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Expected Mean Squares For 4-Way Crossed Model With Balanced Correlated Data

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ABSTRACT

In this study we calculate the expected mean squares for 4-way crossed balanced model with correlated data and notice the effect of the correlated data on F statistic. The expected mean of squares for this study is calculated using an approach based on some aspects of linear algebra.

Key words: The crossed model , Analysis of variance

Introduction

ANOVA has become a popular statistical procedure in a wide variety of disciplines. For all that the analysis of data from experimental designs is often hampered by lack of technique to correct the usual F-test for the effect of correlations. Although the assumption of the independence of the observations in ANOVA may be seen like a reasonable assumption in examining data using experimental design . The assumption of independent however is rarely verified. Gastwirth and Rubin (1971) [2],Smith and Lewis (1980) [1].

Pavur and Liwes (1982) [7] study this case and explain that the correlate caused weakness of results for the analysis of variance , Pavur and Davenport (1985) [6] study the effect of correlated data on the analysis of variance results and on the type I error for 2-way balanced model . Pavur (1988) [5] studied simple linear model with correlated error expression and notice the effect of correlated on multiple comparison procedures for this model , Al-Shahiry (1997) [4] studied the effect of correlation on F statistic and the correction factor for one way model Al-kaabawi (2000) [10] found the expected mean squares for balanced crossed two-way model with correlated data ,Abdullah and Al-kaabawi (2007) [8] found the expected mean squares for balanced crossed 3-way model with correlated data , Al-Kaabawi (2007) studied the effect of dependent data on type I error rates for multiple comparison procedures for 3-way crossed balanced model [9] .

In this study, we have developed a method for adjusting an ANOVA in the presence of correlated data, while their independence is an essential assumption in (ANOVA). In addition, the model should be linear and error terms should be independent and have identical normal distribution.

The task has been accomplished by determining the expected mean squares for error and treatments for the balanced crossed 4-way model and correcting the F statistics for testing the factor effects. Also, this study focuses on the true type I error probability and the effects of departures from independence assumptions on hypothesis in the balanced crossed 4-way model analysis of variance.

DEFINING THE MODEL

Consider the balanced crossed 4- way model with correlated data

$$Y = XB + E \qquad ...(1)$$

where Y is an $(abcdm \times 1)$ vector of observations , X is the design matrix which is equal to

$$1_m \otimes \begin{bmatrix} 1_{abcd} : 1_{bcd} \otimes I_a : 1_{cd} \otimes I_b \otimes 1_a : 1_d \otimes I_c \otimes 1_{ab} : I_d \otimes 1_{abc} : 1_{cd} \otimes I_{ab} : 1_d \otimes I_c \otimes 1_b \otimes I_a : \\ I_d \otimes 1_{bc} \otimes I_a : 1_d \otimes I_c \otimes I_b \otimes 1_a : I_d \otimes 1_c \otimes I_b \otimes 1_a : I_{cd} \otimes 1_{ab} : 1_d \otimes I_{abc} : I_d \otimes 1_c \otimes I_{ab} : \\ I_{cd} \otimes 1_b \otimes I_a : I_{bcd} \otimes 1_a : I_{abcd} \end{bmatrix} \qquad ...(2)$$

E $\approx N_{abcdm}(0, \sigma^2 R)$, where R is a correlation matrix and σ^2 is the variance of each component of observations. Let,

$$B' = [\theta, \alpha_1, ..., \alpha_a, \beta_1, ..., \beta_b, \gamma_1, ..., \gamma_c, \delta_1, ..., \delta_d, (\alpha\beta)_{11}, ..., (\alpha\beta)_{ab}, (\alpha\gamma)_{11}, ..., (\alpha\gamma)_{ac}, (\alpha\delta)_{11}, ..., (\alpha\delta)_{ad},$$
$$(\beta\gamma)_{11}, ..., (\beta\gamma)_{bc}, (\beta\delta)_{11}, ..., (\beta\delta)_{bd}, (\gamma\delta)_{11}, ..., (\gamma\delta)_{cd}, (\alpha\beta\gamma)_{111}, ..., (\alpha\beta\gamma)_{abc}, (\alpha\beta\delta)_{111}, ..., (\alpha\beta\delta)_{abd},$$
$$(\alpha\gamma\delta)_{111}, ..., (\alpha\gamma\delta)_{acd}, (\beta\gamma\delta)_{111}, ..., (\beta\gamma\delta)_{bcd}, (\alpha\beta\gamma\delta)_{1111}, ..., (\alpha\beta\gamma\delta)_{abcd}]$$
$$= [\theta, \alpha', \beta', \gamma', \delta', (\alpha\beta)', (\alpha\gamma)', (\alpha\delta)', (\beta\gamma)', (\beta\delta)', (\gamma\delta)', (\alpha\beta\gamma)', (\alpha\beta\delta)', (\alpha\gamma\delta)', (\beta\gamma\delta)', (\alpha\beta\gamma\delta)'] \qquad ...(3)$$

such that

$$\sum_i \alpha_i = \sum_j \beta_j = \sum_k \gamma_k = \sum_l \delta_l = \sum_i (\alpha\beta)_{ij} = \sum_j (\alpha\beta)_{ij} = \sum_i (\alpha\gamma)_{ik} = \sum_k (\alpha\gamma)_{ik} = \sum_i (\alpha\delta)_{il} = \sum_l (\alpha\delta)_{il}$$
$$= \sum_j (\beta\gamma)_{jk} = \sum_k (\beta\gamma)_{jk} = \sum_j (\beta\delta)_{jl} = \sum_l (\beta\delta)_{jl} = \sum_k (\gamma\delta)_{kl} = \sum_l (\gamma\delta)_{kl} = \sum_i (\alpha\beta\gamma)_{ijk} = \sum_j (\alpha\beta\gamma)_{ijk}$$
$$= \sum_k (\alpha\beta\gamma)_{ijk} = \sum_i (\alpha\beta\delta)_{ijl} = \sum_j (\alpha\beta\delta)_{ijl} = \sum_l (\alpha\beta\delta)_{ijl} = \sum_i (\alpha\gamma\delta)_{ikl} = \sum_k (\alpha\gamma\delta)_{ikl} = \sum_l (\alpha\gamma\delta)_{ikl}$$
$$= \sum_j (\beta\gamma\delta)_{jkl} = \sum_k (\beta\gamma\delta)_{jkl} = \sum_l (\beta\gamma\delta)_{jkl} = \sum_i (\alpha\beta\gamma\delta)_{ijkl} = \sum_j (\alpha\beta\gamma\delta)_{ijkl} = \sum_k (\alpha\beta\gamma\delta)_{ijkl}$$
$$= \sum_l (\alpha\beta\gamma\delta)_{ijkl} = 0 \qquad ...(4)$$

then

$$\mu = XB = \theta 1_{abcdm} + 1_{bcdm} \otimes \alpha + 1_{cdm} \otimes \beta \otimes 1_a + 1_{dm} \otimes \gamma \otimes 1_{ab} + 1_m \otimes \delta \otimes 1_{abc} + 1_{cdm} \otimes (\alpha\beta) +$$
$$(1_m \otimes 1_d \otimes I_c \otimes 1_b \otimes I_a)(\alpha\gamma) + (1_m \otimes I_d \otimes 1_c \otimes 1_b \otimes I_a)(\alpha\delta) + 1_{dm} \otimes (\beta\gamma) \otimes 1_a +$$
$$(1_m \otimes I_d \otimes 1_c \otimes I_b \otimes 1_a)(\beta\delta) + 1_m \otimes (\gamma\delta) \otimes 1_{ab} + 1_{dm} \otimes (\alpha\beta\gamma) + (1_m \otimes I_d \otimes 1_c \otimes I_b \otimes I_a)(\alpha\beta\delta) +$$
$$(1_m \otimes I_d \otimes I_c \otimes 1_b \otimes I_a)(\alpha\gamma\delta) + 1_m \otimes (\beta\gamma\delta) \otimes 1_a + 1_m \otimes (\alpha\beta\gamma\delta) \qquad ...(5)$$

where 1_s is a vector of one and $\otimes$ denotes the kronecker matrix product of two matrices. The kronecker product was defined by Graybill (1983) . Now, we have

$$Y \approx N(\mu, \sigma^2 R) \qquad ...(6)$$

where

$$R=(1-\rho_1)I_{abcdm}+(\rho_1-\rho_2-\rho_3-\rho_4-\rho_5+\rho_6+\rho_7+\rho_8+\rho_9+\rho_{10}+\rho_{11}-\rho_{12}-\rho_{13}-\rho_{14}-\rho_{15}+\rho_{16})J_m\otimes I_{abcd}+(\rho_2-\rho_6-\rho_7-\rho_8+\rho_{12}+\rho_{13}+\rho_{15}-\rho_{16})J_{dm}\otimes I_{abc}+(\rho_3-\rho_6-\rho_9-\rho_{10}+\rho_{12}+\rho_{13}+\rho_{14}-\rho_{16})J_m\otimes I_d\otimes J_c\otimes I_{ab}+(\rho_6-\rho_{12}-\rho_{13}+\rho_{16})J_{cdm}\otimes I_{ab}+(\rho_4-\rho_7-\rho_9-\rho_{11}+\rho_{12}+\rho_{14}+\rho_{15}-\rho_{16})J_m\otimes I_{cd}\otimes J_b\otimes I_a+(\rho_7-\rho_{12}-\rho_{15}+\rho_{16})J_{dm}\otimes I_c\otimes J_b\otimes I_a+(\rho_9-\rho_{12}-\rho_{14}+\rho_{16})J_m\otimes I_d\otimes J_{bc}\otimes I_a+(\rho_{12}-\rho_{16})J_{bcdm}\otimes I_a+(\rho_5-\rho_8-\rho_{10}-\rho_{11}+\rho_{13}+\rho_{14}+\rho_{15}-\rho_{16})J_m\otimes I_{bcd}\otimes J_a+(\rho_8-\rho_{13}-\rho_{15}+\rho_{16})J_{dm}\otimes I_{bc}\otimes J_a+(\rho_{10}-\rho_{13}-\rho_{14}+\rho_{16})J_m\otimes I_d\otimes J_c\otimes I_b\otimes J_a+(\rho_{13}-\rho_{16})J_{cdm}\otimes I_b\otimes J_a+(\rho_{11}-\rho_{14}-\rho_{15}+\rho_{16})J_m\otimes I_{cd}\otimes J_{ab}+(\rho_{15}-\rho_{16})J_{dm}\otimes I_c\otimes J_{ab}+(\rho_{14}-\rho_{16})J_m\otimes I_d\otimes J_{abc}+\rho_{16}J_{abcdm} \quad ...(7)$$

Let

$$M_s=\frac{1}{s}J_s=\frac{1}{s}1_s1_s' \quad and \quad N_s=I_s-M_s \quad ...(8)$$

(see Pavur (1988))

then the correlation matrix can be rewritten as

$$R=\lambda_1M_{abcdm}+\lambda_2M_{bcdm}\otimes N_a+\lambda_3M_{cdm}\otimes N_b\otimes M_a+\lambda_4M_{dm}\otimes N_c\otimes M_{ab}+\lambda_5M_m\otimes N_d\otimes M_{abc}+\lambda_6M_{cdm}\otimes N_{ab}+\lambda_7M_{dm}\otimes N_c\otimes M_b\otimes N_a+\lambda_8M_m\otimes N_d\otimes M_{bc}\otimes N_a+\lambda_9M_{dm}\otimes N_{bc}\otimes M_a+\lambda_{10}M_m\otimes N_d\otimes M_c\otimes N_b\otimes M_a+\lambda_{11}M_m\otimes N_{cd}\otimes M_{ab}+\lambda_{12}M_{dm}\otimes N_{abc}+\lambda_{13}M_m\otimes N_d\otimes M_c\otimes N_{ab}+\lambda_{14}M_m\otimes N_{cd}\otimes M_b\otimes N_a+\lambda_{15}M_m\otimes N_{bcd}\otimes M_a+\lambda_{16}M_m\otimes N_{abcd}+\lambda_{17}N_m\otimes I_{abcd} \quad ...(9)$$

Where

$$\lambda_{17}=(1-\rho_1)$$

$$\lambda_{16}=(1-\rho_1)+m(\rho_1-\rho_2-\rho_3-\rho_4-\rho_5+\rho_6+\rho_7+\rho_8+\rho_9+\rho_{10}+\rho_{11}+\rho_{12}-\rho_{13}-\rho_{14}-\rho_{15}+\rho_{16})$$

$$\lambda_{15}=\lambda_{16}+am(\rho_5-\rho_8-\rho_{10}-\rho_{11}+\rho_{13}+\rho_{14+}\rho_{15}-\rho_{16})$$

$$\lambda_{14}=\lambda_{16}+bm(\rho_4-\rho_7-\rho_9-\rho_{11}+\rho_{12}+\rho_{14}+\rho_{15}-\rho_{16})$$

$$\lambda_{13}=\lambda_{16}+cm(\rho_3-\rho_6-\rho_9-\rho_{10}+\rho_{12}+\rho_{13}+\rho_{14}-\rho_{16})$$

$$\lambda_{12}=\lambda_{16}+dm(\rho_2-\rho_6-\rho_7-\rho_8+\rho_{12}+\rho_{13}+\rho_{15}-\rho_{16})$$

$$\lambda_{11}=\lambda_{15}+bm(\rho_4-\rho_7-\rho_9-\rho_{11}+\rho_{12}+\rho_{14}+\rho_{15}-\rho_{16})+abm(\rho_{11}-\rho_{14}-\rho_{15}+\rho_{16})$$

$$\lambda_{10}=\lambda_{15}+cm(\rho_3-\rho_6-\rho_9-\rho_{10}+\rho_{12}+\rho_{13}+\rho_{14}-\rho_{16})+acm(\rho_{10}-\rho_{13}-\rho_{14}+\rho_{16})$$

$$\lambda_9=\lambda_{15}+dm(\rho_2-\rho_6-\rho_7-\rho_8+\rho_{12}+\rho_{13}+\rho_{15}-\rho_{16})+adm(\rho_8-\rho_{13}-\rho_{15}+\rho_{16})$$

$$\lambda_8=\lambda_{14}+cm(\rho_3-\rho_6-\rho_9-\rho_{10}+\rho_{12}+\rho_{13}+\rho_{14}-\rho_{16})+bcm(\rho_9-\rho_{12}-\rho_{14}+\rho_{16})$$

$$\lambda_7=\lambda_{14}+dm(\rho_2-\rho_6-\rho_7-\rho_8+\rho_{12}+\rho_{13}+\rho_{15}-\rho_{16})+bdm(\rho_7-\rho_{12}-\rho_{15}+\rho_{16})$$

$$\lambda_6=\lambda_{13}+dm(\rho_2-\rho_6-\rho_7-\rho_8+\rho_{12}+\rho_{13}+\rho_{15}-\rho_{16})+cdm(\rho_6-\rho_{12}-\rho_{13}+\rho_{16})$$

$$\lambda_5=\lambda_{11}+cm(\rho_3-\rho_6-\rho_9-\rho_{10}+\rho_{12}+\rho_{13}+\rho_{14}-\rho_{16})+bcm(\rho_9-\rho_{12}-\rho_{14}+\rho_{16})+acm(\rho_{10}-\rho_{13}-\rho_{14}+\rho_{16})+abcm(\rho_{14}-\rho_{16})$$

$$\lambda_4=\lambda_{11}+dm(\rho_2-\rho_6-\rho_7-\rho_8+\rho_{12}+\rho_{13}+\rho_{15}-\rho_{16})+adm(\rho_8-\rho_{13}-\rho_{15}+\rho_{16})+bdm(\rho_7-\rho_{12}-\rho_{15}+\rho_{16})+abdm(\rho_{15}-\rho_{16})$$

$$\lambda_3=\lambda_{10}+dm(\rho_2-\rho_6-\rho_7-\rho_8+\rho_{12}+\rho_{13}+\rho_{15}-\rho_{16})+cdm(\rho_6-\rho_{12}-\rho_{13}+\rho_{16})+adm(\rho_8-\rho_{13}-\rho_{15}+\rho_{16})+acdm(\rho_{13}-\rho_{16})$$

$$\lambda_2=\lambda_7+cm(\rho_3-\rho_6-\rho_9-\rho_{10}+\rho_{12}+\rho_{13}+\rho_{14}-\rho_{16})+cdm(\rho_6-\rho_{12}-\rho_{13}+\rho_{16})+bcm(\rho_9-\rho_{12}-\rho_{14}+\rho_{16})+bcdm(\rho_{12}-\rho_{16})$$

$$\lambda_1 = \lambda_5 + dm(\rho_2 - \rho_6 - \rho_7 - \rho_8 + \rho_{12} + \rho_{13} + \rho_{15} - \rho_{16}) + cdm(\rho_6 - \rho_{12} - \rho_{13} + \rho_{16}) + bdm(\rho_7 - \rho_{12} - \rho_{15} + \rho_{16}) + adm(\rho_8 - \rho_{13} - \rho_{15} + \rho_{16}) + bcdm(\rho_{12} - \rho_{16}) + acdm(\rho_{13} - \rho_{16}) + abdm(\rho_{15} - \rho_{16}) + abcdm\rho_{16} \quad ...(10)$$

$\lambda_1, \lambda_2, ..., \lambda_{17}$ represent the eigen values of the correlation matrix R repeated
$1, (a-1), (b-1), (c-1), (d-1), (a-1)(b-1), (a-1)(c-1), (a-1)(d-1), (b-1)(c-1), (b-1)(d-1),$
$(c-1)\ (d-1), (a-1)(b-1)(c-1), (a-1)\ (b-1)\ (d-1), (a-1)(c-1)(d-1), (b-1)(c-1)(d-1),$
$abcd(m-1)$
respectively. These eigen values are positive constants $(i.e, \lambda_i > 0, \forall i = 1,...,17)$

Since R is positive definite matrix, then these eigen values are positive values. Furthermore, each of the matrices
$M_{abcdm}, M_{bcdm} \otimes N_a, M_{cdm} \otimes N_b \otimes M_a, M_{dm} \otimes N_c \otimes M_{ab}, M_m \otimes N_d \otimes M_{abc}, M_{cdm} \otimes N_{ab},$
$M_{dm} \otimes N_c \otimes M_b \otimes N_a, M_m \otimes N_d \otimes M_{bc} \otimes N_a, M_{dm} \otimes N_{bc} \otimes M_a, M_m \otimes N_d \otimes M_c \otimes N_b \otimes M_a,$
$M_m \otimes N_{cd} \otimes M_{ab}, M_{dm} \otimes N_{abc}, M_m \otimes N_d \otimes M_c \otimes N_{ab}, M_m \otimes N_{cd} \otimes M_b \otimes N_a, M_m \otimes N_{bcd} \otimes M_a,$
$M_m \otimes N_{abcd}$ *and* $N_m \otimes I_{abcd}$
is idempotent the product of any two which is equal to the zero matrix , then there exists a unique square matrix $R^{-\frac{1}{2}}$ where

$$R^{-\frac{1}{2}} = \frac{1}{\sqrt{\lambda_1}} M_{abcdm} + \frac{1}{\sqrt{\lambda_2}} M_{bcdm} \otimes N_a + \frac{1}{\sqrt{\lambda_3}} M_{cdm} \otimes N_b \otimes M_a + \frac{1}{\sqrt{\lambda_4}} M_{dm} \otimes N_c \otimes M_{ab} + \frac{1}{\sqrt{\lambda_5}} M_m \otimes N_d \otimes M_{abc} + \frac{1}{\sqrt{\lambda_6}} M_{cdm} \otimes N_{ab} + \frac{1}{\sqrt{\lambda_7}} M_{dm} \otimes N_c \otimes M_b \otimes N_a + \frac{1}{\sqrt{\lambda_8}} M_m \otimes N_d \otimes M_{bc} \otimes N_a + \frac{1}{\sqrt{\lambda_9}} M_{dm} \otimes N_{bc} \otimes M_a + \frac{1}{\sqrt{\lambda_{10}}} M_m \otimes N_d \otimes M_c \otimes N_b \otimes M_a + \frac{1}{\sqrt{\lambda_{11}}} M_m \otimes N_{cd} \otimes M_{ab} + \frac{1}{\sqrt{\lambda_{12}}} M_{dm} \otimes N_{abc} + \frac{1}{\sqrt{\lambda_{13}}} M_m \otimes N_d \otimes M_c \otimes N_{ab} + \frac{1}{\sqrt{\lambda_{14}}} M_m \otimes N_{cd} \otimes M_b \otimes N_a + \frac{1}{\sqrt{\lambda_{15}}} M_m \otimes N_{bcd} \otimes M_a + \frac{1}{\sqrt{\lambda_{16}}} M_m \otimes N_{abcd} + \frac{1}{\sqrt{\lambda_{17}}} N_m \otimes I_{abcd} \quad ...(11)$$

Now , by using $R^{-\frac{1}{2}}$ to transform the model (1) to an ordinary Linear model

$$R^{-\frac{1}{2}} Y = R^{-\frac{1}{2}} XB + R^{-\frac{1}{2}} E \quad ...(12)$$

$$Y^* = R^{-\frac{1}{2}} XB + E^* \quad ...(13)$$

Where

$$E^* \approx N_{abcdm}(0, \sigma^2 I) \quad ...(14)$$

and

$$\mu^* = R^{-\frac{1}{2}} XB = XB^* \quad Y^* = R^{-\frac{1}{2}} Y \quad , \quad E^* = R^{-\frac{1}{2}} E \quad ...(15)$$

Such that

$$B^{*'} = \begin{bmatrix} \frac{1}{\sqrt{\lambda_1}}\theta, \frac{1}{\sqrt{\lambda_2}}\alpha', \frac{1}{\sqrt{\lambda_3}}\beta', \frac{1}{\sqrt{\lambda_4}}\gamma', \frac{1}{\sqrt{\lambda_5}}\delta', \frac{1}{\sqrt{\lambda_6}}(\alpha\beta)', \frac{1}{\sqrt{\lambda_7}}(\alpha\gamma)', \frac{1}{\sqrt{\lambda_8}}(\alpha\delta)', \frac{1}{\sqrt{\lambda_9}}(\beta\gamma)', \\ \frac{1}{\sqrt{\lambda_{10}}}(\beta\delta)', \frac{1}{\sqrt{\lambda_{11}}}(\gamma\delta)', \frac{1}{\sqrt{\lambda_{12}}}(\alpha\beta\gamma)', \frac{1}{\sqrt{\lambda_{13}}}(\alpha\beta\delta)', \frac{1}{\sqrt{\lambda_{14}}}(\alpha\gamma\delta)', \frac{1}{\sqrt{\lambda_{15}}}(\beta\gamma\delta)', \\ \frac{1}{\sqrt{\lambda_{16}}}(\alpha\beta\gamma\delta)' \end{bmatrix} \quad ...(16)$$

So

$$Y^* = XB^* + E^* \quad ...(17)$$

Now , define $e_s = (0,...,0,1,...,0)'$, (see Al-shahiry (1997))

$$Y_i = (I_m \otimes I_d \otimes I_c \otimes I_b \otimes e_i')Y \;;\; i = 1,...,a$$

$$Y = \left(I_m \otimes I_d \otimes I_c \otimes I_b \otimes [e_1 : e_2 : ... : e_a] \right) \begin{pmatrix} Y_1 \\ Y_2 \\ \cdot \\ \cdot \\ \cdot \\ Y_a \end{pmatrix} \quad ...(18)$$

$$Y_j = (I_m \otimes I_d \otimes I_c \otimes e_j' \otimes I_a)Y \;;\; j = 1,...,b$$

$$Y = \left(I_m \otimes I_d \otimes I_c \otimes [e_1 \otimes I_a : e_2 \otimes I_a : ... : e_b \otimes I_a] \right) \begin{pmatrix} Y_1 \\ Y_2 \\ \cdot \\ \cdot \\ \cdot \\ Y_b \end{pmatrix} \quad ...(19)$$

$$Y_k = (I_m \otimes I_d \otimes e_k' \otimes I_b \otimes I_a)Y \;;\; k = 1,...,c$$

$$Y = \left(I_m \otimes I_d \otimes [e_1 \otimes (I_b \otimes I_a) : ... : e_c \otimes (I_b \otimes I_a)] \right) \begin{pmatrix} Y_1 \\ Y_2 \\ \cdot \\ \cdot \\ \cdot \\ Y_c \end{pmatrix} \quad ...(20)$$

$$Y_l = (I_m \otimes e_l' \otimes I_c \otimes I_b \otimes I_a)Y \qquad ; \quad l = 1,\ldots,d$$

$$Y = \left(I_m \otimes \left[e_1 \otimes (I_c \otimes I_b \otimes I_a) : \ldots : e_d \otimes (I_c \otimes I_b \otimes I_a) \right] \right) \begin{pmatrix} Y_1 \\ Y_2 \\ \cdot \\ \cdot \\ \cdot \\ Y_d \end{pmatrix} \qquad \ldots(21)$$

$$Y_{ij} = (I_m \otimes I_d \otimes I_c \otimes e_j' \otimes e_i')Y \qquad ; i = 1,\ldots,a, \; j = 1,\ldots,b$$

$$Y = \left(I_m \otimes I_d \otimes I_c \otimes \begin{bmatrix} e_1 \otimes e_1 : e_2 \otimes e_1 : \ldots : e_b \otimes e_1 : e_1 \otimes e_2 : e_2 \otimes e_2 : \ldots : e_b \otimes e_2 : \ldots : \\ e_1 \otimes e_a : \ldots : e_b \otimes e_a \end{bmatrix} \right) \begin{pmatrix} Y_{11} \\ \cdot \\ \cdot \\ \cdot \\ Yab \end{pmatrix} \ldots(22)$$

$$Y_{ik} = (I_m \otimes I_d \otimes e_k' \otimes I_b \otimes e_i')Y \;\; ; i = 1,\ldots,a \;\; , \; k = 1,\ldots,c$$

$$Y = (I_m \otimes I_d \otimes \begin{bmatrix} e_1 \otimes I_b \otimes e_1 : e_2 \otimes I_b \otimes e_1 : \ldots : e_c \otimes I_b \otimes e_1 : e_1 \otimes I_b \otimes e_2 : e_2 \otimes I_b \otimes e_2 \\ : \ldots : e_c \otimes I_b \otimes e_2 : \ldots : e_1 \otimes I_b \otimes e_a : e_2 \otimes I_b \otimes e_a : \ldots : e_c \otimes I_b \otimes e_a \end{bmatrix}) \begin{pmatrix} Y_{11} \\ \cdot \\ \cdot \\ \cdot \\ Y_{ac} \end{pmatrix} \ldots(23)$$

$$Y_{il} = (I_m \otimes e_l' \otimes I_c \otimes I_b \otimes e_i')Y \qquad ; \quad i = 1,\ldots,a \;\; , \; l = 1,\ldots,d$$

$$Y = \left(I_m \otimes \begin{bmatrix} e_1 \otimes (I_c \otimes I_b) \otimes e_1 : e_2 \otimes (I_c \otimes I_b) \otimes e_1 : \ldots : e_d \otimes (I_c \otimes I_b) \otimes e_1 : \\ e_1 \otimes (I_c \otimes I_b) \otimes e_2 : e_2 \otimes (I_c \otimes I_b) \otimes e_2 : \ldots : e_d \otimes (I_c \otimes I_b) \otimes e_2 : \ldots : \\ e_1 \otimes (I_c \otimes I_b) \otimes e_a : e_2 \otimes (I_c \otimes I_b) \otimes e_a : \ldots : e_d \otimes (I_c \otimes I_b) \otimes e_a \end{bmatrix} \right) \begin{pmatrix} Y_{11} \\ \cdot \\ \cdot \\ \cdot \\ Y_{ad} \end{pmatrix} \quad \ldots(24)$$

$$Y_{jk} = (I_m \otimes I_d \otimes e_k' \otimes e_j' \otimes I_a)Y \;\; ; \;\; j = 1,\ldots,b \; , \; k = 1,\ldots,c$$

$$Y = \left(I_m \otimes I_d \otimes \begin{bmatrix} e_1 \otimes e_1 \otimes I_a : e_2 \otimes e_1 \otimes I_a : \ldots : e_c \otimes e_1 \otimes I_a : e_1 \otimes e_2 \otimes I_a : \\ e_2 \otimes e_2 \otimes I_a : \ldots : e_c \otimes e_2 \otimes I_a : \ldots : e_1 \otimes e_b \otimes I_a : \cdots : e_c \otimes e_b \otimes I_a \end{bmatrix} \right) \begin{pmatrix} Y_{11} \\ \cdot \\ \cdot \\ \cdot \\ Y_{bc} \end{pmatrix} \ldots(25)$$

$$Y_{jl} = (I_m \otimes e_l' \otimes I_c \otimes e_j' \otimes I_a)Y \qquad : j = 1,\ldots,b \; , \; l = 1,\ldots,d$$

$$Y = (I_m \otimes \begin{bmatrix} e_1 \otimes I_c \otimes e_1 \otimes I_a : e_2 \otimes I_c \otimes e_1 \otimes I_a : \ldots : e_d \otimes I_c \otimes e_1 \otimes I_a : e_1 \otimes I_c \\ \otimes e_2 \otimes I_a : e_2 \otimes I_c \otimes e_2 \otimes I_a : \ldots : e_d \otimes I_c \otimes e_2 \otimes I_a : \ldots : e_1 \otimes I_c \otimes e_b \otimes I_a \\ : e_2 \otimes I_c \otimes e_b \otimes I_a : \ldots : e_d \otimes I_c \otimes e_b \otimes I_a \end{bmatrix}) \begin{pmatrix} Y_{11} \\ \cdot \\ \cdot \\ \cdot \\ Y_{bd} \end{pmatrix} \quad \ldots(26)$$

$$Y_{kl} = (I_m \otimes e_l' \otimes e_k' \otimes I_b \otimes I_a)Y \; ; k = 1,\ldots,c \; , \; l = 1,\ldots,d$$

$$Y = (I_m \otimes \begin{bmatrix} e_1 \otimes e_1 \otimes (I_b \otimes I_a) : e_2 \otimes e_1 \otimes (I_b \otimes I_a) : \ldots : e_d \otimes e_1 \otimes (I_b \otimes I_a) : \\ e_1 \otimes e_2 \otimes (I_b \otimes I_a) : e_2 \otimes e_2 \otimes (I_b \otimes I_a) : \ldots : e_d \otimes e_2 \otimes (I_b \otimes I_a) \\ : \ldots : e_1 \otimes e_c \otimes (I_b \otimes I_a) : e_2 \otimes e_c \otimes (I_b \otimes I_a) : \ldots : e_d \otimes e_c \otimes (I_b \otimes I_a) \end{bmatrix}) \begin{pmatrix} Y_{11} \\ . \\ . \\ . \\ Y_{cd} \end{pmatrix} \quad \ldots(27)$$

$$Y_{ijk} = (I_m \otimes I_d \otimes e_k' \otimes e_j' \otimes e_i')Y \; ; i = 1,\ldots,a \; , \; j = 1,..,b \; , \; k = 1,\ldots,c$$

$$Y = (I_m \otimes I_d \otimes \begin{bmatrix} e_1 \otimes e_1 \otimes e_1 : e_2 \otimes e_1 \otimes e_1 : \ldots : e_c \otimes e_1 \otimes e_1 : e_1 \otimes e_2 \otimes e_1 : e_2 \otimes e_2 \otimes e_1 \\ : \ldots : e_c \otimes e_2 \otimes e_1 : \ldots : e_1 \otimes e_b \otimes e_1 : e_2 \otimes e_b \otimes e_1 : \ldots : e_c \otimes e_b \otimes e_1 : \ldots : \\ e_1 \otimes e_1 \otimes e_a : e_2 \otimes e_1 \otimes e_a : \ldots : e_c \otimes e_1 \otimes e_a : \ldots : e_1 \otimes e_b \otimes e_a : e_2 \otimes e_b \\ \otimes e_a : \ldots : e_c \otimes e_b \otimes e_a \end{bmatrix}) \begin{pmatrix} Y_{111} \\ . \\ . \\ . \\ Y_{abc} \end{pmatrix} \ldots(28)$$

$$Y_{ijl} = (I_m \otimes e_l' \otimes I_c \otimes e_j' \otimes e_i')Y \; ; \; i = 1,\ldots,a, \; j = 1,\ldots,b, l = 1,\ldots,d$$

$$Y = I_m \otimes \begin{bmatrix} e_1 \otimes I_c \otimes e_1 \otimes e_1 : \ldots : e_d \otimes I_c \otimes e_1 \otimes e_1 : e_1 \otimes I_c \otimes e_2 \otimes e_1 : \ldots : e_d \otimes I_c \otimes e_2 \\ \otimes e_1 : \ldots : e_1 \otimes I_c \otimes e_b \otimes e_1 : \ldots : e_d \otimes I_c \otimes e_b \otimes e_1 : e_1 \otimes I_c \otimes e_1 \otimes e_2 : \ldots : e_d \otimes \\ I_c \otimes e_1 \otimes e_2 : e_1 \otimes I_c \otimes e_2 \otimes e_2 : \ldots : e_d \otimes I_c \otimes e_2 \otimes e_2 : \ldots : e_1 \otimes I_c \otimes e_b \otimes e_2 : \\ \ldots : e_d \otimes I_c \otimes e_b \otimes e_2 : \ldots : e_1 \otimes I_c \otimes e_b \otimes e_a : \ldots : e_d \otimes I_c \otimes e_b \otimes e_a \end{bmatrix}) \begin{pmatrix} Y_{111} \\ . \\ . \\ . \\ Y_{abd} \end{pmatrix} \ldots(29)$$

$$Y_{ikl} = (I_m \otimes e_l' \otimes e_k' \otimes I_b \otimes e_i')Y \; ; i = 1,\ldots,a \; , \; k = 1,\ldots,c \; , \; l = 1,\ldots,d$$

$$Y = (I_m \otimes \begin{bmatrix} e_1 \otimes e_1 \otimes I_b \otimes e_1 : \ldots : e_d \otimes e_1 \otimes I_b \otimes e_1 : \ldots : e_1 \otimes e_c \otimes I_b \otimes e_1 : e_2 \otimes e_c \otimes \\ I_b \otimes e_1 : \ldots : e_d \otimes e_c \otimes I_b \otimes e_1 : \ldots : e_d \otimes e_c \otimes I_b \otimes e_a \end{bmatrix}) \begin{pmatrix} Y_{111} \\ . \\ . \\ . \\ Y_{acd} \end{pmatrix} ..(30)$$

$$Y_{jkl} = (I_m \otimes e_l' \otimes e_k' \otimes e_j' \otimes I_a)Y \; ; \; j = 1,\ldots,b \; , k = 1,\ldots,c \; ; l = 1,\ldots,d$$

$$Y = (I_m \otimes \begin{bmatrix} e_1 \otimes e_1 \otimes e_1 \otimes I_a : \ldots : e_d \otimes e_1 \otimes e_1 \otimes I_a : \ldots : e_1 \otimes e_c \otimes e_1 \otimes I_a : \ldots : \\ e_d \otimes e_c \otimes e_1 \otimes I_a : e_1 \otimes e_1 \otimes e_b \otimes I_a : \ldots : e_d \otimes e_1 \otimes e_b \otimes I_a : e_1 \otimes e_2 \\ \otimes e_b \otimes I_a : \ldots : e_d \otimes e_2 \otimes e_b \otimes I_a : \ldots : e_1 \otimes e_c \otimes e_b \otimes I_a : \ldots : e_d \otimes e_c \otimes e_b \otimes I_a \end{bmatrix}) \begin{pmatrix} Y_{111} \\ . \\ . \\ . \\ Y_{bcd} \end{pmatrix} \ldots(31)$$

ANALYSIS OF VARIANCE

The observation Y_{ijklm} has normal distribution with μ_{ijklm} mean and σ^2 variance then from Cockran"s theorem the total sum of squares (SSTO) to the model can be written as
Where

SSTO=SSA+SSB+SSC+SSD+SSAB+SSAC+SSAD+SSBC+SSBD+SSCD+SSABC

+SSABD+SSACD+SSBCD+SSABCD+SSE …(32)

$$SSTO=\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c}\sum_{l=1}^{d}\sum_{h=1}^{m}Y_{ijklh}^{2}-abcdm\overline{Y}_{.....}^{2} \quad ...(33)$$

$$SSA=bcdm\sum_{i=1}^{a}\overline{Y}_{i....}^{2}-abcdm\overline{Y}_{.....}^{2} \quad ...(34)$$

$$SSB=acdm\sum_{j=1}^{b}\overline{Y}_{.j...}^{2}-abcdm\overline{Y}_{.....}^{2} \quad ...(35)$$

$$SSC=abdm\sum_{k=1}^{c}\overline{Y}_{..k..}^{2}-abcdm\overline{Y}_{.....}^{2} \quad ...(36)$$

$$SSD=abcm\sum_{l=1}^{d}\overline{Y}_{...l.}^{2}-abcdm\overline{Y}_{.....}^{2} \quad ...(37)$$

$$SSAB=cdm\sum_{i=1}^{a}\sum_{j=1}^{b}\overline{Y}_{ij...}^{2}-bcdm\sum_{i=1}^{a}\overline{Y}_{i....}^{2}-acdm\sum_{j=1}^{b}\overline{Y}_{.j...}^{2}+abcdm\overline{Y}_{.....}^{2} \quad ...(38)$$

$$SSAC=bdm\sum_{i=1}^{a}\sum_{k=1}^{c}\overline{Y}_{i.k..}^{2}-bcdm\sum_{i=1}^{a}\overline{Y}_{i....}^{2}-abdm\sum_{k=1}^{c}\overline{Y}_{..k..}^{2}+abcdm\overline{Y}_{.....}^{2} \quad ...(39)$$

$$SSAD=bcm\sum_{i=1}^{a}\sum_{l=1}^{d}\overline{Y}_{i..l.}^{2}-bcdm\sum_{i=1}^{a}\overline{Y}_{i....}^{2}-abcm\sum_{l=1}^{d}\overline{Y}_{...l.}^{2}+abcdm\overline{Y}_{.....}^{2} \quad ...(40)$$

$$SSBC=adm\sum_{j=1}^{b}\sum_{k=1}^{c}\overline{Y}_{.jk..}^{2}-acdm\sum_{j=1}^{b}\overline{Y}_{.j...}^{2}-abdm\sum_{k=1}^{c}\overline{Y}_{..k..}^{2}+abcdm\overline{Y}_{.....}^{2} \quad ...(41)$$

$$SSBD=acm\sum_{j=1}^{b}\sum_{l=1}^{d}\overline{Y}_{.j.l.}^{2}-acdm\sum_{j=1}^{b}\overline{Y}_{.j...}^{2}abcm\sum_{l=1}^{d}\overline{Y}_{...l.}^{2}+abcdm\overline{Y}_{.....}^{2} \quad ...(42)$$

$$SSCD=abm\sum_{k=1}^{c}\sum_{l=1}^{d}\overline{Y}_{..kl.}^{2}-abdm\sum_{k=1}^{c}\overline{Y}_{..k..}^{2}-abcm\sum_{l=1}^{d}\overline{Y}_{...l.}^{2}+abcdm\overline{Y}_{.....}^{2} \quad ...(43)$$

$$SSABC=dm\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c}\overline{Y}_{ijk..}^{2}-cdm\sum_{i=1}^{a}\sum_{j=1}^{b}\overline{Y}_{ij...}^{2}-bdm\sum_{i=1}^{a}\sum_{k=1}^{c}\overline{Y}_{i.k..}^{2}-adm\sum_{j=1}^{b}\sum_{k=1}^{c}\overline{Y}_{.jk..}^{2}+bcdm\sum_{i=1}^{a}\overline{Y}_{i....}^{2}+$$
$$acdm\sum_{j=1}^{b}\overline{Y}_{.j...}^{2}+abdm\sum_{k=1}^{c}\overline{Y}_{..k..}^{2}-abcdm\overline{Y}_{.....}^{2} \quad ...(44)$$

$$SSABD=cm\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{l=1}^{d}\overline{Y}_{ij.l.}^{2}-cdm\sum_{i=1}^{a}\sum_{j=1}^{b}\overline{Y}_{ij...}^{2}-bcm\sum_{i=1}^{a}\sum_{l=1}^{d}\overline{Y}_{i..l.}^{2}-acm\sum_{j=1}^{b}\sum_{l=1}^{d}\overline{Y}_{.j.l.}^{2}+bcdm\sum_{i=1}^{a}\overline{Y}_{i....}^{2}+$$
$$acdm\sum_{j=1}^{b}\overline{Y}_{.j...}^{2}+abcm\sum_{l=1}^{d}\overline{Y}_{...l.}^{2}-abcdm\overline{Y}_{.....}^{2} \quad ...(45)$$

$$SSACD=bm\sum_{i=1}^{a}\sum_{k=1}^{c}\sum_{l=1}^{d}\overline{Y}_{i.kl.}^{2}-bdm\sum_{i=1}^{a}\sum_{k=1}^{c}\overline{Y}_{i.k..}^{2}-bcm\sum_{i=1}^{a}\sum_{l=1}^{d}\overline{Y}_{i..l.}^{2}-abm\sum_{k=1}^{c}\sum_{l=1}^{d}\overline{Y}_{..kl.}^{2}+bcdm\sum_{i=1}^{a}\overline{Y}_{i....}^{2}+$$
$$abdm\sum_{k=1}^{c}\overline{Y}_{..k..}^{2}+abcm\sum_{l=1}^{d}\overline{Y}_{...l.}^{2}-abcdm\overline{Y}_{.....}^{2} \quad ...(46)$$

$$SSBCD=am\sum_{j=1}^{b}\sum_{k=1}^{c}\sum_{l=1}^{d}\overline{Y}_{.jkl.}^{2}-adm\sum_{j=1}^{b}\sum_{k=1}^{c}\overline{Y}_{.jk..}^{2}-acm\sum_{j=1}^{b}\sum_{l=1}^{d}\overline{Y}_{.j.l.}^{2}-abm\sum_{k=1}^{c}\sum_{l=1}^{d}\overline{Y}_{..kl.}^{2}+acdm\sum_{j=1}^{b}\overline{Y}_{.j...}^{2}+$$
$$abdm\sum_{k=1}^{c}\overline{Y}_{..k..}^{2}+abcm\sum_{l=1}^{d}\overline{Y}_{...l.}^{2}-abcdm\overline{Y}_{.....}^{2} \quad ...(47)$$

$$SSABCD = m\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c}\sum_{l=1}^{d}\bar{Y}_{ijkl.}^{2} - dm\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c}\bar{Y}_{ijk..}^{2} - cm\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{l=1}^{d}\bar{Y}_{ij.l.}^{2} - bm\sum_{i=1}^{a}\sum_{k=1}^{c}\sum_{l=1}^{d}\bar{Y}_{i.kl.}^{2} - am\sum_{j=1}^{b}\sum_{k=1}^{c}\sum_{l=1}^{d}\bar{Y}_{.jkl.}^{2}$$
$$+ cdm\sum_{i=1}^{a}\sum_{j=1}^{b}\bar{Y}_{ij...}^{2} + bdm\sum_{i=1}^{a}\sum_{k=1}^{c}\bar{Y}_{i.k..}^{2} + bcm\sum_{i=1}^{a}\sum_{l=1}^{d}\bar{Y}_{i..l.}^{2} + adm\sum_{j=1}^{b}\sum_{k=1}^{c}\bar{Y}_{.jk..}^{2} + acm\sum_{j=1}^{b}\sum_{l=1}^{d}\bar{Y}_{.j.l.}^{2} +$$
$$abm\sum_{k=1}^{c}\sum_{l=1}^{d}\bar{Y}_{..kl.}^{2} - bcdm\sum_{i=1}^{a}\bar{Y}_{i....}^{2} - acdm\sum_{j=1}^{b}\bar{Y}_{.j...}^{2} - abdm\sum_{k=1}^{c}\bar{Y}_{..k..}^{2} - abcm\sum_{l=1}^{d}\bar{Y}_{...l.}^{2} + abcdm\bar{Y}_{.....}^{2} \quad (48)$$

$$SSE = \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c}\sum_{l=1}^{d}\sum_{h=1}^{m}Y_{ijklh}^{2} - m\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c}\sum_{l=1}^{d}\bar{Y}_{ijkl.}^{2} \quad \cdots(49)$$

The degrees of freedom for SSA,SSB,SSC,SSD,SSAB,SSAC,SSAD,SSBC, SSBD,SSCD,SSABC,SSABD,SSACD,SSBCD, SSABCD,SSE and SSTO are (a-1) , (b-1) ,(c-1) ,(d-1) , (a-1) (b-1) , (a-1) (c-1),(a-1)(d-1),(b-1)(c-1) ,(b-1) (d-1) , (c-1)(d-1),(a-1)(b-1)(c-1),(a-1)(b-1)(d-1),(a-1) (c-1)(d-1),(b-1)(c-1)(d-1),(a-1)(b-1)(c-1) (d-1), abcd(m-1) and (abcdm-1) respectively

EXPECTED MEAN SQUARES E(MS)

Now we calculate the expected mean squares with correlated data for MSE , MSA , MSB , MSC , MSD , MSAB , MSAC , MSAD , MSBC , MSBD , MSCD , MSABC , MSABD , MSACD , MSBCD and MSABCD

$$MSE = \frac{SSE}{abcd(m-1)} \quad ...(50)$$

$$= \frac{1}{abcd(m-1)}\left(\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c}\sum_{l=1}^{d}\sum_{h=1}^{m}Y_{ijklh}^{2} - m\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c}\sum_{l=1}^{d}\bar{Y}_{ijkl.}^{2}\right) \quad ...(51)$$

Since

$$\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c}\sum_{l=1}^{d}\sum_{h=1}^{m}Y_{ijklh}^{2} = Y'Y = Y'(I_m \otimes I_d \otimes I_c \otimes I_b \otimes I_a)Y \quad ...(52)$$

and

$$\bar{Y}_{ijkl.} = \frac{1}{m}1_m' Y_{ijkl} = \frac{1}{m}Y_{ijkl}' 1_m \quad ...(53)$$

$$\bar{Y}_{ijkl.}^{2} = \frac{1}{m^2}Y_{ijkl}' 1_m 1_m' Y_{ijkl}$$

$$\bar{Y}_{ijkl.}^{2} = \frac{1}{m^2}Y_{ijkl}' J_m Y_{ijkl} = \frac{1}{m}Y_{ijkl}' M_m Y_{ijkl} \quad ...(54)$$

$$\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c}\sum_{l=1}^{d}\bar{Y}_{ijkl.}^{2}=\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c}\sum_{l=1}^{d}\frac{1}{m}Y'_{ijkl}M_{m}Y_{ijkl}=\frac{1}{m}\left[Y'_{1111}M_{m}Y_{1111}+...+Y'_{abcd}M_{m}Y_{abcd}\right]$$

$$=\frac{1}{m}(Y'_{1111},...,Y'_{abcd})\begin{pmatrix}M_m & O & \cdots O\\ \vdots & & \\ O & O & \cdots M_m\end{pmatrix}_{abcd\times abcd}\begin{pmatrix}Y_{1111}\\ Y_{1112}\\ \vdots\\ Y_{abcd}\end{pmatrix}=\frac{1}{m}Y'(M_m\otimes I_{abcd})Y$$

$$\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c}\sum_{l=1}^{d}\bar{Y}_{ijkl.}^{2}=\frac{1}{m}Y'(M_m\otimes I_d\otimes I_c\otimes I_b\otimes I_a)Y \qquad ...(55)$$

So

$$SSE=Y'I_{abcdm}Y-m(\frac{1}{m})Y'(M_m\otimes I_{abcd})Y$$

$$SSE=Y'(I_m-M_m\otimes I_{abcd})Y$$

$$SSE=Y'(N_m\otimes I_{abcd})Y \qquad \cdots(56)$$

by using converter model and using the relations (6),(17),(15)

$$Y^{*}\approx N(R^{-\frac{1}{2}}\mu,\sigma^{2}I) \qquad \cdots(57)$$

by using the relation (56) but for converter model

$$SSE^{*}=Y^{*'}(N_m\otimes I_d\otimes I_c\otimes I_b\otimes I_a)Y^{*} \qquad \cdots(58)$$

by substituting the relation (15) in the relation (58) we get on

$$SSE^{\bullet}=Y'R^{-\frac{1}{2}}(N_m\otimes I_d\otimes I_c\otimes I_b\otimes I_a)R^{-\frac{1}{2}}Y \qquad \cdots(59)$$

by the substituting the relation (11) in the relation (59) and by using the relation (8)

$$SSE^{*}=\frac{1}{\lambda_{17}}SSE \qquad \cdots(60)$$

$$SSE=\lambda_{17}SSE^{*} \qquad \cdots(61)$$

By using relations (57), (58) , and by using theorem 1 and corollary in the appendix and since $(N_m\otimes I_{abcd})$ is idempotent matrix so

$$SSE^{*}\approx\sigma^{2}X^{2}_{abcd(m-1)}(\eta_1) \qquad \cdots(62)$$

$$\eta_1=\frac{\mu^{*'}(N_m\otimes I_{abcd})\mu^{*}}{\sigma^{2}}=(\frac{1}{\sigma^{2}})\left\|(N_m\otimes I_{abcd})\mu^{*}\right\|^{2} \qquad \cdots(63)$$

η_1 represents the non central parameter for chi-square distribution

$$M_s1_s=1_s\,,N_s1_s=\underline{0},1_s'N_s=\underline{0} \qquad \cdots(64)$$

see pavure (1988)

by using relations (5),(8),(64),(11),(15),(63),and since $1_a\perp\alpha,1_b\perp\beta,1_c\perp\gamma,1_d\perp\delta$

$$\eta_1 = (\frac{1}{\sigma^2})\mu^{*\prime}(N_m \otimes I_{abcd})\mu^*$$

$$= (\frac{1}{\sigma^2})\mu' R^{-\frac{1}{2}}(N_m \otimes I_{abcd})R^{-\frac{1}{2}}\mu$$

$$= (\frac{1}{\sigma^2\lambda_{17}})(\underline{0}) = \underline{0}$$

$$SSE^* \approx \sigma^2 X^2_{abcd(m-1)}(\underline{0}) \qquad \cdots(65)$$

$$E(SSE^*) = abcd(m-1)\sigma^2 \qquad \ldots(66)$$

by using the relation (65) and (66)

$$E(SSE) = abcd(m-1)\lambda_{17}\sigma^2 \qquad \cdots(67)$$

$$E(MSE) = E\left(\frac{SSE}{abcd(m-1)}\right) = \frac{E(SSE)}{abcd(m-1)} = \frac{abcd(m-1)\lambda_{17}\sigma^2}{abcd(m-1)} = \sigma^2\lambda_{17} = \sigma^2(1-\rho_1) \qquad \cdots(68)$$

Also

$$MSA = \frac{SSA}{(a-1)} \qquad \cdots(69)$$

$$= \frac{1}{(a-1)}\left[bcdm\sum_{i=1}^{a}\overline{Y}_{i....}^2 - abcdm\overline{Y}_{.....}^2\right]$$

Since

$$\overline{Y}_{i....} = (\frac{1}{bcdm})1'_{bcdm}Y_i = \frac{1}{bcdm}Y_i'1_{bcdm}$$

$$\overline{Y}_{i....}^2 = \frac{1}{(bcdm)^2}Y_i'1_{bcdm}1'_{bcdm}Y_i$$

$$= \frac{1}{(bcdm)^2}Y_i'J_{bcdm}Y_i$$

$$= (\frac{1}{bcdm})Y_i'\frac{1}{bcdm}J_{bcdm}Y_i$$

$$= (\frac{1}{bcdm})Y_i'M_{bcdm}Y_i \qquad \cdots(70)$$

Now

$$\sum_{i=1}^{a}\overline{Y}_{i....}^2 = \sum_{i=1}^{a}(\frac{1}{bcdm})Y_i'M_{bcdm}Y_i = (\frac{1}{bcdm})(Y_1'M_{bcdm}Y_1 + \cdots + Y_a'M_{bcdm}Y_a)$$

$$= (\frac{1}{bcdm})(Y_1', \cdots, Y_a')\begin{pmatrix} M_{bcdm} & O & \cdots O \\ \vdots & & \\ O & O & \cdots M_{bcdm} \end{pmatrix}\begin{pmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_a \end{pmatrix} = \frac{1}{bcdm}Y'(M_{bcdm} \otimes I_a)Y \qquad \cdots(71)$$

and also

$$\bar{Y}_{.....} = \frac{1}{abcdm} 1'_{abcdm} Y = \frac{1}{abcdm} Y' 1_{abcdm} \quad \cdots(72)$$

$$\bar{Y}^2_{.....} = \frac{1}{(abcdm)^2} Y' 1_{abcdm} 1'_{abcdm} Y = \frac{1}{(abcdm)^2} Y' J_{abcdm} Y = \frac{1}{abcdm} Y' \frac{1}{abcdm} J_{abcdm} Y$$

$$= (\frac{1}{abcdm}) Y' M_{abcdm} Y \quad \cdots(73)$$

by substituting relations (71),(73)in the relation (34) and by using the relation (8) then

$$SSA = Y'(M_m \otimes M_d \otimes M_c \otimes M_b \otimes N_a) Y \quad \cdots(74)$$

by using the relation (74) but for converter model and using the relations (17),(57),

$$SSA^* = Y^{*'}(M_m \otimes M_d \otimes M_c \otimes M_b \otimes N_a) Y^* = \left\| (M_m \otimes M_d \otimes M_c \otimes M_b \otimes N_a) Y^* \right\|^2 \quad \cdots(75)$$

by substituting relation (15) in the relation (75) and by using the relations (11), (8), (64) then

$$SSA^* = Y' R^{-\frac{1}{2}} (M_m \otimes M_d \otimes M_c \otimes M_b \otimes N_a) R^{-\frac{1}{2}} Y = (\frac{1}{\lambda_2}) Y'(M_m \otimes M_d \otimes M_c \otimes M_b \otimes N_a) Y$$

$$= (\frac{1}{\lambda_2}) SSA \quad \cdots(76)$$

$$SSA = \lambda_2 SSA^* \quad \cdots(77)$$

by using the relations (57) , (75) , theorem 1, corollary in the appendix and since the matrix $(M_{bcdm} \otimes N_a)$ is idempotent matrix

$$SSA^* \approx \sigma^2 X^2_{(a-1)}(\eta_2) \quad \cdots(78)$$

$$\eta_2 = \frac{\mu^{*'}(M_m \otimes M_d \otimes M_c \otimes M_b \otimes N_a)\mu^*}{\sigma^2} = (\frac{1}{\sigma^2}) \left\| (M_m \otimes M_d \otimes M_c \otimes M_b \otimes N_a)\mu^* \right\|^2 \quad \cdots(79)$$

η_2 represent non central parameter for chi-square distribution . by using the relations (5), (8), (64), (11), (15), (79)

$$\eta_2 = (\frac{1}{\sigma^2}) \mu^{*'} (M_m \otimes M_d \otimes M_c \otimes M_b \otimes N_a) \mu^* = (\frac{1}{\sigma^2}) \mu' R^{-\frac{1}{2}} (M_m \otimes M_d \otimes M_c \otimes M_b \otimes N_a) R^{-\frac{1}{2}} \mu$$

$$= (\frac{bcdm}{\sigma^2 \lambda_2}) \alpha' \alpha = (\frac{1}{\sigma^2 \lambda_2}) bcdm \sum_{i=1}^{a} \alpha_i^2 \quad \cdots(80)$$

So

$$SSA^* \approx \sigma^2 X^2_{(a-1)}((\frac{bcdm}{\sigma^2 \lambda_2}) \sum_{i=1}^{a} \alpha_i) \quad \cdots(81)$$

buy using Lemma 1 and Lemma 2 in the appendix

$$E(SSA^*) = (\frac{bcdm}{\lambda_2}) \sum_{i=1}^{a} \alpha_i^2 + \sigma^2 (a-1) \quad \cdots(82)$$

by using relations (77) and (82)

$$E(SSA^*) = bcdm \sum_{i=1}^{a} \alpha_i^2 + \sigma^2 \lambda_2 (a-1) \quad \cdots(83)$$

$$E(MSA) = E(\frac{SSA}{a-1}) = \frac{E(SSA)}{a-1} = \left(\frac{bcdm}{a-1}\right) \sum_{i=1}^{a} \alpha_i^2 + \sigma^2 \lambda_2 \quad \cdots(84)$$

Similarly we obtain that

$$E(MSB)=\left(\frac{acdm}{b-1}\right)\sum_{j=1}^{b}\beta_j^2+\sigma^2\lambda_3 \quad \cdots(85)$$

$$E(MSC)=\left(\frac{abdm}{c-1}\right)\sum_{k=1}^{c}\gamma_k^2+\sigma^2\lambda_4 \quad \cdots(86)$$

$$E(MSD)=\left(\frac{abcm}{d-1}\right)\sum_{l=1}^{d}\delta_l^2+\sigma^2\lambda_5 \quad \cdots(87)$$

$$E(MSAB)=\left(\frac{cdm}{(a-1)(b-1)}\right)\sum_{i=1}^{a}\sum_{j=1}^{b}(\alpha\beta)_{ij}^2+\sigma^2\lambda_6 \quad \cdots(88)$$

$$E(MSAC)=\left(\frac{bdm}{(a-1)(c-1)}\right)\sum_{i=1}^{a}\sum_{k=1}^{c}(\alpha\gamma)_{ik}^2+\sigma^2\lambda_7 \quad \cdots(89)$$

$$E(MSAD)=\left(\frac{bcm}{(a-1)(d-1)}\right)\sum_{i=1}^{a}\sum_{l=1}^{d}(\alpha\delta)_{il}^2+\sigma^2\lambda_8 \quad \cdots(90)$$

$$E(MSBC)=\left(\frac{adm}{(b-1)(c-1)}\right)\sum_{j=1}^{b}\sum_{k=1}^{c}(\beta\gamma)_{jk}^2+\sigma^2\lambda_9 \quad \cdots(91)$$

$$E(MSBD)=\left(\frac{acm}{(b-1)(d-1)}\right)\sum_{j=1}^{b}\sum_{l=1}^{d}(\beta\delta)_{jl}^2+\sigma^2\lambda_{10} \quad \cdots(92)$$

$$E(MSCD)=\left(\frac{abm}{(c-1)(d-1)}\right)\sum_{k=1}^{c}\sum_{l=1}^{d}(\gamma\delta)_{kl}^2+\sigma^2\lambda_{11} \quad \cdots(93)$$

$$E(MSABC)=\left(\frac{dm}{(a-1)(b-1)(c-1)}\right)\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c}(\alpha\beta\gamma)_{ijk}^2+\sigma^2\lambda_{12} \quad \cdots(94)$$

$$E(MSABD)=\left(\frac{cm}{(a-1)(b-1)(c-1)}\right)\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{l=1}^{d}(\alpha\beta\delta)_{ijl}^2+\sigma^2\lambda_{13} \quad \cdots(95)$$

$$E(MSACD)=\left(\frac{bm}{(a-1)(c-1)(d-1)}\right)\sum_{i=1}^{a}\sum_{k=1}^{c}\sum_{l=1}^{d}(\alpha\gamma\delta)_{ikl}^2+\sigma^2\lambda_{14} \quad \cdots(96)$$

$$E(MSBCD)=\left(\frac{am}{(b-1)(c-1)(d-1)}\right)\sum_{j=1}^{b}\sum_{k=1}^{c}\sum_{l=1}^{d}(\beta\gamma\delta)_{jkl}^2+\sigma^2\lambda_{15} \quad \cdots(97)$$

$$E(MSABCD)=\left(\frac{n}{(a-1)(b-1)(c-1)(d-1)}\right)\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c}\sum_{l=1}^{d}(\alpha\beta\gamma\delta)_{ijkl}+\sigma^2\lambda_{16} \quad \cdots(98)$$

Therefore the analysis of variance table (ANOVA) can be written as table (1)

F-TEST

After finding (ANOVA) for studying models we can discuss Fifteen cases for null hypotheses to know that if the factor levels mean is equal and these hypotheses are

$$H_0:\alpha_i=0 \quad \forall\, i=1,\cdots,a$$
$$H_1:\alpha_i\neq 0 \ \ for\ some\ i \quad \cdots(99)$$

$$H_0:\beta_j=0 \quad \forall\, j=1,\cdots,b$$
$$H_1:\beta_j\neq 0 \ \ for\ some\ j \quad \cdots(100)$$

$H_0 : \gamma_k = 0 \quad \forall\, k = 1, \cdots, c$

$H_1 : \gamma_k \neq 0 \;\; for\, some\, k$ $\cdots(101)$

$H_0 : \delta_l = 0 \quad \forall\, l = 1, \cdots, d$

$H_1 : \delta_l \neq 0 \;\; for\, some\, l$ $\cdots(102)$

$H_0 : (\alpha\beta)_{ij} = 0 \quad \forall\, i, j \ni i = 1, \cdots, a \;, j = 1, \cdots, b$

$H_1 : (\alpha\beta)_{ij} \neq 0 \;\; for\, some\, i \;\; or \;\; j$ $\cdots(103)$

$H_0 : (\alpha\gamma)_{ik} = 0 \quad \forall\, i, k \ni i = 1, \cdots, a \,, k = 1, \cdots, c$

$H_1 : (\alpha\gamma)_{ik} \neq 0 \;\; for\, some\, i \;\; or \;\; k$ $\cdots(104)$

$H_0 : (\alpha\delta)_{il} = 0 \quad \forall\, i, l \ni i = 1, \cdots, a \;,\; l = 1, \cdots, d$

$H_1 : (\alpha\delta)_{il} \neq 0 \;\; for\, some\, i \;\; or \;\; l$ $\cdots(105)$

$H_0 : (\beta\gamma)_{jk} = 0 \quad \forall\, j, k \ni j = 1, \cdots, b \;,\; k = 1, \cdots, c$

$H_1 : (\beta\gamma)_{jk} \neq 0 \;\; for\, some\, j \;\; or \;\; k$ $\cdots(106)$

$H_0 : (\beta\delta)_{jl} = 0 \quad \forall\, j, l \ni j = 1, \cdots, b \;,\; l = 1, \cdots, d$

$H_1 : (\beta\delta)_{jl} \neq 0 \;\; for\, some\, j \;\; or \;\; l$ $\cdots(107)$

$H_0 : (\gamma\delta)_{kl} = 0 \quad \forall\, k, l \ni k = 1, \cdots, c \;,\; l = 1, \cdots, d$

$H_1 : (\gamma\delta)_{kl} \neq 0 \;\; for\, some\, k \;\; or \;\; l$ $\cdots(108)$

$H_0 : (\alpha\beta\gamma)_{ijk} = 0 \quad \forall\, i, j, k \ni i = 1, \cdots, a \;,\; j = 1, \cdots, b \;,\; k = 1, \cdots, c$

$H_1 : (\alpha\beta\gamma)_{ijk} \neq 0 \;\; for\, some\, i \;\; or \;\; j \;\; or \;\; k$ $\cdots(109)$

$H_0 : (\alpha\beta\delta)_{ijl} = 0 \quad \forall\, i, j, l \ni i = 1, \cdots, a \;, j = 1, \cdots, b \;, l = 1, \cdots, d$

$H_1 ; (\alpha\beta\delta)_{ijl} \neq 0 \quad for \;\; some \;\; i \;\; or \;\; j \;\; or \;\; l$ $\cdots(110)$

$H_0 : (\alpha\gamma\delta)_{ikl} = 0 \quad \forall\, i, k, l \ni i = 1, \cdots, a \;,\; k = 1, \cdots, c \;,\; l = 1, \cdots, d$

$H_1 : (\alpha\gamma\delta)_{ikl} \neq 0 \;\; for\, some\, i \;\; or \;\; k \;\; or \;\; l$ $\cdots(111)$

$H_0 : (\beta\gamma\delta)_{jkl} = 0 \quad \forall\, j, k, l \ni j = 1, \cdots, b \;,\; k = 1, \cdots, c \;,\; l = 1, \cdots, d$

$H_1 : (\beta\gamma\delta)_{jkl} \neq 0 \;\; for\, some\, j \;\; or \;\; k \;\; or \;\; l$ $\cdots(112)$

$H_0 : (\alpha\beta\gamma\delta)_{ijkl} = 0 \quad \forall\, i, j, k, l \ni i = 1, \cdots, a \;,\; j = 1, \cdots, b \;,\; k = 1, \cdots, c \;,\; l = 1, \cdots, d$

$H_1 : (\alpha\beta\gamma\delta)_{ijkl} \neq 0 \;\; for\, some\, i \;\; or \;\; j \;\; or \;\; k \;\; or \;\; l$ $\cdots(113)$

from the following relations

$$SSE^* \approx \sigma^2 X^2_{abcd(m-1)}(\underline{0})$$

$$SSA^* \approx \sigma^2 X^2_{(a-1)}((\frac{bcdm}{\lambda_2\sigma^2})\sum_{i=1}^{a}\alpha_i^2)$$

$$SSB^* \approx \sigma^2 X^2_{(b-1)}((\frac{acdm}{\lambda_3\sigma^2})\sum_{i=1}^{b}\beta_j^2)$$

$$SSC^* \approx \sigma^2 X^2_{(c-1)}((\frac{abdm}{\lambda_4\sigma^2})\sum_{k=1}^{c}\gamma_k^2)$$

$$SSD^* \approx \sigma^2 X^2_{(d-1)}((\frac{abcm}{\lambda_5\sigma^2})\sum_{l=1}^{d}\delta_l^2)$$

$$SS(AB)^* \approx \sigma^2 X^2_{(a-1)(b-1)}((\frac{cdm}{\lambda_6\sigma^2})\sum_{i=1}^{a}\sum_{j=1}^{b}(\alpha\beta)_{ij}^2)$$

$$SS(AC)^* \approx \sigma^2 X^2_{(a-1)(c-1)}((\frac{bdm}{\sigma^2\lambda_7})\sum_{i=1}^{a}\sum_{k=1}^{c}(\alpha\gamma)^2_{ik})$$

$$SS(AD)^* \approx \sigma^2 X^2_{(a-1)(d-1)}((\frac{bcm}{\lambda_8\sigma^2})\sum_{i=1}^{a}\sum_{l=1}^{d}(\alpha\delta)^2_{il})$$

$$SS(BC)^* \approx \sigma^2 X^2_{(b-1)(c-1)}((\frac{adm}{\lambda_9\sigma^2})\sum_{j=1}^{b}\sum_{k=1}^{c}(\beta\gamma)^2_{jk})$$

$$SS(BD)^* \approx \sigma^2 X^2_{(b-1)(d-1)}((\frac{acm}{\lambda_{10}\sigma^2})\sum_{j=1}^{b}\sum_{l=1}^{d}(\beta\delta)^2_{jl})$$

$$SS(CD)^* \approx \sigma^2 X^2_{(c-1)(d-1)}((\frac{abm}{\lambda_{11}\sigma^2})\sum_{k=1}^{c}\sum_{l=1}^{d}(\gamma\delta)^2_{kl})$$

$$SS(ABC)^* \approx \sigma^2 X^2_{(a-1)(b-1)(c-1)}((\frac{dm}{\lambda_{12}\sigma^2})\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c}(\alpha\beta\gamma)^2_{ijk})$$

$$SS(ABD)^* \approx \sigma^2 X^2_{(a-1)(b-1)(d-1)}((\frac{cm}{\lambda_{13}\sigma^2})\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{l=1}^{d}(\alpha\beta\delta)^2_{ijl})$$

$$SS(ACD)^* \approx \sigma^2 X^2_{(a-1)(c-1)(d-1)}((\frac{bm}{\lambda_{14}\sigma^2})\sum_{i=1}^{a}\sum_{k=1}^{c}\sum_{l=1}^{d}(\alpha\gamma\delta)^2_{ikl})$$

$$SS(BCD)^* \approx \sigma^2 X^2_{(b-1)(c-1)(d-1)}((\frac{am}{\lambda_{15}\sigma^2})\sum_{j=1}^{b}\sum_{k=1}^{c}\sum_{l=1}^{d}(\beta\gamma\delta)^2_{jkl})$$

$$SS(ABCD)^* \approx \sigma^2 X^2_{(a-1)(b-1)(c-1)(d-1)}((\frac{m}{\lambda_{16}\sigma^2})\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c}\sum_{l=1}^{d}(\alpha\beta\gamma\delta)^2_{ijkl})$$

Also from the relations

$$SSE^* = Y^{*'}(N_m \otimes I_{abcd})Y^*$$

$$SSA^* = Y^{*'}(M_{bcdm} \otimes N_a)Y^*$$

$$SSB^* = Y^{*'}(M_{cdm} \otimes N_b \otimes M_a)Y^*$$

$$SSC^* = Y^{*'}(M_{dm} \otimes N_c \otimes M_{ab})Y^*$$

$$SSD^* = Y^{*'}(M_m \otimes N_d \otimes M_{abc})Y^*$$

$$SS(AB)^* = Y^{*'}(M_{cdm} \otimes N_{ab})Y^*$$

$$SS(AC)^* = Y^{*'}(M_{dm} \otimes N_c \otimes M_b \otimes N_a)Y^*$$

$$SS(AD)^* = Y^{*'}(M_m \otimes N_d \otimes M_{bc} \otimes N_a)Y^*$$

$$SS(BC)^* = Y^{*'}(M_{dm} \otimes N_c \otimes N_b \otimes M_a)Y^*$$

$$SS(BD)^* = Y^{*'}(M_m \otimes N_d \otimes M_c \otimes N_b \otimes M_a)Y^*$$

$$SS(CD)^* = Y^{*'}(M_m \otimes N_{cd} \otimes M_{ab})Y^*$$

$$SS(ABC)^* = Y^{*'}(M_{dm} \otimes N_{abc})Y^*$$

$$SS(ABD)^* = Y^{*'}(M_m \otimes N_d \otimes M_c \otimes N_{ab})Y^*$$
$$SS(ACD)^* = Y^{*'}(M_m \otimes N_{cd} \otimes M_b \otimes N_a)Y^*$$
$$SS(BCD)^* = Y^{*'}(M_m \otimes N_{bcd} \otimes M_a)Y^*$$
$$SS(ABCD)^* = Y^{*'}(M_m \otimes N_{abcd})Y^*$$

and Since the matrices

$M_{abcdm}, M_{bcdm} \otimes N_a, M_{cdm} \otimes N_b \otimes M_a, M_{md} \otimes N_c \otimes M_{ab}, M_m \otimes N_d \otimes M_{abc}, M_{cdm} \otimes N_{ab},$
$M_{dm} \otimes N_c \otimes M_b \otimes N_a, M_m \otimes N_d \otimes M_{bc} \otimes N_a, M_{dm} \otimes N_{bc} \otimes M_a, M_m \otimes N_d \otimes M_c \otimes N_b \otimes M_a,$
$M_m \otimes N_{cd} \otimes M_{ab}, M_{dm} \otimes N_{abc}, M_m \otimes N_d \otimes M_c \otimes N_{ab}, M_m \otimes N_{cd} \otimes M_b \otimes N_a, M_m \otimes N_{bcd} \otimes M_a$
$M_m \otimes N_{abcd}, N_m \otimes I_{abcd}$

are idempotent and the product of any two is zero matrix , by using point 3 from theorem (2), then
$SSE^*, SSA^*, SSB^*, SSC^*, SSD^*, SS(AB)^*, SS(AC)^*, SS(AD)^*, SS(BC)^*, SS(BD)^*, SS(CD)^*,$ in
$SS(ABC)^*, SS(ABD)^*, SS(ACD)^*, SS(BCD)^*, SS(ABCD)^*$
dependent therefore we can write the F^* distribution for test to equal factor levels mean as

$$F_1^* = \frac{SSA^*/\sigma^2(a-1)}{SSE^*/\sigma^2 abcd(m-1)} = (\frac{\lambda_{17}}{\lambda_2})\frac{SSA/\sigma^2(a-1)}{SSE/\sigma^2 abcd(m-1)} = C_1F_1 \approx F((a-1), abcd(m-1)) \text{ , where } C_1 = \frac{\lambda_{17}}{\lambda_2}$$

$$F_2^* = \frac{SSB^*/\sigma^2(b-1)}{SSE^*/\sigma^2 abcd(m-1)} = (\frac{\lambda_{17}}{\lambda_3})\frac{SSB/\sigma^2(b-1)}{SSE/\sigma^2 abcd(m-1)} = C_2F_2 \approx F((b-1), abcd(m-1)) \text{ , where } C_2 = \frac{\lambda_{17}}{\lambda_3}$$

$$F_3^* = \frac{SSC^*/\sigma^2(c-1)}{SSE^*/\sigma^2 abcd(m-1)} = (\frac{\lambda_{17}}{\lambda_4})\frac{SSC/\sigma^2(c-1)}{SSE/\sigma^2 abcd(m-1)} = C_3F_3 \approx F((c-1), abcd(m-1)) \text{ , where } C_3 = \frac{\lambda_{17}}{\lambda_4}$$

$$F_4^* = \frac{SSD^*/\sigma^2(d-1)}{SSE^*/\sigma^2 abcd(m-1)} = (\frac{\lambda_{17}}{\lambda_5})\frac{SSD/\sigma^2(d-1)}{SSE/\sigma^2 abcd(m-1)} = C_4F_4 \approx F((d-1), abcd(m-1)) \text{ , where } C_4 = \frac{\lambda_{17}}{\lambda_5}$$

$$F_5^* = \frac{SS(AB)^*/\sigma^2(a-1)(b-1)}{SSE^*/\sigma^2 abcd(m-1)} = (\frac{\lambda_{17}}{\lambda_6})\frac{SSAB/\sigma^2(a-1)(b-1)}{SSE/\sigma^2 abcd(m-1)} = C_5F_5 \approx F((a-1)(b-1), abcd(m-1)) \text{ , where } C_5 = \frac{\lambda_{17}}{\lambda_6}$$

$$F_6^* = \frac{SS(AC)^*/\sigma^2(a-1)(c-1)}{SSE^*/\sigma^2 abcd(m-1)} = (\frac{\lambda_{17}}{\lambda_7})\frac{SSAC/\sigma^2(a-1)(c-1)}{SSE/\sigma^2 abcd(m-1)} = C_6F_6 \approx F((a-1)(c-1), abcd(m-1)) \text{ , where } C_6 = \frac{\lambda_{17}}{\lambda_7}$$

$$F_7^* = \frac{SS(AD)^*/\sigma^2(a-1)(d-1)}{SSE^*/\sigma^2 abcd(m-1)} = (\frac{\lambda_{17}}{\lambda_8})\frac{SSAD/\sigma^2(a-1)(d-1)}{SSE/\sigma^2 abcd(m-1)} = C_7F_7 \approx F((a-1)(d-1), abcd(m-1)), \text{where } C_7 = \frac{\lambda_{17}}{\lambda_8}$$

$$F_8^* = \frac{SS(BC)^*/\sigma^2(b-1)(c-1)}{SSE^*/\sigma^2 abcd(m-1)} = (\frac{\lambda_{17}}{\lambda_9})\frac{SSBC/\sigma^2(b-1)(c-1)}{SSE/\sigma^2 abcd(m-1)} = C_8F_8 \approx F((b-1)(c-1), abcd(m-1)), \text{where } C_8 = \frac{\lambda_{17}}{\lambda_9}$$

$$F_9^* = \frac{SS(BD)^* / \sigma^2 (b-1)(d-1)}{SSE^* / \sigma^2 abcd(m-1)} = (\frac{\lambda_{17}}{\lambda_{10}}) \frac{SSBD / \sigma^2 (b-1)(d-1)}{SSE / \sigma^2 abcd(m-1)} = C_9 F_9 \approx F((b-1)(d-1), abcd(m-1)), where\ C_9 = \frac{\lambda_{17}}{\lambda_{10}}$$

$$F_{10}^* = \frac{SS(CD)^* / \sigma^2 (c-1)(d-1)}{SSE^* / \sigma^2 abcd(m-1)} = (\frac{\lambda_{17}}{\lambda_{11}}) \frac{SSCD / \sigma^2 (c-1)(d-1)}{SSE / \sigma^2 abcd(m-1)} = C_{10} F_{10} \approx F((c-1)(d-1), abcd(m-1)), where\ C_{10} = \frac{\lambda_{17}}{\lambda_{11}}$$

$$F_{11}^* = \frac{SS(ABC)^* / \sigma^2 (a-1)(b-1)(c-1)}{SSE^* / \sigma^2 abcd(m-1)} = (\frac{\lambda_{17}}{\lambda_{12}}) \frac{SSABC / \sigma^2 (a-1)(b-1)(c-1)}{SSE / \sigma^2 abcd(m-1)} = C_{11} F_{11} \approx F((a-1)(b-1)(c-1), abcd(m-1)), where\ C_{11} = \frac{\lambda_{17}}{\lambda_{12}}$$

$$F_{12}^* = \frac{SS(ABD)^* / \sigma^2 (a-1)(b-1)(d-1)}{SSE^* / \sigma^2 abcd(m-1)} = (\frac{\lambda_{17}}{\lambda_{13}}) \frac{SSABD / \sigma^2 (a-1)(b-1)(d-1)}{SSE / \sigma^2 abcd(m-1)} = C_{12} F_{12} \approx F((a-1)(b-1)(d-1), abcd(m-1)), where\ C_{12} = \frac{\lambda_{17}}{\lambda_{13}}$$

$$F_{13}^* = \frac{SS(ACD)^* / \sigma^2 (a-1)(c-1)(d-1)}{SSE^* / \sigma^2 abcd(m-1)} = (\frac{\lambda_{17}}{\lambda_{14}}) \frac{SSACD / \sigma^2 (a-1)(c-1)(d-1)}{SSE / \sigma^2 abcd(m-1)} = C_{13} F_{13} \approx F((a-1)(c-1)(d-1), abcd(m-1)), where\ C_{13} = \frac{\lambda_{17}}{\lambda_{14}}$$

$$F_{14}^* = \frac{SS(BCD)^* / \sigma^2 (b-1)(c-1)(d-1)}{SSE^* / \sigma^2 abcd(m-1)} = (\frac{\lambda_{17}}{\lambda_{15}}) \frac{SSBCD / \sigma^2 (b-1)(c-1)(d-1)}{SSE / \sigma^2 abcd(m-1)} = C_{14} F_{14} \approx F((b-1)(c-1)(d-1), abcd(m-1)), where\ C_{14} = \frac{\lambda_{17}}{\lambda_{15}}$$

$$F_{15}^* = \frac{SS(ABCD)^* / \sigma^2 (a-1)(b-1)(c-1)(d-1)}{SSE^* / \sigma^2 abcd(m-1)} = (\frac{\lambda_{17}}{\lambda_{16}}) \frac{SSABCD / \sigma^2 (a-1)(b-1)(c-1)(d-1)}{SSE / \sigma^2 abcd(m-1)} = C_{15} F_{15}$$

$$\approx F((a-1)(b-1)(c-1)(d-1), abcd(m-1)) \quad , where\ C_{15} = \frac{\lambda_{17}}{\lambda_{16}}$$

After finding F^* statistics we find table values from F-distribution table with significance level α and degree of freedom and compare between these values.

CORRECTING FOR CORRELATION

The corrector factors $C_1, C_2, \ldots, C_{15}$ may take one of the following cases

1- corrector factor=1
2- corrector factor>1
3- corrector factor<1

when the value of corrector factor is equal to one then there is not need to the correction F test . But when the corrector factor is not equal to one then we need correction.(see Al-Shahiry (1997)).
Tables (2),(3),(4),(5),(6),(7),(8),(9),(10),(11),(12),(13),(14),(15) shows the values of true α for a variety of values for significance level of α was calculated for some hypothetical values for $C_1, C_2, \ldots, C_{15}$.To calculate the true α in tables from 2 to 16 we used the formula

$$X = X_1 + (Y - Y_1)\left[\frac{(X_2 - X_1)}{(Y_2 - Y_1)}\right] \text{where}$$

X_1, X_2 represents significance levels which take from the tables in the statistical distribution F and be known $, Y_1, Y_2, Y$ represents the values corresponding to spreadsheet X_1, X_2 and

$X = True\ alpha$ respectively and be known also where Y represents the value of a statistical spreadsheet F at the level α multiplied by correction factor

CONCLUSIONS

1- This model is an extension of some models studied earlier by many authors. But , under certain condition we may get the same models discussed before by pavur and Davenport (1985) [6], Al-kaabawi (2000) [10] and Abdullah, Al-kaabawi (2007) [8].

2- Tables 2, …, 16, show that the true alpha level inflate (deflate) when the correction constant <(or >) 1 , and this leads to have a smaller (bigger) rejection region for the complete null hypothesis on testing factors.

3- This study may be extended to n-way model.

4- The model which is under studying can be extended to unbalanced model where i=1,…,a , j=1,…,b , k=1,…,c , l=1,…,d and h=1,2,…, m_{ijkl}

APPENDIX

Lemma 1: Let X be a random vector with mean vector μ variance matrix Σ.

a- let $Y = AX + b$. The $E(Y) = A\mu + b$ and $COV(Y) = A\Sigma A'$.

b- $E\|X\|^2 = \|\mu\|^2 + tr\Sigma$ (where $tr\Sigma$ is the trace of Σ).

Lemma 2: Let $Y \approx N_n(\mu, \sigma^2 I)$, where $Y' = (Y_1, \cdots, Y_n)$, $\mu' = (\mu_1, \cdots, \mu_2)$ and $\sigma^2 > 0$ is a scalar. Then the Y_i are independent , $Y_i \approx N_i(\mu_i, \sigma^2)$ and

$$\frac{\|Y\|}{\sigma^2} = \frac{Y'Y}{\sigma^2} \approx X_n^2(\frac{\mu'\mu}{\sigma^2})$$

Theorem1: Let $Y \approx N_n(\mu, \Sigma), \Sigma > 0$. Then

a- $Y'\Sigma^{-1}Y \approx X_n^2(\mu'\Sigma^{-1}\mu)$.

b- $(Y-\mu)'\Sigma^{-1}(Y-\mu) \approx X_n^2(0)$.

Theorem2: Let $N_n \approx (\mu, \sigma^2 I), \sigma^2 > 0$, let A and B be $p \times n$ and $s \times n$ matrices and let C and D be $n \times n$ nonnegative definite matrices. Then

a- AY and BY are independent if and only if $AB' = 0$

b-If $AC = 0$, then AY and $Y'CY$ are independent.

c-If CD=0 , then $Y'CY$ and $Y'DY$ are independent.

Corollary: Let $Y \approx N_n(\mu, \sigma^2 I), \sigma^2 > 0$. If A is idempotent and rank $(A) = P$, then

$$Y'AY \approx \sigma^2 X_p^2(\frac{\mu' A\mu}{\sigma^2}).$$

Lemma 1,lemma2 , theorem1,theorem2, corollary took from Arnold (1981).

Table (6) true α for different values of C_5

C_5	0.1	0.05	0.025
0.75	0.2473	0.1846	0.1495
0.9	0.159	0.1039	0.0748
1	0.1	0.05	0.025
1.4	0.0665	0.0266	0.0090
1.6	0.0497	0.0149	0.0011

a=5,b=3,c=d=n=2

Table (7) true α for different values of C_6

C_6	0.1	0.05	0.025
0.7	0.2404	0.1816	0.1486
0.85	0.1702	0.1158	0.0868
1	0.1	0.05	0.025
1.5	0.0460	0.0293	0.0104
1.8	0.0135	0.0169	0.0016

a=5,b=c=d=n=2

Table (8) true α for different values of C_7

C_7	0.1	0.05	0.025
0.5	0.2453	0.2009	0.1761
0.88	0.1349	0.0862	0.0613
1	0.1	0.05	0.025
1.85	0.0503	0.0291	0.0089
2.2	0.0298	0.0205	0.0022

a=2,b=5,c=3,d=n=2

Table (9) true α for different values of C_8

C_8	0.1	0.05	0.025
0.55	0.2284	0.1836	0.1588
0.63	0.2056	0.1598	0.1350
1	0.1	0.05	0.025
1.99	0.0466	0.0289	0.0087
2.4	0.0245	0.0201	0.0019

a=b=c=2,d=3,n=2

Table (10) true α for different values of C_9

Table (2) true α for different values of C_1

α / C_1	0.1	0.05	0.025
0.75	0.2204	0.1634	0.0852
0.95	0.1241	0.0727	0.0332
1	0.1	0.05	0.025
1.5	0.0667	0.02599	0.0082
1.7	0.0534	0.0164	0.0014

a=5,b=c=d=2,n=4

Table (3) true α for different values of C_2

C_2	0.1	0.05	0.025
0.85	0.1436	0.0751	0.0407
0.99	0.1029	0.0517	0.0260
1	0.1	0.05	0.025
1.75	0.0547	0.0307	0.0101
1.95	0.0426	0.0255	0.0062

a=5,b=c=d=2,n=4

Table (4) true α for different values of C_3

C_3	0.1	0.05	0.025
0.6	0.2424	0.1909	0.0978
0.96	0.1142	0.0641	0.0323
1	0.1	0.05	0.025
1.77	0.0427	0.0284	0.0091
2	0.0255	0.02195	0.0044

a=b=2,c=3,d=n=2

Table (5) true α for different values of C_4

C_4	0.1	0.05	0.025
0.8	0.1927	0.138	0.0831
0.99	0.1046	0.0544	0.0292
1	0.1	0.05	0.025
1.3	0.0658	0.0391	0.0156
1.6	0.0316	0.0234	0.0062

a=b=2,c=3,d=5,n=2

C_{13}	0.1	0.05	0.025
0.6	0.2478	0.1957	0.1659
0.7	0.2109	0.1593	0.1307
1	0.1	0.05	0.025
1.6	0.0502	0.0300	0.0103
1.7	0.0253	0.0199	0.0029

a=2,b=5,c=3,d=n=2

Table (15) true α for different values of C_{14}

C_{14}	0.1	0.05	0.025
0.85	0.1885	0.1308	0.0996
0.99	0.1059	0.0554	0.030
1	0.1	0.05	0.025
1.2	0.0585	0.0267	0.0170
1.3	0.0378	0.0155	0.01301

a=2,b=5,c=3,d=n=2

Table (16) true α for different values of C_{15}

C_{15}	0.1	0.05	0.025
0.7	0.2068	0.1557	0.1273
0.85	0.1534	0.1029	0.0762
1	0.1	0.05	0.025
1.5	0.0628	0.0360	0.0147
1.8	0.0404	0.0276	0.0085

a=2,b=3,c=d=n=2

C_9	0.1	0.05	0.025
0.68	0.1921	0.146	0.0754
0.88	0.1345	0.086	0.0439
1	0.1	0.05	0.025
1.68	0.0439	0.0341	0.0127
1.88	0.0294	0.0274	0.0091

a=b=2,c=5,d=n=2

Table (11) true α for different values of C_{10}

C_{10}	0.1	0.05	0.025
0.5	0.2403	0.1953	0.1715
0.9	0.1281	0.0793	0.0543
1	0.1	0.05	0.025
1.9	0.0549	0.033	0.0118
2.5	0.0249	0.0216	0.0030

a=b=c=d=n=2

Table (12) true α for different values of C_{11}

C_{11}	0.1	0.05	0.025
0.75	0.2393	0.1782	0.1441
0.95	0.1279	0.0757	0.0488
1	0.1	0.05	0.025
1.1	0.0805	0.0388	0.0210
1.3	0.0415	0.0165	0.0131

a=4,b=3,c=2,d=5,n=2

Table (13) true α for different values of C_{12}

C_{12}	0.1	0.05	0.025
0.5	0.2427	0.1984	0.1736
0.9	0.1285	0.0797	0.0547
1	0.1	0.05	0.025
1.9	0.0515	0.0308	0.0101
2.4	0.0245	0.0201	0.0019

a=b=c=2,d=3,n=2

Table (14) true α for different values of C_{13}

Fig (1) the relation between true alpha and Alpha with df(4,120)

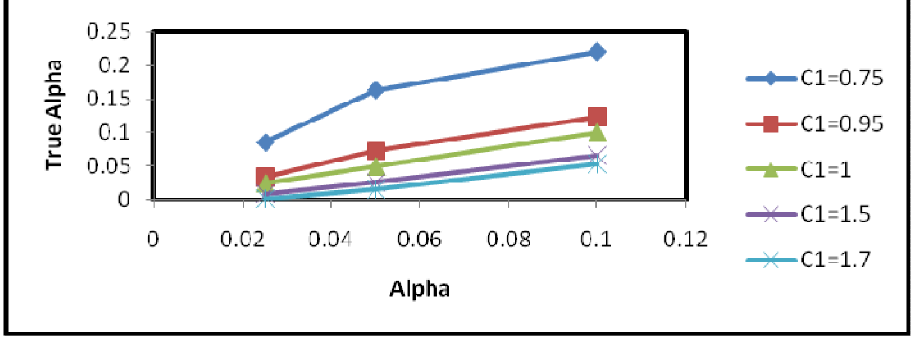
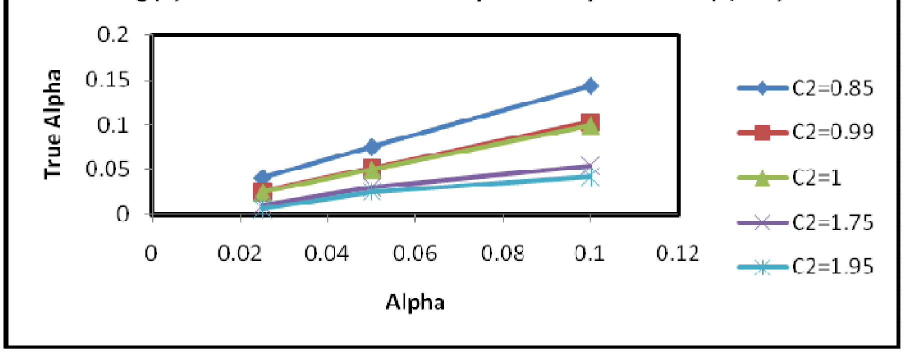
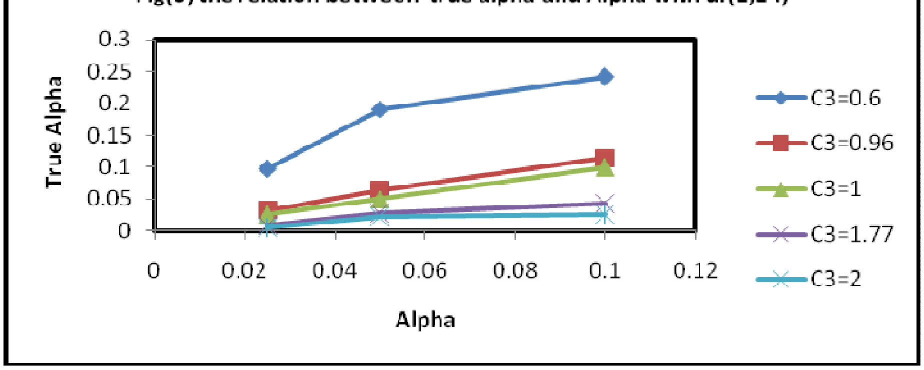


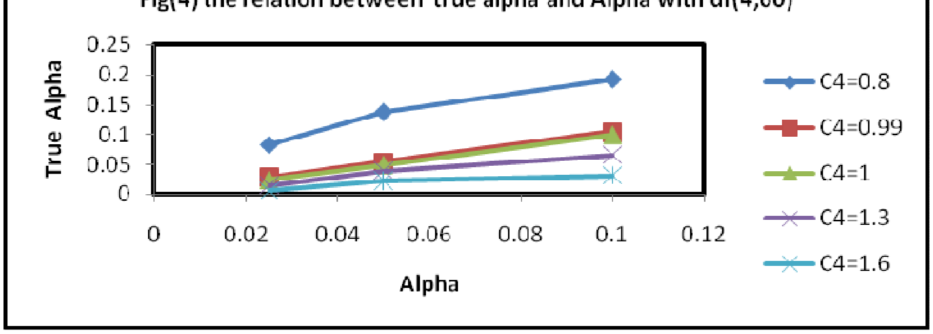
Fig (2) the relation between true alpha and Alpha with df (1,120)

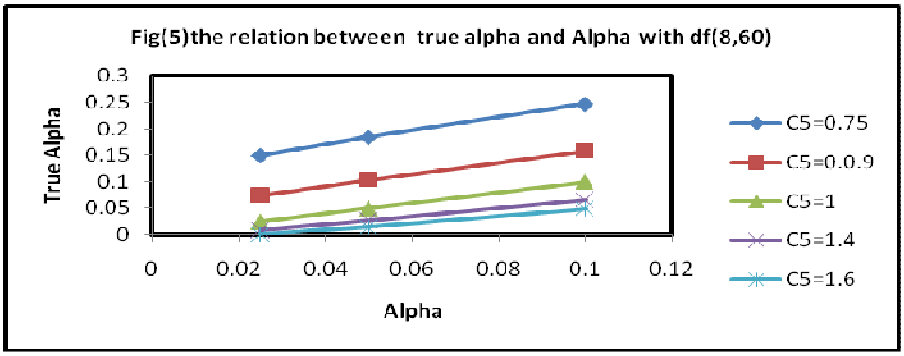


Fig(3) the relation between true alpha and Alpha with df(2,24)

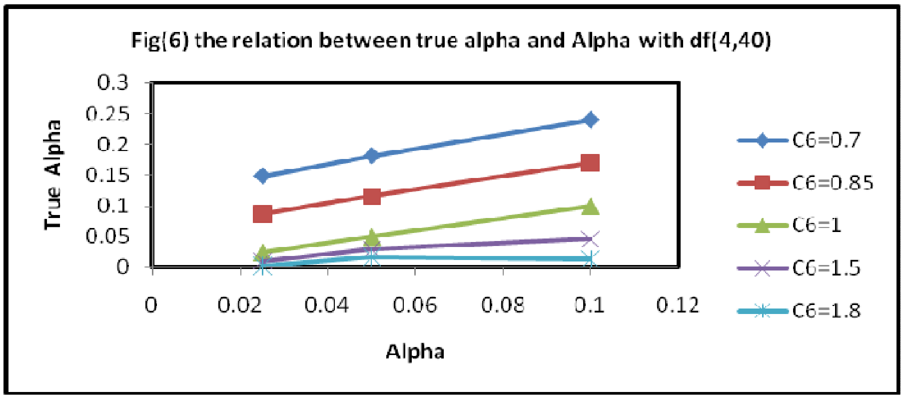


Fig(4) the relation between true alpha and Alpha with df(4,60)

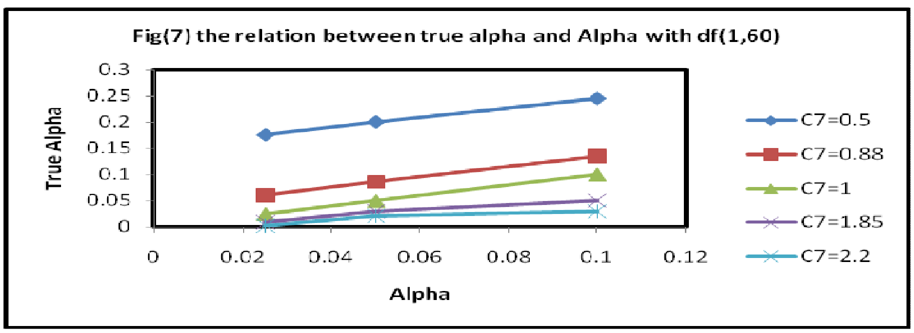




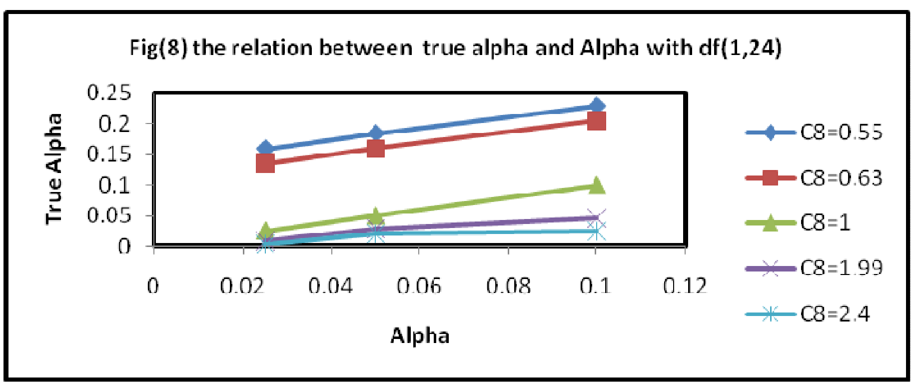
Fig(5)the relation between true alpha and Alpha with df(8,60)



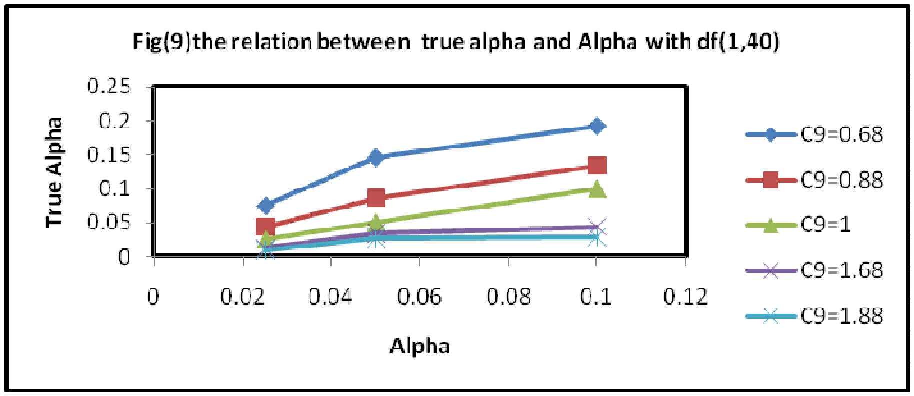
Fig(6) the relation between true alpha and Alpha with df(4,40)

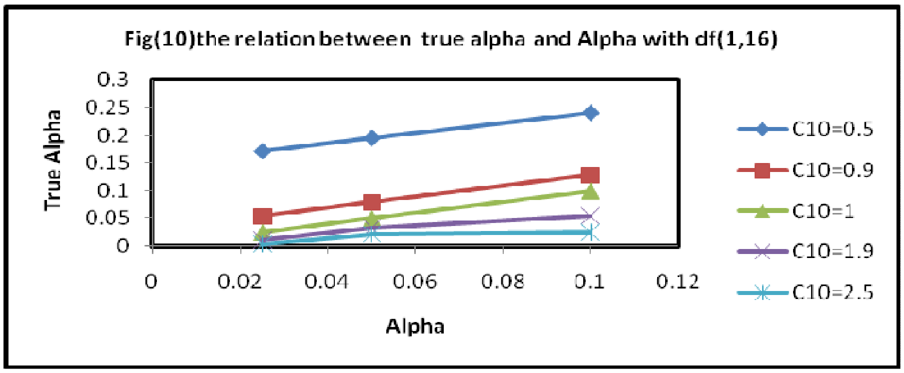


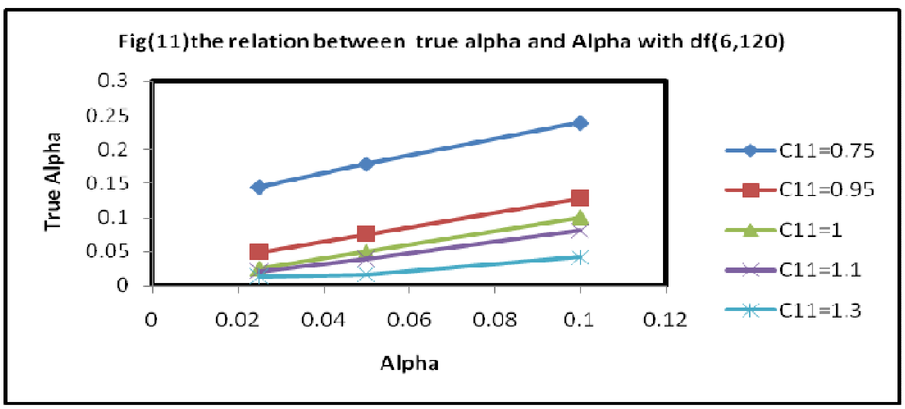
Fig(7) the relation between true alpha and Alpha with df(1,60)

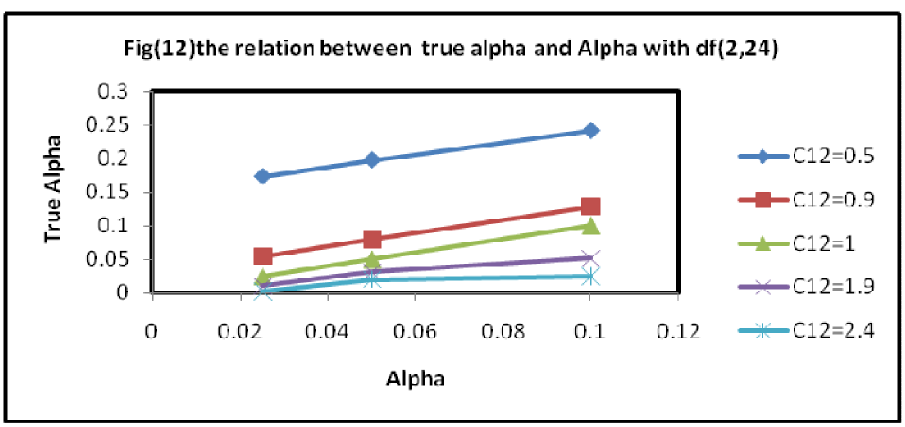


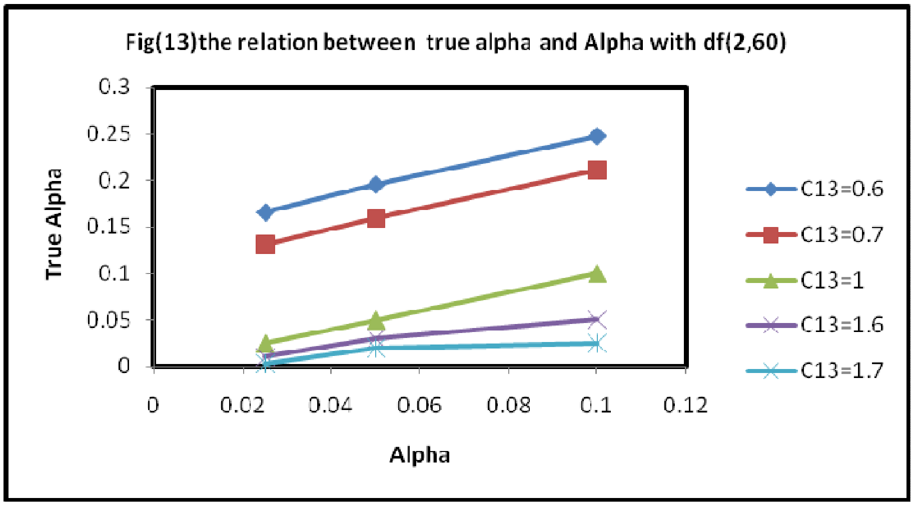
Fig(8) the relation between true alpha and Alpha with df(1,24)

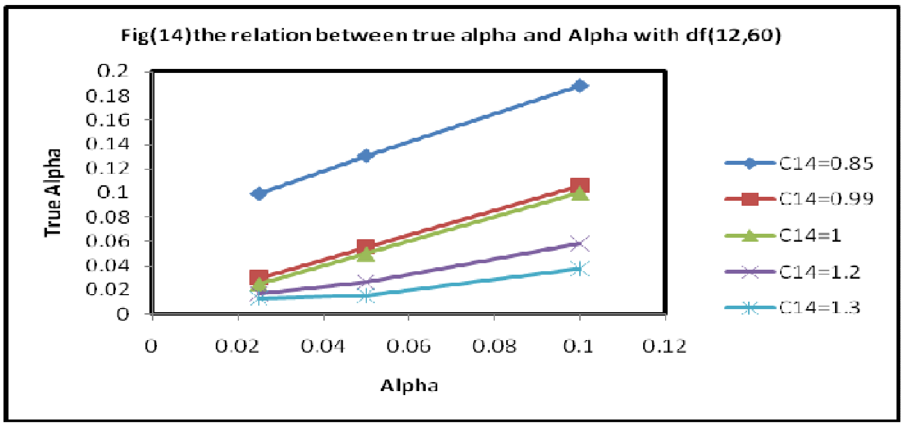

Fig(9)the relation between true alpha and Alpha with df(1,40)
True Alpha
Alpha
0.25
0.2
0.15
0.1
0.05
0
0
0.02
0.04
0.06
0.08
0.1
0.12
C9=0.68
C9=0.88
C9=1
C9=1.68
C9=1.88

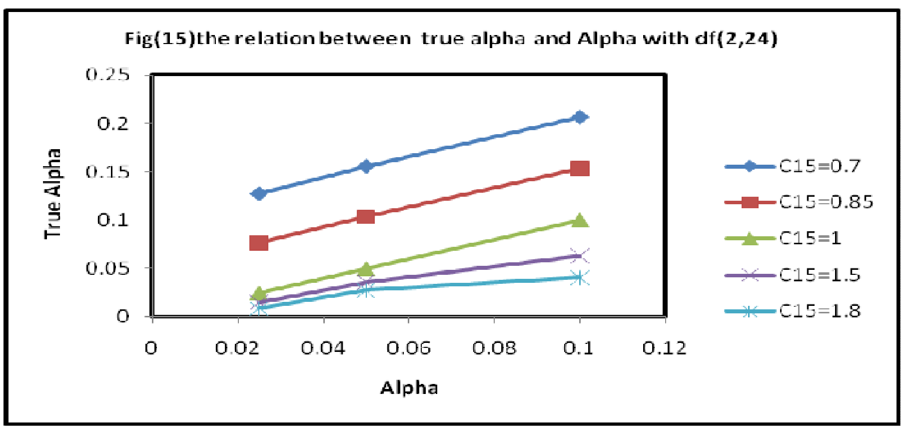

Fig(10)the relation between true alpha and Alpha with df(1,16)
True Alpha
Alpha
0.3
0.25
0.2
0.15
0.1
0.05
0
0
0.02
0.04
0.06
0.08
0.1
0.12
C10=0.5
C10=0.9
C10=1
C10=1.9
C10=2.5


Fig(11)the relation between true alpha and Alpha with df(6,120)
True Alpha
Alpha
0.3
0.25
0.2
0.15
0.1
0.05
0
0
0.02
0.04
0.06
0.08
0.1
0.12
C11=0.75
C11=0.95
C11=1
C11=1.1
C11=1.3


Fig(12)the relation between true alpha and Alpha with df(2,24)
True Alpha
Alpha
0.3
0.25
0.2
0.15
0.1
0.05
0
0
0.02
0.04
0.06
0.08
0.1
0.12
C12=0.5
C12=0.9
C12=1
C12=1.9
C12=2.4


Fig(13)the relation between true alpha and Alpha with df(2,60)
True Alpha
Alpha
0.3
0.25
0.2
0.15
0.1
0.05
0
0
0.02
0.04
0.06
0.08
0.1
0.12
C13=0.6
C13=0.7
C13=1
C13=1.6
C13=1.7


Fig(14)the relation between true alpha and Alpha with df(12,60)
True Alpha
Alpha
0.2
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
0
0.02
0.04
0.06
0.08
0.1
0.12
C14=0.85
C14=0.99
C14=1
C14=1.2
C14=1.3


Fig(15)the relation between true alpha and Alpha with df(2,24)
True Alpha
Alpha
0.25
0.2
0.15
0.1
0.05
0
0
0.02
0.04
0.06
0.08
0.1
0.12
C15=0.7
C15=0.85
C15=1
C15=1.5
C15=1.8

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حساب توقع معدلات المربعات لنموذج ذي أربعة اتجاهات متقاطع متوازن لبيانات مترابطة

الخلاصة

في هذا البحث تم حساب توقع معدلات المربعات لنموذج ذي أربعة اتجاهات متقاطع متوازن لبيانات مترابطة ولاحظنا التأثير للترابط بين البيانات على الإحصائية F . وتم عمل كل ذلك باستخدام بعض مفاهيم الجبر الخطي .