^{3} u^{2-i} v^{i-1}, u-v\right\rangle-\beta\langle-(u-v), u-v\rangle \tag{13}
\end{align*}
$$

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By definition of scalar product and the properties of the differential operator $\frac{\partial^{2}}{\partial x^{2}}$ in $H$, then there exist $\delta_{1}>0$ such that

$$
\begin{equation*}
\left\langle\frac{\partial^{2}}{\partial x^{2}}(u-v), u-v\right\rangle \geq \delta_{1}\|u-v\|^{2} \tag{14}
\end{equation*}
$$

By Schwartz inequality and the definition of scalar product, we have

$$
\begin{aligned}
\left\langle(u-v) \sum_{i=1}^{2} u^{2-i} v^{i-1}, u-v\right\rangle & \leq\left\|(u-v) \sum_{i=1}^{2} u^{2-i} v^{i-1}\right\|\|u-v\| \\
& \leq 2 M\|u-v\|^{2} .
\end{aligned}
$$

where $\|u\| \leq M,\|v\| \leq M$. Therefore:

$$
\begin{equation*}
\left\langle-(u-v) \sum_{i=1}^{2} u^{2-i} v^{i-1}, u-v\right\rangle \geq-2 M\|u-v\|^{2} \tag{15}
\end{equation*}
$$

Similarly,

$$
\begin{equation*}
\left\langle-(u-v) \sum_{i=1}^{3} u^{3-i} v^{i-1}, u-v\right\rangle \geq-3 M^{2}\|u-v\|^{2} \tag{16}
\end{equation*}
$$

Substituting (14)-(16) into (13) gives

$$
\begin{align*}
\langle L(u)-L(v), u-v\rangle & \geq\left[\delta_{1}+2(\rho-\beta) M-3 \rho M^{2}-\beta\right]\|u-v\|^{2} \\
& =K\|u-v\|^{2}, \quad(K>0) \tag{17}
\end{align*}
$$

where,

$$
\begin{equation*}
K=\delta_{1}+2(\rho-\beta) M-3 \rho M^{2}-\beta>0 \tag{18}
\end{equation*}
$$

which implies that

$$
\begin{equation*}
\delta_{1}>\beta-2(\rho-\beta) M+3 \rho M^{2} \tag{19}
\end{equation*}
$$

Thus, the hypothesis $\mathrm{H}_{1}$ holds.
Secondly, we verify convergence hypothesis $\mathrm{H}_{2}$ for the operator $L(u)$. Using the Schwartz inequality, we have

$$
\begin{align*}
\langle L(u)-L(v), w\rangle & =\left\langle\frac{\partial^{2}}{\partial x^{2}}(u-v), w\right\rangle+(\rho-\beta)\left\langle(u-v) \sum_{i=1}^{2} u^{2-i} v^{i-1}, w\right\rangle \\
& -\rho\left\langle(u-v) \sum_{i=1}^{3} u^{2-i} v^{i-1}, w\right\rangle+\beta\langle(u-v), w\rangle  \tag{20}\\
\leq & \delta_{2}\|u-v\|\|w\|+2(\rho-\beta) M\|u-v\|\|w\|-3 \rho M^{2}\|u-v\|\|w\| \\
& +\beta\|u-v\|\|w\| \\
= & {\left[\delta_{2}+2(\rho-\beta) M-3 \rho M^{2}+\beta\right]\|u-v\|\|w\| } \\
= & C(M)\|u-v\|\|w\| \tag{21}
\end{align*}
$$

where

$$
\begin{equation*}
C(M)=\delta_{2}+2(\rho-\beta) M-3 \rho M^{2}+\beta, \tag{22}
\end{equation*}
$$

The hypothesis $\mathrm{H}_{2}$ is satisfied.

## 4-Examples

In this section, three cases for $\rho$ and $\beta$ in (1) order to demonstrate the effectiveness of ADM. For the parameter values as in the following cases:

Case 1: Let $\rho=0$ and $\beta=1$, then (1) write as follows [3]
$u_{t}=u_{x x}+u(1-u)$
with initial condition

$$
u(x, 0)=\frac{1}{\left(1+\exp \left(\frac{x}{\sqrt{6}}\right)\right)^{2}},
$$

By using (8), we have

$$
\begin{align*}
& A_{0}=u_{0}-u_{0}^{2} \\
& A_{1}=u_{1}-2 u_{0} u_{1} \\
& A_{2}=-u_{1}^{2}+u_{2}-2 u_{0} u_{2}  \tag{23}\\
& A_{3}=-2 u_{1} u_{2}+u_{3}-2 u_{0} u_{3} \\
& \vdots
\end{align*}
$$

The $A_{n}$ 's have been known, so the $\left\{u_{n}\right\}$ terms can be determine by using the recursive relations (9). Simple calculations lead to
the exact solution is given by

$$
\left.\begin{array}{l}
\left.\begin{array}{l}
u(x, t)=\left[1+\exp \left(\frac{x-5 / \sqrt{6} t}{\sqrt{6}}\right)\right]^{-2} \\
u_{0}=\frac{1}{(1+\exp (x / \sqrt{6}))^{2}} \\
u_{1}=\frac{5 \exp (x / \sqrt{6}) t}{3(1+\exp (x / \sqrt{6}))^{3}} \\
u_{2}=\frac{25 \exp (x / \sqrt{6})(-1+2 \exp (x / \sqrt{6})) t^{2}}{36(1+\exp (x / \sqrt{6}))^{4}} \\
u_{3}=\frac{125 \exp (x / \sqrt{6})(1+4 \exp (\sqrt{2} x / \sqrt{3})-7 \exp (x / \sqrt{6})) t^{3}}{648(1+\exp (x / \sqrt{6}))^{5}}
\end{array}\right\}, \\
\vdots
\end{array}\right\}
$$

Putting these individual terms in (4), we have

$$
\begin{align*}
& u(x, t)=\frac{1}{(1+\exp (x / \sqrt{6}))^{2}}+\frac{5 \exp (x / \sqrt{6}) t}{3(1+\exp (x / \sqrt{6}))^{3}}+\frac{25 \exp (x / \sqrt{6})(-1+2 \exp (x / \sqrt{6})) t^{2}}{36(1+\exp (x / \sqrt{6}))^{4}}+ \\
& \frac{125 \exp (x / \sqrt{6})(1+4 \exp (\sqrt{2} x / \sqrt{3})-7 \exp (x / \sqrt{6})) t^{3}}{648(1+\exp (x / \sqrt{6}))^{5}}+\cdots \tag{25}
\end{align*}
$$

Table 1: Comparison of the exact and Adomian decomposition solutions.

| t | $\mathbf{x}$ | Exact solution | ADM solution eq. (25) | Absolute error |
| :---: | :---: | :---: | :---: | :---: |
| 0.2 | 0.25 | $2.6641 \mathrm{e}-001$ | $2.6641 \mathrm{e}-001$ | $1.8061 \mathrm{e}-007$ |
|  | 0.5 | $2.4072 \mathrm{e}-001$ | 2.4072e-001 | 5.7288e-008 |
|  | 0.75 | $2.1639 \mathrm{e}-001$ | $2.1639 \mathrm{e}-001$ | $6.2282 \mathrm{e}-008$ |
|  | 1 | 1.9351e-001 | 1.9351e-001 | 1.6995e-007 |
| 0.4 | 0.25 | 3.1087e-001 | 3.1087e-001 | $6.8122 \mathrm{e}-006$ |
|  | 0.5 | $2.8330 \mathrm{e}-001$ | $2.8330 \mathrm{e}-001$ | $2.9059 \mathrm{e}-006$ |
|  | 0.75 | $2.5683 \mathrm{e}-001$ | $2.5683 \mathrm{e}-001$ | $9.5842 \mathrm{e}-007$ |
|  | 1 | $2.3163 \mathrm{e}-001$ | $2.3163 \mathrm{e}-001$ | $4.5120 \mathrm{e}-006$ |
| 0.6 | 0.25 | $3.5783 \mathrm{e}-001$ | 3.5777e-001 | $5.9153 \mathrm{e}-005$ |
|  | 0.5 | $3.2883 \mathrm{e}-001$ | 3.2880e-001 | 3.0064e-005 |
|  | 0.75 | 3.0064e-001 | 3.0063e-001 | $7.0561 \mathrm{e}-007$ |
|  | 1 | $2.7345 \mathrm{e}-001$ | $2.7347 \mathrm{e}-001$ | 2.6851e-005 |

Case 2: Let $\rho=0$ and $\beta=6$, then (1) write as follows [9]
$u_{t}=u_{x x}+6 u(1-u)$
with initial condition
$u(x, 0)=\frac{1}{(1+\exp (x))^{2}}$,
the exact solution is given by

$$
u(x, t)=[1+\exp (x-5 t)]^{-2}
$$

By using (8), we obtain

$$
\begin{equation*}
u_{2}=\frac{25 \exp (x)(-1+2 \exp (x)) t^{2}}{(1+\exp (x))^{4}} \tag{27}
\end{equation*}
$$

$$
\begin{aligned}
& A_{0}=6 u_{0}-6 u_{0}^{2} \\
& A_{1}=6 u_{1}-12 u_{0} u_{1} \\
& A_{2}=-6 u_{1}^{2}+6 u_{2}-12 u_{0} u_{2} \\
& A_{3}=-12 u_{1} u_{2}+6 u_{3}-12 u_{0} u_{3}
\end{aligned}
$$

Substituting the relations (26) into recurrent relations (9) yields

$$
\begin{aligned}
& u_{0}=\frac{1}{(1+\exp (x))^{2}} \\
& u_{1}=\frac{10 \exp (x) t}{(1+\exp (x))^{3}}
\end{aligned}
$$

$$
\begin{aligned}
& u_{3}=\frac{125 \exp (x)(1+4 \exp (2 x)-7 \exp (x)) t^{3}}{3(1+\exp (x))^{5}} \\
& \vdots
\end{aligned}
$$

The $\left\{u_{n}\right\}$ terms are known, so the solution is given by

$$
\begin{align*}
u(x, t)= & \frac{1}{(1+\exp (x))^{2}}+\frac{10 \exp (x) t}{(1+\exp (x))^{3}}+\frac{25 \exp (x)(-1+2 \exp (x)) t^{2}}{(1+\exp (x))^{4}}+ \\
& \frac{125 \exp (x)(1+4 \exp (2 x)-7 \exp (x)) t^{3}}{3(1+\exp (x))^{5}}+\cdots \tag{28}
\end{align*}
$$

Table 2: Comparison of the exact and Adomian decomposition solutions.

| t | $\mathbf{x}$ | Exact solution | ADM solution eq. (28) | Absolute error |
| :---: | :---: | :---: | :---: | :---: |
| 0.2 | 0.25 | $4.6128 \mathrm{e}-001$ | $4.6128 \mathrm{e}-001$ | 1.3136e-010 |
|  | 0.5 | 3.8746e-001 | 3.8746e-001 | 3.5974e-010 |
|  | 0.75 | 3.1604e-001 | 3.1604e-001 | 1.0525e-011 |
|  | 1 | $2.5000 \mathrm{e}-001$ | $2.5000 \mathrm{e}-001$ | $1.7022 \mathrm{e}-010$ |
| 0.4 | 0.25 | $7.2582 \mathrm{e}-001$ | $7.2605 \mathrm{e}-001$ | $3.1764 \mathrm{e}-004$ |
|  | 0.5 | $6.6843 \mathrm{e}-001$ | $6.6812 \mathrm{e}-001$ | $2.2809 \mathrm{e}-004$ |
|  | 0.75 | $6.0420 \mathrm{e}-001$ | 6.0411e-001 | 3.1138e-004 |
|  | 1 | $5.3445 \mathrm{e}-001$ | $5.3461 \mathrm{e}-001$ | 8.8348e-005 |
| 0.6 | 0.25 | 8.8344e-001 | $1.7237 \mathrm{e}+000$ | 8.4025e-001 |
|  | 0.5 | 8.5404e-001 | $8.7829 \mathrm{e}-002$ | $7.6621 \mathrm{e}-001$ |
|  | 0.75 | 8.1839e-001 | $4.0131 \mathrm{e}-001$ | $4.1709 \mathrm{e}-001$ |
|  | 1 | $7.7580 \mathrm{e}-001$ | $1.1895 \mathrm{e}+000$ | $4.1372 \mathrm{e}-001$ |

Case 3: Let $\rho=1$ and $\beta=-q$, then (1) write as follows [10]

$$
u_{t}=u_{x x}+u(1-u)(u-q), \quad 0<q<\frac{1}{2} . \quad u(x, t)=\frac{1}{1+\exp \left(\left(-x-v_{0} t\right) / \sqrt{2}\right)}, \quad v_{0}=\frac{1-2 q}{\sqrt{2}}
$$

The exact solution is given by
with initial condition
By using (8), we have
$u(x, 0)=\frac{1}{1+\exp (-x / \sqrt{2})}$,
$A_{0}=-q u_{0}+(1+q) u_{0}^{2}-u_{0}^{3}$,
$A_{1}=-q u_{1}+2(1+q) u_{0} u_{1}-3 u_{0}^{2} u_{1}$,
$A_{2}=-q u_{2}+(1+q)\left(u_{1}^{2}+2 u_{0} u_{2}\right)-3\left(u_{0} u_{1}^{2}+u_{0}^{2} u_{2}\right)$,
$A_{3}=-q u_{3}+2(1+q)\left(u_{0} u_{3}+u_{1} u_{2}\right)-3\left(u_{0} u_{1} u_{2}+u_{0}^{2} u_{3}\right)$,
!

Substituting relations (29) into recursive relations (9) yields

$$
\left.\begin{array}{rl}
u_{0} & =\frac{1}{1+\exp (-x / \sqrt{2})} \\
u_{1} & =\frac{(2 q-1) \exp (-x / \sqrt{2}) t}{2(1+\exp (-x / \sqrt{2}))^{2}} \\
u_{2} & =\frac{(2 q-1)^{2}(1-\exp (-x / \sqrt{2})) \exp (-x / \sqrt{2}) t^{2}}{8(1+\exp (-x / \sqrt{2}))^{3}}  \tag{30}\\
u_{3} & =\frac{(2 q-1)^{3}(1-4 \exp (-x / \sqrt{2})+\exp (-\sqrt{2} x)) \exp (-x / \sqrt{2}) t^{3}}{48(1+\exp (-x / \sqrt{2}))^{4}}
\end{array}\right\}
$$

Substituting relations (30) into recursive relations (4) yields

$$
\begin{align*}
& u(x, t)= \frac{1}{1+\exp (-x / \sqrt{2})}+\frac{(2 q-1) \exp (-x / \sqrt{2}) t}{2(1+\exp (-x / \sqrt{2}))^{2}}+ \\
& \frac{(2 q-1)^{2}(1-\exp (-x / \sqrt{2})) \exp (-x / \sqrt{2}) t^{2}}{8(1+\exp (-x / \sqrt{2}))^{3}}+ \\
& \frac{(2 q-1)^{3}(1-4 \exp (-x / \sqrt{2})+\exp (-\sqrt{2} x)) \exp (-x / \sqrt{2}) t^{3}}{48(1+\exp (-x / \sqrt{2}))^{4}}+\cdots \tag{31}
\end{align*}
$$

Table 3: Comparison of the exact and Adomian decomposition solutions at $\mathbf{q}=\mathbf{0 . 2}$.

| t | $\mathbf{x}$ | Exact solution | ADM solution eq. (31) | Absolute error |
| :---: | :---: | :---: | :---: | :---: |
| 0.2 | 0.25 | 3.1281e-001 | 3.1281e-001 | $8.5165 \mathrm{e}-013$ |
|  | 0.5 | $3.8403 \mathrm{e}-001$ | $3.8403 \mathrm{e}-001$ | $9.6778 \mathrm{e}-013$ |
|  | 0.75 | $4.5782 \mathrm{e}-001$ | $4.5782 \mathrm{e}-001$ | $7.8743 \mathrm{e}-013$ |
|  | 1 | $5.3107 \mathrm{e}-001$ | 5.3107e-001 | $4.3054 \mathrm{e}-013$ |
| 0.4 | 0.25 | $3.0958 \mathrm{e}-001$ | 3.0958e-001 | 2.7183e-011 |
|  | 0.5 | 3.8061e-001 | 3.8061e-001 | 3.0977e-011 |
|  | 0.75 | $4.5435 \mathrm{e}-001$ | $4.5435 \mathrm{e}-001$ | $2.5271 \mathrm{e}-011$ |
|  | 1 | $5.2770 \mathrm{e}-001$ | $5.2770 \mathrm{e}-001$ | $1.3873 \mathrm{e}-011$ |
| 0.6 | 0.25 | $3.0637 \mathrm{e}-001$ | $3.0637 \mathrm{e}-001$ | $2.0589 \mathrm{e}-010$ |
|  | 0.5 | $3.7720 \mathrm{e}-001$ | $3.7720 \mathrm{e}-001$ | $2.3530 \mathrm{e}-010$ |
|  | 0.75 | $4.5088 \mathrm{e}-001$ | $4.5088 \mathrm{e}-001$ | $1.9246 \mathrm{e}-010$ |
|  | 1 | $5.2431 \mathrm{e}-001$ | 5.2431e-001 | $1.0609 \mathrm{e}-010$ |

Table 4: Comparison of the exact and Adomian decomposition solutions at $\mathbf{q}=\mathbf{0 . 4}$.

| t | $\mathbf{x}$ | Exact solution | ADM solution eq. (31) | Absolute error |
| :---: | :---: | :---: | :---: | :---: |
| 0.2 | 0.25 | 3.0958e-001 | 3.0958e-001 | 1.4470e-011 |
|  | 0.5 | $3.8061 \mathrm{e}-001$ | 3.8061e-001 | $2.7183 \mathrm{e}-011$ |
|  | 0.75 | $4.5435 \mathrm{e}-001$ | $4.5435 \mathrm{e}-001$ | 3.0977e-011 |
|  | 1 | $5.2770 \mathrm{e}-001$ | $5.2770 \mathrm{e}-001$ | $2.5271 \mathrm{e}-011$ |
| 0.4 | 0.25 | 3.0317e-001 | 3.0317e-001 | $8.6535 \mathrm{e}-010$ |
|  | 0.5 | $3.7379 \mathrm{e}-001$ | 3.7379e-001 | $9.9179 \mathrm{e}-010$ |
|  | 0.75 | $4.4740 \mathrm{e}-001$ | $4.4740 \mathrm{e}-001$ | $8.1331 \mathrm{e}-010$ |
|  | 1 | $5.2092 \mathrm{e}-001$ | $5.2092 \mathrm{e}-001$ | $4.5015 \mathrm{e}-010$ |
| 0.6 | 0.25 | $2.9681 \mathrm{e}-001$ | $2.9681 \mathrm{e}-001$ | $6.5360 \mathrm{e}-009$ |
|  | 0.5 | $3.6700 \mathrm{e}-001$ | 3.6700e-001 | $7.5344 \mathrm{e}-009$ |
|  | 0.75 | $4.4046 \mathrm{e}-001$ | $4.4046 \mathrm{e}-001$ | $6.2107 \mathrm{e}-009$ |
|  | 1 | 5.1411e-001 | 5.1411e-001 | $3.4655 \mathrm{e}-009$ |

## 5-Coclusions

The Adomian decomposition method is powerful method for solving many kinds of nonlinear Fisher's problems. However, the important part of this method is calculating Adomian polynomials for nonlinear operator.
In this paper, we apply ADM, as seen in Tables 14 , of the three cases of the generalized nonlinear

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Fisher's equation are shown that, the absolute errors are very small. This is confirming the convergence of ADM. Moreover, it is simplicity, efficiency and very easy to implement.
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## تطبيق طريقة تحليل أدومين لحل معادلة فيشر

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> (المستخلص
> في هذا البحث تم تطبيق طريقة تحلبل أدومين لحل معادلة فيشر اللاخطية العامة مع بر هان تقاربها. نتائج الاختبار ات العددية لثلاث أنواع من معادلة فيشر أثبتت كفاءة وسهولة تطبيقها.

