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Phase Transitions and Two neutron bosons separation energies in the IBM

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Abstract

In the framework of the IBM the three transitional regions (vibrational, rotational , rotational γ-unstable and vibrational $\gamma-$unstable) are analyzed. A new kind of plot is presented for studying phase transition in finite systems such as atomic nuclei. The importance of analyzing binding energies and not only energy spectra and electromagnetic transitions, describing transitional regions is emphasized and a new method is discussed in order to provide a consistent description of both , ground state and excite state properties.

Keywords: Interacting boson model (IBM), Energy levels, Transition probability

1-Introduction

In the last few years, interest in the study of phase transitions and phase coexistence in atomic nuclei has been revived [1--7] in particular making use of the Interacting Boson Model (IBM) [8]. In the chart of nuclei, three transitional regions can be distinguished: (a) where a change is observed from spherical to well deformed nuclei when moving from the lighter to the heavier nuclei; (b) where one notices that the lighter isotopes are spherical while the heavier ones indicate a γ-unstable character (c) where the lighter isotopes are well-deformed while the heavier show γ-unstable properties. Although these three transitional regions have been studied extensively in the framework of IBM, the discussion of phase transitions has not always been treated in a proper way. In particular, one of the weak points is how to define an appropriate control parameter in a system such as the atomic nucleus, where this parameter is fixed (for given N and Z) and cannot be extremely fitted [9].This problem has been recently considered in a study by Casten et al., [3]. In the present work, we follow the approach of reference [10] but, additionally, we show that it is convenient to explore nuclear binding energies (BE), together with excited states properties, in order to obtain a consistent description, essentially when one is dealing with chains of isotopes.

2-Transitional regions and phase transitions in the IBM: Traditional and new points of view

In order to deal with transitional regions in the framework of the IBM, a very convenient Hamiltonian can be written as follows:

$$H = \kappa\left(N\frac{1-\xi}{\xi}\hat{n}_d - \hat{Q}\ .\hat{Q}\ \right)\kappa^{\sim} + \hat{L}.\hat{L} \quad \text{.........................(1)}$$

With $0 \le \xi \le 1$, where $\hat{L}\ = \sqrt{10}(d^{+} \times \tilde{d})^{(1)}$ and $\hat{Q}\ = s^{+}\tilde{d}\ + d^{+}\tilde{s}\ + \chi(d^{+} \times \tilde{d}\ )^{(2)}$, with this parameterization one can move easily through the three legs of Casten's triangle modifying the values of χ and ξ. The three dynamical limits of the IBM correspond to the following values $(\chi,\xi): U(5),(\chi,0); SU(3),(-\sqrt{7}/2);$ and $O(6),(0,1)$.

One of the most strong facts that characterize the transitional regions is the possibility of observation of phase transition. Although phase transitions are strictly well- defined in macroscopic systems and the atomic nucleus in a finite system, a number of studies have shown that the concepts of a phase transition retains its validity and usefulness in small systems too [11,12]. A useful tool to discuss phase transition in finite system is the coherent states which, in the case of the IBM, are also known as intrinsic states [13,14].

$$|c\rangle = \frac{1}{N}(\Gamma_c^+)^N|0\rangle. \quad \text{where} \qquad \Gamma_c^+ = \frac{1}{\sqrt{1+\beta^2}}(s^+ + \beta d_o^+) \ldots\ldots\ldots\ldots\ldots(2)$$

Here the parameter β will act as an order parameter.

The appearance of a phase transition is denoted by discontinuity in the first or second order derivative of the BE with respect to the control parameter and, at the same time, by an abrupt change in the value of the order parameter [9,10]. In the case of the transitional regions U(5)-SU(3) and U(5)-O(6) the control parameter is ξ , giving rise to a first order phase in the first case, and a second order phase transition in the second one. The transitional region O(6)-SU(3) should be described by using χ as a control parameter although no phase transition appears.

Though this analysis provides a very straightforward way of treating phase transitions, there exists a snag on it: the control parameter ξ (or χ) is not a genuine control parameter, because it cannot be modified extremely. The control parameter important only when moving along a chain of nuclei and also assumes a change in the number of bosons. So, a consistent treatment of phase transitions should go beyond the analysis presented above and vary, at the same time, which the control parameter and the number of bosons.

The appropriate way of treating phase transitions and transitional regions is to plot in the same figure the value of the binding energy versus the control parameter for different values of N . In such a plot, a transition will develop through changes in the control parameter, but at the same time going through curves with a different number of bosons . In order to illustrate this new procedure, we discuss the transitional regions U(5)-SU(3), U(5)-O(6) and, SU(3)-O(6) in figure 1a, 1b, 1c respectively. In these figures, only the laboratory results are presented. As an example, trajectories for real nuclei are also plotted in each figure. The more interesting cases appear in figures-1a and 1b because in these two cases, phase transitions indeed happen. So, inspecting these figures, one can easily see if a given nucleus is situated in the spherical region (ξ near to 0), in the deformed region (ξ near to 1) or at the critical point (ξ around 0.2).

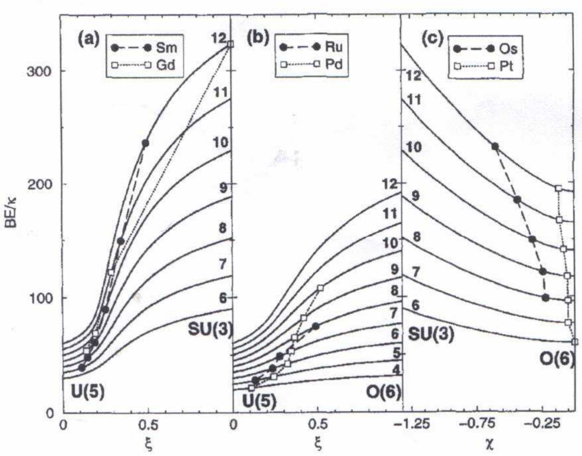

FIGURE 1. Binding energies in the laboratory system for different numbers of bosons, N, as a function of the control parameter, in the transitional regions (a) $U(5)$-$SU(3)$, (b) $U(5)$-$O(6)$, and (c) $SU(3)$-$O(6)$. Full circles and open squares correspond to the theoretical positions of different chains of isotopes.

The next question will be: which observable can inform us about the existence of phase transition? The natural answer should be BE or equivalently two-neutron separation energies (S_{2n}). However, it is not clear, a priori , how S_{2n} will behave when crossing a phase transition point. Let us start with the definition of S_{2n}.

$$S_{2n} = A + BN + BE_{IBM}(N) - BE_{IBM}(N-1) \qquad (3)$$

Where A and B take into account the bulk contribution to the nuclear interaction and come from the linear and quadratic U(6) Casimir invariants. The coefficients A and B remain as constants for a given major closed shell [15] . When the linear dependence is excluded from S_{2n}, we will refer to the remaining as S_{2n} . Next, we will simulate phase transitions establishing a connection between the control parameter and the number of bosons. We thereby focus on the U(5) to SU(3) transition in a chain of isotopes with 5 protons, and variable number of neutrons pairs, ranging from $N_\nu = 0$ to $N_\nu = 10$. Two functional dependences will be used, firstly linear and secondly quadratic,

$$\xi = 0.099 N_\nu + 0.01 \qquad (4)$$

$$\xi = 0.099 N_\nu^2 + 0.01 \qquad (5)$$

with $\chi = -\sqrt{7}/2$. In order to establish the correspondence, we use the empirical observation that more ,the system moves from the U(5) to the SU(3) limit as the number of bosons increase. For convenience, a linear dependence equal to $S_{2n}^{lin}/\kappa = 200 - 20N_\nu$ has been chosen as a reference value in order to obtain a realistic behavior in S_{2n} .We depict the results in Fig.2 and it can be observed that an anomaly in the linear dependence of S_{2n} and S'_{2n} right at the place where the phase transition takes place . This anomaly is clearly observed in the Nd-Sm-Gd region [8].

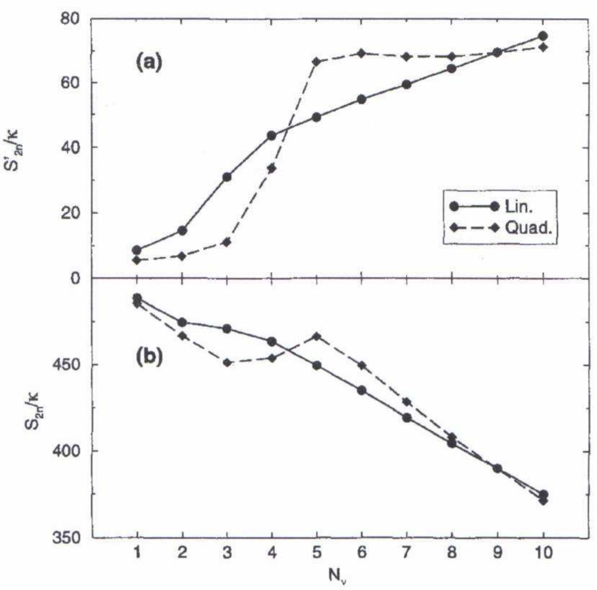

FIGURE 2. Schematic plot of (a) S'_{2n} and (b) S_{2n} in the $U(5)$-$SU(3)$ transitional region. Full lines corresponds to a linear variation of ξ with respect to N_ν while dashed lines corresponds to a quadratic dependence.

3-Results and Discussion

Within framework of IBM , the energy spectra and transition probabilities of many medium-mass and heavy nuclei have been successfully analyzed. However, in many of these studies the binding energies have been ignored. Here, we calculated the BE through the knowledge of the excitation energies and transition probabilities.
At the same time we show that the description of BE is not a trivial task in the framework of the IBM.
Let us consider the U(5)-SU(3) transitional region and in particular the Gd isotopes which form a clear example of nuclei becoming deformed (SU(3) limit). So it seems natural to take $\chi = -\sqrt{7}/2$ for the whole chain. The remaining parameters are taken from [10]. Another possible parameterization can be obtained following references [10,16] where a value $\chi = -0.6$ was used. Both parameterizations provide reasonable descriptions of excitation energies (see reference [10]), but one of the parameterizations (the one with $\chi = -\sqrt{7}/2$) clearly fails in describing S_{2n} values as shown in fig.(3).

4- Conclusion

In this contribution we present a new approach for treating phase transitions in finite systems, taking into account that a change in the control parameter implies a change of nucleus. On the other hand, we study how the crossing of a phase transition point can induce an anomalous behavior in the value of S_{2n}. Finally, we show that the study of S_{2n} for long chains of isotopes can help in the fixing of the parameters of the IBM Hamiltonian.

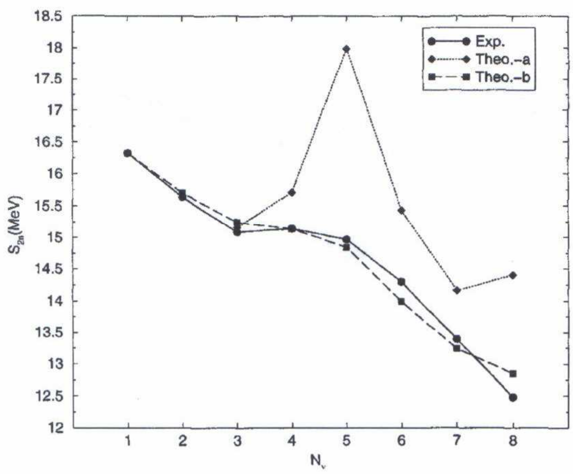

FIGURE 3. Two-neutron separation energies for Gd isotopes. Full lines correspond to experimental data, while dotted lines correspond to $\chi = -\sqrt{7}/2$ and dashed lines correspond to $\chi = -0.6$.

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انتقال ألطور و طاقة الفصل للبوزونات النيوترونية في أنموذج IBM

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المستخلص

في إطار العمل في أنموذج البوزونات المتفاعلة IBM هناك ثلاث مناطق انتقالية قد درست هي : الاهتزازية- الدورانية- الدورانية باتجاه كاما غير المستقر والاهتزازية باتجاه كاما غير المستقر. نوع جديد من المنحنيات قدمت في دراسة انتقال الطور المحدود في نواة الذرة. والمهم هنا هو تحليل طاقة الربط وليس فقط مستويات الطاقة والانتقالات الكهرومغناطيسية في عملية توضيح مناطق الانتقال. وتم أيضا مناقشة طريقة جديدة لتهيئة تفسير مناسب لخواص كل من الحالة الأرضية ومستويات الطاقة المتهيجة.