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# Solving Definite Double Integrals by Simulation Methods

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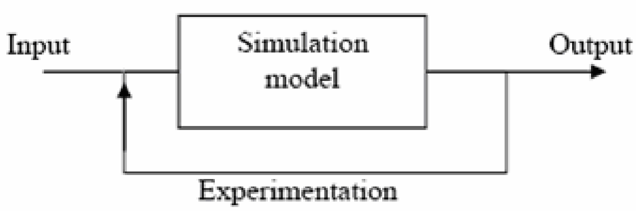
## *Abstract*

This paper introduces a new approach to find a satisfactory solution for definite double integration (DI) by using computer simulation method. The model, the roles and dynamic behavior for this system are built. This method uses this model by applying several experimentations. The results are good with small error because the model is statistical type.

## *1.Introduction*

Computer simulation methods have developed since the early 1960s and may well be the most commonly used of all the analytical tools of management science. The analyst builds a model of the system and writes computer programs which embody the model and uses a computer to imitate the system's behavior when subject to a variety of operating policies. Thus, the most desirable policy may be selected [1,2,3].

The simulation approach to any problem involves the developments of a model of the system. The model is simply an unambiguous statement of the way in which the various components of the system interact to produce the behavior of the system. Computer simulation involves experimentation on a computer-based model of same system; figure (1) shows the basic idea.




Figure(1): Computer simulation idea

When use simulation, the following advantages can be gained.

1. Cost: the cost when use simulation is low.
2. Time: the computer simulation takes little time to complete its operation.
3. Replication: the simulation provided another coy of system, when it rarely kind.
4. Safety: because the simulation may be not estimate the effect of extreme condition. The real system is in the safe side.

## *2.Double Integrals*

In calculus, the definite integral for $f(x)$ of a single variable

$$\int_a^b f(x)dx \quad , \ f(x) \geq 0 \qquad \text{.. ... (1)}$$

is the area under the curve $y = f(x)$ from $x = a$ to $x = b$ . The definite integral can be extended to the functions of more than one variable [4,5]. Consider $z = f(x, y)$ a function of two variables that is continuous and nonnegative on a region $R$

The definite double integral is denoted by

$$\iint_R f(x, y)dA \qquad \text{.......... (2)}$$

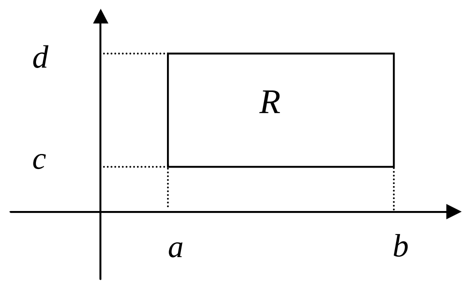
and this definite double integral represents the volume of the solid under the surface $z = f(x, y)$, so we can write

$$V = \iint_R f(x, y)dA \qquad \text{............. (3)}$$

There are two kinds of the region $R$ [4] :

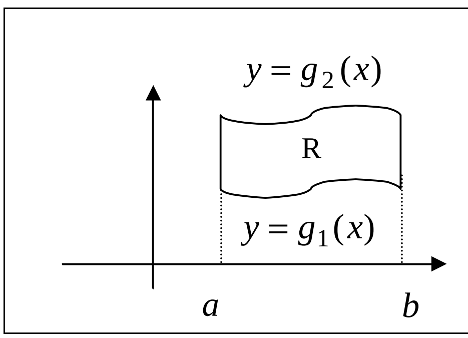
2.1 <u>*The rectangular region*:</u> Here $R$ is the rectangle defined by the inequalities $a \le x \le b$ and $c \le y \le d$ . Then the volume can be written as:

$$V = \int_a^b \int_c^d f(x, y)dydx = \int_c^d \int_a^b f(x, y)dxdy \qquad \text{..............(4)}$$

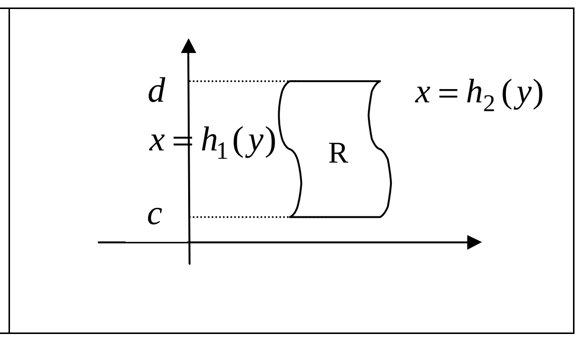



2.2 <u>*The nonrectangular region*</u>: This region falls into two types:

a. <u>A type I region</u>: Here $R$ is bounded on the left and right by vertical lines $x = a$ and $x = b$, and is bounded below and above by continuous curves $y = g_1(x)$ and $y = g_2(x)$ where $g_1(x) \le g_2(x)$ for all $a \le x \le b$.




**Type I region**




**Type II region**

b. <u>A type II region:</u> Here $R$ is bounded below and above by horizontal lines $y = c$ and $y = d$ , and is bounded on the left and right by continuous curves $x = h_1(y)$ and $x = h_2(y)$ where $h_1(y) \le h_2(y)$ for all $c \le y \le d$ .

Now, if $R$ is a type I region on which $f(x, y)$ is continuous, then

$$\iint_R f(x, y)dA = \int_a^b \int_{g_1(x)}^{g_2(x)} f(x, y)dydx \qquad \text{... (5)}$$

and if $R$ is a type II region on which $f(x, y)$ is continuous, then

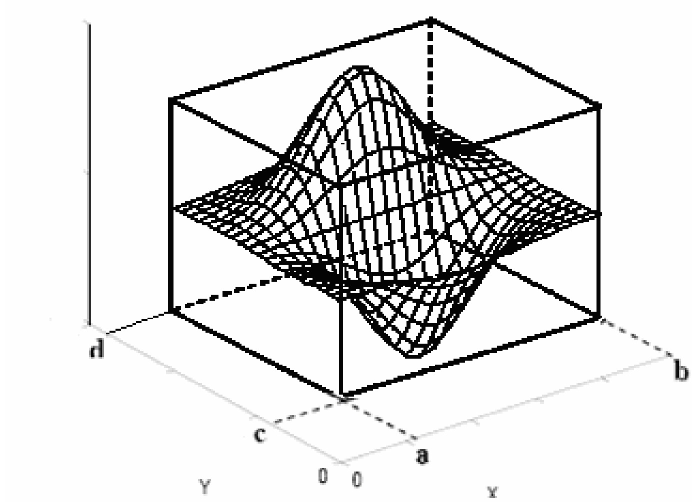
$$\iint_R f(x,y)dA = \int_c^d \int_{h_1(y)}^{h_2(y)} f(x,y)dxdy \quad \text{... (6)}$$

### *3. Double integral modeling:*

Before producing a simulation model, the analyst must decide what will be the principal elements of that model. In doing so, two aspects should be borne in mind. The first, good representation of the system to allow a model to be best . The second aspect, that is, what are the objectives of the study?

Because double integral system is a type of dynamic system, then the model is a set of different equations whose variable is to change their value through time. For nonnegative $f(x,y)$, the definite double integral is equal to the volume under the surface $z = f(x,y)$ and above $xy-$ plane for $x$ and $y$ in $R$. Figure (2) shows a function on a rectangular region.

$$V = \int_a^b \int_c^d f(x,y)dydx \quad \text{........ (1)}$$



Figure(2). The Volume under the surface f(x,y).

The volume is a portion of the total volume enclosed by box with base $((b-a),(d-c))$ and height $\max f(x,y)$. If a point p is dropped randomly on box (this can be visualized as the throwing of a dart p from a distance onto the volume $V$ so that the probability of hitting any point on $V$ is the same), then the probability $P$ of the point p falling in the volume $V$ is given by [5]:

$$p = \frac{V}{(b-a).(d-c).Z\max} \quad \text{........ (8)}$$

where Zmax computed in the **algorithm** below, on the random chosen points p.

If the point p is dropped $N$ times and $k$ times it fell within the volume V, then

$$p = \frac{k}{N} \qquad N \to \infty \quad \text{........ (9)}$$

From (8) and (9) :

$$\frac{V}{(b-a)(d-c).Z\max} = \frac{k}{N} \quad \Rightarrow$$

$$V = \frac{k.(b-a)(d-c).Z\max}{N} \qquad N \to \infty \quad \text{..... (10)}$$

Equation (10) represents a model for computing the double definite integral.

**Algorithm for computing Zmax**

Step1: let Zmax = 0

Step2: generate $x$ and $y$ randomly in the region $R$

Step3: put $Z = f(x, y)$

Step4: if $Z \geq$ Zmax then Zmax $= Z$

Step5: repeat steps 2-4 N times

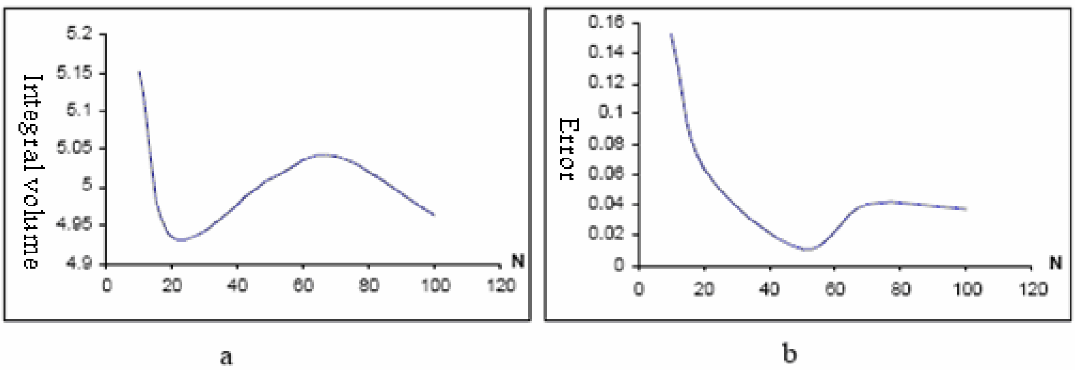
## 4. Experiments:

In order to evaluate our approach, we take five examples. In each example, its result is computed by this approach. Through processing, we determine the best numbers $N$ (which represents the number of the dropped points), and mean error between the original result (computed by integral method) and the simulation result.

1- The following integral was tried to compute its value by simulation method.

$$V = \int_0^3 \int_1^2 (1 + 8xy)dxdy$$

The result for this integral is (56.4774) when (N=100), figure(3-a) showed this value. The value of this integral when using integral method is (57), for that the mean error by using simulation method is shown in figure(3-b).

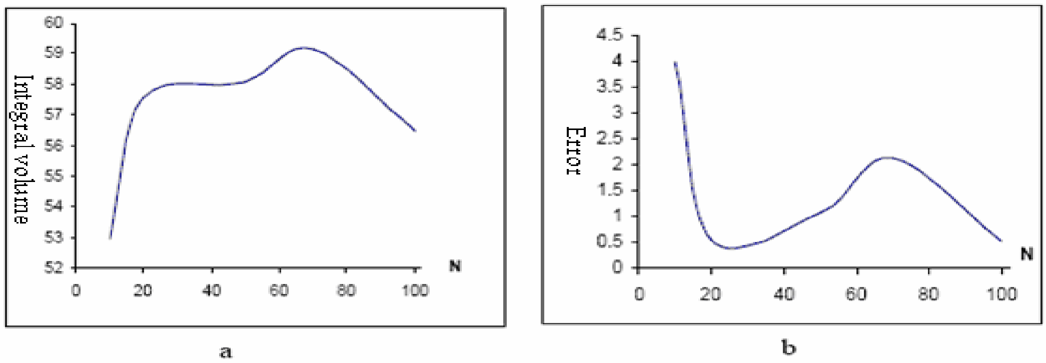



Figure(3): a)- the best value for integral, b)- the simulation error when compared with original value for integral.

2- The following integral was tried to compute its value by simulation method.

$$V = \int_0^2 \int_0^1 (4 - x - y)dxdy$$

The result for this integral is (5.0112) when ($N$ =50), figure (4-a) showed this value. The value of this integral when using integral method is (5), for that, the mean error by use simulation method is shown in figure(4-b).

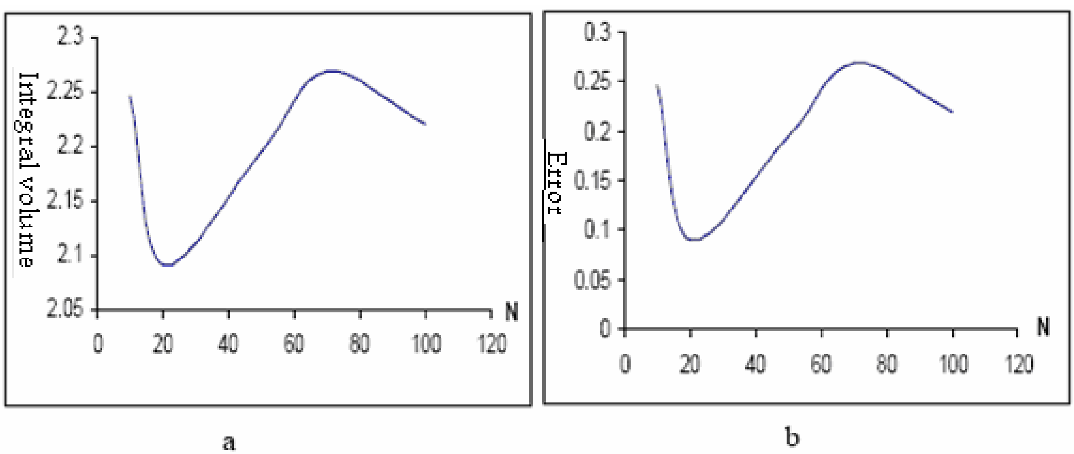



Figure(4) : a)– the best value for integral, b)– the simulation error when compared with original value for integral.

3- The following integral was tried to compute its value by simulation method.

$$V = \int_0^{\ln 3}\int_0^{\ln 2} e^{x+y}\ dxdy$$

The result for this integral is (2.091) when ( $N$ =20), figure (5-a) showed this value. The value of this integral when using integral method is (2), for that, the mean error by use simulation method is shown in figure (5-b).

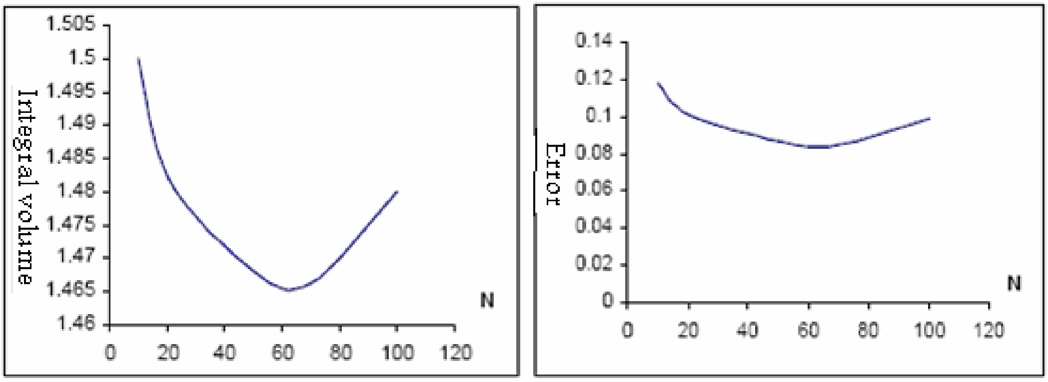



Figure(5): a)– the best value for integral, b)– the simulation error when compared with original value for integral.

4- The following integral was tried to compute its value by simulation method.

$$V = \int_0^{2}\int_0^{1} \sin(\sqrt{x^3 + y^3})dxdy$$

The result for this integral is (1.4663) when ( $N$ =70), figure (6-a) showed the this value. The value of this integral when using integral method is (1.3817), for that, the mean error by use simulation method is shown in figure (6-b).



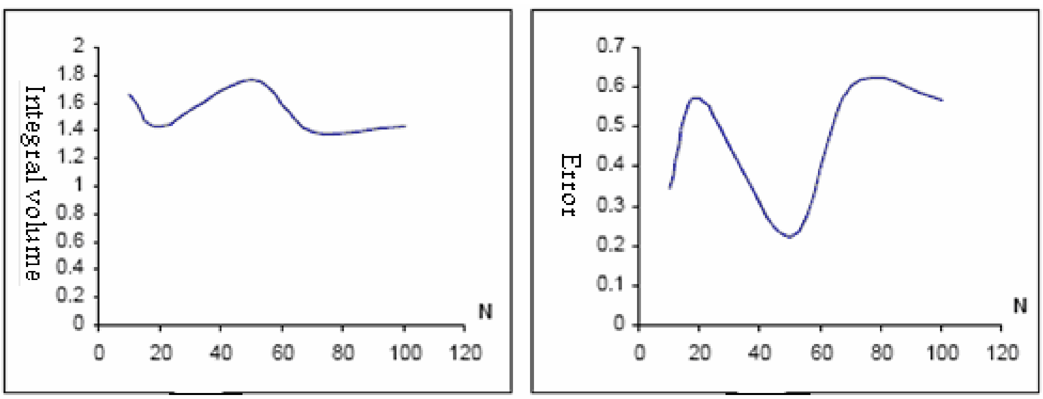

a b

Figure(6): a)- the best value for integral, b)- the simulation error when compared with original value for integral.

5- The following integral was tried to compute its value by simulation method.

$$V = \int_0^1 \int_0^{2-2x} (4-4x-2y)\,dxdy$$

The result for this integral is (1.3933) when ($N$ =70), figure(7-a) showed the this value. The value of this integral when using integral method is (1.333), for that the mean error by use simulation method is shown in figure (7-b).




a b

Figure(7) : a)- the best value for integral, b)- the simulation error when compared with original value for integral.

## *5.Conclusions:*

This approach might help to reach a satisfactory solution to any definite double integral. Ideally, this approach covers two aspects: the building of a model and the application of simulation method. While experimental results have small error when compared with exact results, there are some concerns that should be noted: First, this research represents an initial attempt to solve double integrals without using integral methods. Second, this system (DI) and its model have a discrete behavior, and for that the results may have errors. But this research remains good idea for solving any DI.

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# حل التكامل الثنائي المحدد باستخدام طرائق المحاكاة

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***المستخلص***

هذا البحث يقدم طريقة جديدة تستخدم طرائق المحاكاة بوساطة الحاسوب لإيجاد حل مُقنع للتكامل الثنائي المحدد (DI). الأنموذج لهذا النظام و القوانين و سلوكه الديناميكي تم بنائهم. هذه الطريقة استخدمت هذا الأنموذج بأجراء مجموعة من الاختبارات. كانت النتائج جيدة مع القليل من الخطأ لان هذا الأنموذج من نوع احصائي.