Finding the Optimal Solution of Fuzzy Transportation Problems

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ABSTRACT

The paper investigates fuzzy transportations problem with the aid of trapezoidal fuzzy numbers. For finding the initial solution of this problem we have preferred the fuzzy least costs method and ranking method, also the optimal solution by using fuzzy multipliers method has been carried out. A new relevant numerical example was also included.

أيجاد الحل الأمثل لمسائل النقل الضبابية

الخلاصة

في هذه الدراسة فأننا بحثنا في مسائل النقل الضبابية بمساعدة اعداد شبه المنحرف الضبابية . لأيجاد الحل الابتدائي لهذه المسألة فقد فضلنا طريقة اقل الكلف الضبابية وطريقة الترتيب , أيضا" وجدنا الحل الأمثل بأستخدام طريقة المضاعفات الضبابية .كما ادرج مثال عددي جديد ذو صلة بالموضوع.

INTRODUCTION

The basic transportation problem was originally stated by HitchCock [8] and later discussed in detail by Koopman [12]. An earlier approach was given by Kantrovich [11].The linear programming formulation and the associated systematic method for solution were first given by Dantzig [6] then after 1960's Bellman and Zadeh [3] proposed the concept of decision making in fuzzy environment. Lai and Hwang [13] others considered the situation where all parameters are fuzzy. The task of distributor's decision optimization can be reformulated as the generalation of classical transportation problem.

Conventional transportation problem is the special type of linear programming problem where special mathematical structure of restrictions is used. In classical approach, transportation costs from M wholesalers to the N consumers are minimized. In 1979, Isermann [10] introduced algorithm for solving this problem which provides effective solutions. The Ringuest and Rinks [14] proposed two iterative algorithms for solving linear, multi criteria transportation problem.

Similar solution interval and fuzzy coefficients had been elaborated. The further development of this approach introduced in work [10]. In works by S.Chanas and D.Kuchta [5] the approach based on interval and fuzzy coefficients had been elaborated The further development of this approach introduced in work [13].In this work, the fuzzy transportation problems using trapezoidal fuzzy numbers are

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discussed, we have found the initial solution of fuzzy transportation problem by using fuzzy least costs method and ranking method, then the optimal solution by using fuzzy multipliers method.

BASIC DEFINITIONS

Here, we briefly give some fundamental definitions that will be needed later.

Definition [15]:- A real fuzzy number $\tilde{a}$ is a fuzzy subset of the real number R with membership function $\mu_{\tilde{a}}$ satisfying the following conditions.

(1) $\mu_{\tilde{a}}$ is continuous from R to the closed interval [0, 1].

(2) $\mu_{\tilde{a}}$ is strictly increasing and continuous on $[a_1, a_2]$.

(3) $\mu_{\tilde{a}}$ is strictly decreasing and continuous on $[a_3, a_4]$ where a_1, a_2, a_3 & a_4 are real numbers, and the fuzzy number denoted by $\tilde{a} = [a_1, a_2, a_3, a_4]$ is called a trapezoidal fuzzy number**.**

Definition [2]:- The fuzzy number $a = [a_1, a_2, a_3, a_4]$ is a trapezoidal number, denoted by

$$\mu_a(x) = \begin{cases} (x - a_1) / (a_2 - a_1), & a_1 \le x \le a_2 \\ 1, & a_2 \le x \le a_3 \\ (x - a_4)/(a_3 - a_4), & a_3 \le x \le a_4 \\ 0, & \text{otherwise} \end{cases} \quad \ldots\ldots (1 - 1)$$

$[a_1, a_2, a_3, a_4]$ its membership function μ_a is given below the figure.

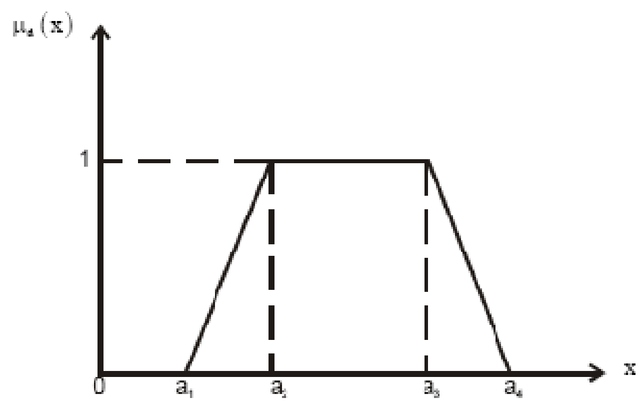

Figure (1) Membership function of a fuzzy number $\tilde{a}$.

Definition [15]:- (**i**) Fuzzy feasible solution: Any set of fuzzy non negative allocations $X_{ij} > [-2\partial, -1\partial, 1\partial, 2\partial]$ where ∂ is small +ve number, which satisfies the row and column sum is a fuzzy feasible solution.

(**ii**) Fuzzy basic feasible solution: A feasible solution is a fuzzy basic feasible solution if the number of non negative allocation is at most (m + n - 1) where m is the number of rows and n is the number of columns in a transportation table.

(**iii**) Fuzzy non degenerate basic feasible solution: Any fuzzy feasible solution to a transportation problem containing (m) origins and (n) destinations is said to be fuzzy non degenerate, if it contains exactly (m + n - 1) occupied cells.
(**iv**) Fuzzy degenerate basic feasible solution: If a fuzzy basic feasible solution contains less than (m + n - 1) non negative allocations, it is said to be degenerate.

ARITHMETIC OPERATIONS [15]
Let $\tilde{a}$ = [a_1, a_2, a_3, a_4] and $\tilde{b}$= [b_1, b_2, b_3, b_4] two trapezoidal fuzzy numbers then the arithmetic operations on $\tilde{a}$ and $\tilde{b}$
As follows:-

Addition: $\tilde{a}+\tilde{b}=(a_1+b_1, a_2+b_2, a_3+b_3, a_4+b_4)$ ……(1-2)

Subtraction: $\tilde{a}-\tilde{b}=(a_1-b_4, a_2-b_3, a_3-b_2, a_4-b_1)$ ……(1-3)

Multiplication: $\tilde{a}\bullet\tilde{b}=(a_1/4(b_1+b_2+b_3+b_4), a_2/4(b_1+b_2+b_3+b_4), a_3/4(b_1+b_2+b_3+b_4), a_4/4(b_1+b_2+b_3+b_4))$…. (1-4)

Ranking method
In this section we shall introduce ranking method [1, 4] for trapezoidal fuzzy numbers that will be needed for later work.
For any trapezoidal fuzzy number $\tilde{a}=(a_1,a_2,a_3,a_4)$, we define:-
$\underline{a}=a_m-1/2h_{\underline{a}}$ $\overline{a}=a_m+1/2h_a$, where $a_{m}=a_2+a_3/2$ and $h_{\underline{a}}=a_1/a_1+a_4$, $h_a=a_4/\underline{a_1+}a_4$ … (2-1)
Now assume that $\tilde{a}=(a_1, a_2, a_3, a_4)$, $\tilde{b}=(b_1, b_2, b_3, b_4)$ be two trapezoidal fuzzy numbers.
Let

$\overline{R}(\tilde{a},\tilde{b})=\overline{a}-$ ……… (2-2)
$\underline{R}(\tilde{a},\tilde{b})=\underline{a}-\underline{b}$

Where **$\underline{a}$, $\overline{a}$, $\underline{b}$, $\overline{b}$ are** defined in (2-1).

Lemma
Assume that $\tilde{a}=(a_1,a_2,a_3,a_4)$, $\tilde{b}=(b_1,b_2,b_3,b_4)$ be two trapezoidal fuzzy numbers. Then, we have

$$\overline{R}(\tilde{a},\tilde{b})=-\overline{R}(\tilde{b},\tilde{a})$$
$$\underline{R}(\tilde{a},\tilde{b})=-\underline{R}(\tilde{b},\tilde{a})$$

Proof:
It is straightforward from (2-2).
Definition
Assume that $\tilde{a}=(a_1,a_2,a_3,a_4)$, $\tilde{b}=(b_1,b_2,b_3,b_4)$ be two trapezoidal fuzzy numbers and $\underline{R}(\tilde{b},\tilde{a})\geq 0$. Define the relations ≈ and as given below:

i) $a^{\sim} \approx b^{\sim}$ if and only if $\underline{R}(a^{\sim},b^{\sim})= R(a^{\sim},b^{\sim})$.

ii) $a^{\sim}$ $b^{\sim}$ if and only if $\underline{R}(a^{\sim},b^{\sim})$ $R(a^{\sim},b^{\sim})$.

Remark

We denote $a^{\sim}$ $b^{\sim}$ if and only if $a^{\sim} \approx b^{\sim}$ or $a^{\sim}$ $b^{\sim}$. Then $a^{\sim}$ $b^{\sim}$ if and only if $\underline{R}(a^{\sim},b^{\sim})\geq R(a^{\sim},b^{\sim})$.

Also $a^{\sim}$ $b^{\sim}$ if and only if $b^{\sim}$ $a^{\sim}$.

We let $0^{\sim}$ = (0,0,0,0) as a zero trapezoidal fuzzy numbers. Thus any $a^{\sim}$ such that $a^{\sim} \approx 0^{\sim}$, is a zero too.

Remark::-The fuzzy trapezoidal number$[-2\partial,-1\partial,1\partial,2\partial]\approx[0,0,0,0]$.

Fuzzy Transportation problem[9,15]

Consider a transportation with m fuzzy origins (rows) and n fuzzy destinations (columns). Let Cij $=[c_{ij}^{(1)}, c_{ij}^{(2)}, c_{ij}^{(3)}, c_{ij}^{(4)}]$
be the cost of transporting one unit of the product from ith fuzzy origin to jth fuzzy destination. ai = $[ai^{(1)}, ai^{(2)}, ai^{(3)}, ai^{(4)}]$

be the quantity of commodity available at fuzzy origin i , bj= $[bj^{(1)}, bj^{(2)}, bj^{(3)}, bj^{(4)}]$ the quantity of commodity needed at

fuzzy destination j. Xij = $[x_{ij}^{(1)}, x_{ij}^{(2)}, x_{ij}^{(3)}, x_{ij}^{(4)}]$ is quantity transported from ith fuzzy origin to jth fuzzy destination.

The above fuzzy transportation problem can be stated in the below tabular form.

	1	2 …...	n	Fuzzy capacity
1	C_{11} X_{11}	C_{12}…... X_{12}…..	C_{1n} X_{1n}	a_1
2 :	C_{21} :	C_{22}…... : X_2	C_{2n} :	a_2 :
m	C_{m1} X_{m1}	C_{m2}….. X_{m2}…..	C_{mn} X_{mn}	a_m
fuzzy demand	b_1	b_2…...	b_n	$\sum_{i=1}^{m} a_i=\sum_{j=1}^{n} b_j$

Where $C_{ij} =[c_{ij}^{(1)}, c_{ij}^{(2)}, c_{ij}^{(3)}, c_{ij}^{(4)}]$, $a_i = [ai^{(1)}, ai^{(2)}, ai^{(3)}, ai^{(4)}]$, $b_j= [bj^{(1)}, bj^{(2)}, bj^{(3)}, bj^{(4)}]$, $X_{ij} = [x_{ij}^{(1)}, x_{ij}^{(2)}, x_{ij}^{(3)}, x_{ij}^{(4)}]$

The linear programming model representing the fuzzy transportation is given by:

Minimize $\boldsymbol{Z}=\sum\sum [c_{ij}^{(1)}, c_{ij}^{(2)}, c_{ij}^{(3)}, c_{ij}^{(4)}]\ [x_{ij}^{(1)}, x_{ij}^{(2)}, x_{ij}^{(3)}, x_{ij}^{(4)}]$

Subject to the constraints

$$\sum[x_{ij}^{(1)}, x_{ij}^{(2)}, x_{ij}^{(3)}, x_{ij}^{(4)}]= [ai^{(1)}, ai^{(2)}, ai^{(3)}, ai^{(4)}] \quad \text{for } i = 1,2,\dots,m \text{ (Row sum)}$$

$$\sum[x_{ij}^{(1)}, x_{ij}^{(2)}, x_{ij}^{(3)}, x_{ij}^{(4)}] = [bj^{(1)}, bj^{(2)}, bj^{(3)}, bj^{(4)}] \quad \text{for } j = 1,2,\dots,n \text{ (Column sum)} \qquad \dots\dots\dots(3\text{-}1)$$

$$[x_{ij}^{(1)}, x_{ij}^{(2)}, x_{ij}^{(3)}, x_{ij}^{(4)}]\geq 0$$

The given fuzzy transportation problem is said to be balanced if

$$\sum\ [ai^{(1)}, ai^{(2)}, ai^{(3)}, ai^{(4)}]= \sum[bj^{(1)}, bj^{(2)}, bj^{(3)}, bj^{(4)}]$$

i.e., if the total fuzzy capacity is equal to the total fuzzy demand.

SOLUTION OF A FUZZY TRANSPORTATION PROBLEM

The solution of a fuzzy transportation problem can be solved two stages, namely initial solution and optimal solution. In this section we find the initial solution by using fuzzy least cost method and ranking method, also we have found the optimal solution by using fuzzy multipliers method.

FUZZY LEAST COST METHOD

The procedure is as follows. Assign as much as possible to the variable with the smallest fuzzy unit cost in the entire tableau. Where we choose the smallest fuzzy cost by using ranking method. Cross out the satisfied row or column (As in the north west-corner method [7] if both a column and a row are satisfied simultaneously, only one may be crossed out.). After adjusting the supply and demand for all uncrossed –out rows and columns .repeat the process by assigning as much as much as possible to the variable with smallest uncrossed –out unit cost. The procedure is complete when exactly one row or one column is left uncrossed –out.

FUZZY MULTIPLIERS METHOD

This method is used for finding the optimal basic feasible solution in fuzzy environment and the procedure is as follows:-

Step (1):- Find out a set of numbers $[ui^{(1)}, ui^{(2)}, ui^{(3)}, ui^{(4)}]$ and $[vj^{(1)}, vj^{(2)}, vj^{(3)}, vj^{(4)}]$ for each fuzzy basic solution $[x_{ij}^{(1)}, x_{ij}^{(2)}$, $x_{ij}^{(3)}, x_{ij}^{(4)}]$ such that :-

$$[ui^{(1)}, ui^{(2)}, ui^{(3)}, ui^{(4)}]+ [vj^{(1)}, vj^{(2)}, vj^{(3)}, vj^{(4)}]= [c_{ij}^{(1)}, c_{ij}^{(2)}, c_{ij}^{(3)}, c_{ij}^{(4)}]\dots..(3\text{-}2).$$

These equations yield (m+n-1) equations (because there are only (m+n-1) fuzzy basic variables in (m+n) unknowns),

Step (2):- Find the values of the multipliers in equation (3-2) by assuming an arbitrary value for only one of the multipliers (usually $[u1^{(1)}, u1^{(2)}, u1^{(3)}, u1^{(4)}]$ is set equal to [0,0,0,0]) and then solving the (m+n-1) equations in the remaining (m+n-1) unknown multipliers.

Step (3):- determine the value $[\underline{c}_{pq}^{(1)}, \underline{c}_{pq}^{(2)}, \underline{c}_{pq}^{(3)}, \underline{c}_{pq}^{(4)}]$ of each non basic fuzzy variable$[\underline{x}_{pq}^{(1)}, \underline{x}_{pq}^{(2)}, \underline{x}_{pq}^{(3)}, \underline{x}_{pq}^{(4)}]$ such that $[\underline{c}_{pq}^{(1)}, \underline{c}_{pq}^{(2)}, \underline{c}_{pq}^{(3)}, \underline{c}_{pq}^{(4)}]= [c_{pq}^{(1)}, c_{pq}^{(2)}, c_{pq}^{(3)}, c_{pq}^{(4)}] -[up^{(1)}, up^{(2)}, up^{(3)}, up^{(4)}]- [vq^{(1)}, vq^{(2)}, vq^{(3)}, vq^{(4)}]$ …..(3-3).

Step (4):- if all values of $[\underline{c}_{pq}^{(1)}, \underline{c}_{pq}^{(2)}, \underline{c}_{pq}^{(3)}, \underline{c}_{pq}^{(4)}]$ of eq(3-2) are positive that is to say $([\underline{c}_{pq}^{(1)}, \underline{c}_{pq}^{(2)}, \underline{c}_{pq}^{(3)}, \underline{c}_{pq}^{(4)}]>[0,0,0,0]$

Then the initial fuzzy basic feasible solution $[x_{ij}^{(1)}, x_{ij}^{(2)}, x_{ij}^{(3)}, x_{ij}^{(4)}]$ is optimal, where i=1,…,m , j=1,..,n . If not there exists a non basic variable $[\underline{x}_{pq}^{(1)}, \underline{x}_{pq}^{(2)}, \underline{x}_{pq}^{(3)}, \underline{x}_{pq}^{(4)}]$ such that :-

$[\underline{c}_{pq}^{(1)}, \underline{c}_{pq}^{(2)}, \underline{c}_{pq}^{(3)}, \underline{c}_{pq}^{(4)}]=\min \{ [\underline{c}_{pq}^{(1)}, \underline{c}_{pq}^{(2)}, \underline{c}_{pq}^{(3)}, \underline{c}_{pq}^{(4)}]<[0,0,0,0]\}$,and $[\underline{x}_{pq}^{(1)}, \underline{x}_{pq}^{(2)}, \underline{x}_{pq}^{(3)}, \underline{x}_{pq}^{(4)}]$, is made a basic variable to improve the value of $[z^{(1)}, z^{(2)}, z^{(3)}, z^{(4)}]$, Such that we construct a closed loop for the current entering variable $[\underline{x}_{pq}^{(1)}, \underline{x}_{pq}^{(2)}, \underline{x}_{pq}^{(3)}, \underline{x}_{pq}^{(4)}]$ in the cell that contain this value, The loop starts and ends at the designate non basic variable. It consists of successive horizontal and vertical segments (assign + and – alternately) whose end points must be fuzzy basic variables , except for the end points that are associated with the entering variable .this means that every corner element of the loop must be a cell containing a fuzzy basic variable.

Step (4):- determine the leaving variable $[x_{ij}^{(1)}, x_{ij}^{(2)}, x_{ij}^{(3)}, x_{ij}^{(4)}]$ where the leaving variable is selected from among the
Corner variables of the loop which have the smallest allocation value, where we subtract and add this value to each corner.

Step (5):- find the value of $[z^{(1)}, z^{(2)}, z^{(3)}, z^{(4)}]$ at the new fuzzy basic variables $[x_{ij}^{(1)}, x_{ij}^{(2)}, x_{ij}^{(3)}, x_{ij}^{(4)}]$, where i=1,…,m , j=1,..,n.

NUMERICAL EXAMPLE

To solve the following fuzzy transportation problem starting with the fuzzy initial fuzzy basic feasible solution obtained by fuzzy least cost method whose fuzzy cost and fuzzy requirement table is given below:-

Table (3-1)

	FD1	FD2	FD3	FD4	Fuzzy capacity
FS1	[0,2,4,8]	[1,2,4,9]	[0,2,4,8]	[0,3,4,5]	[0,2,4,6]
FS2	[4,8,12,16]	[4,7,9,12]	[2,4,6,8]	[1,3,5,7]	[2,4,9,13]
FS3	[[0,0,0,0]	[4,8,10,15]	[2,4,6,8]	[0,5,7,9]	[2,4,6,8]
Fuzzy demand	[1,3,5,7]	[1,2,4,6]	[1,3,5,7]	[1,2,5,7]	[4,10,19,27]

Since $\sum a_i = \sum b_j$ =[4,10,19,27]=[4,10,19,27],The problem is balanced fuzzy transportation problem. There exists a fuzzy initial basic feasible solution.

Table (3-2)

	FD1	FD2	FD3	FD4	Fuzzy capacity
FS1	[0,2,4,8]	[1,2,4,9]	[0,2,4,8]	[0,3,4,5] [0,2,4,6]	[0,2,4,6]
FS2	[4,8,12,16]	[4,7,9,12] [-12,-4,8,17]	[2,4,6,8] [1,3,5,7]	[1,3,5,7] [-5,-2,3,7]	[2,4,9,13]
FS3	[0,0,0,0] [1,3,5,7]	[4,8,10,15] [-5,-1,3,7]	[2,4,6,8]	[0,5,7,9]	[2,4,6,8]
Fuzzy demand	[1,3,5,7]	[-5,-1,7,12]	[1,3,5,7]	[1,2,5,7]	[4,10,19,27]

Note :- " by definition(2.1) [-5,-1,7,12]≈[1,2,4,6]"

Since the number of occupied cell having m + n – 1 = 6 and are also independent, there exist a non degenerate fuzzy basic feasible solution.

Therefore, the initial fuzzy transportation minimum cost is,

$[z^{(1)}, z^{(2)}, z^{(3)}, z^{(4)}]= [c_{14}^{(1)}, c_{14}^{(2)}, c_{14}^{(3)}, c_{14}^{(4)}][x_{14}^{(1)}, x_{14}^{(2)}, x_{14}^{(3)}, x_{14}^{(4)}]+[c_{22}^{(1)}, c_{22}^{(2)}, c_{22}^{(3)}, c_{22}^{(4)}][x_{22}^{(1)}, x_{22}^{(2)}, x_{22}^{(3)}, x_{22}^{(4)}]+[c_{23}^{(1)}, c_{23}^{(2)}, c_{23}^{(3)}, c_{23}^{(4)}][x_{23}^{(1)}, x_{23}^{(2)}, x_{23}^{(3)}, x_{23}^{(4)}]+[c_{24}^{(1)}, c_{24}^{(2)}, c_{24}^{(3)}, c_{24}^{(4)}][x_{24}^{(1)}, x_{24}^{(2)}, x_{24}^{(3)}, x_{24}^{(4)}]+[c_{31}^{(1)}, c_{31}^{(2)}, c_{31}^{(3)}, c_{31}^{(4)}][x_{31}^{(1)}, x_{31}^{(2)}, x_{31}^{(3)}, x_{31}^{(4)}]+[c_{32}^{(1)}, c_{32}^{(2)}, c_{32}^{(3)}, c_{32}^{(4)}][x_{32}^{(1)}, x_{32}^{(2)}, x_{32}^{(3)}, x_{32}^{(4)}]$

$[z^{(1)}, z^{(2)}, z^{(3)}, z^{(4)}]$= [0,3,4,5] [0,2,4,6]+ [4,7,9,12] [-12,-4,8,17]+ [2,4,6,8] [1,3,5,7]+ [1,3,5,7] [-5,-2,3,7]+ [0,0,0,0] [1,3,5,7]+ [4,8,10,15] [-5,-1,3,7]

$[z^{(1)}, z^{(2)}, z^{(3)}, z^{(4)}]$=[21.75,51,70,94.25] ……………….(*)

Now we find the optimal solution by using fuzzy multipliers method. First we determine a set number $[ui^{(1)}, ui^{(2)}, ui^{(3)}, ui^{(4)}]$ and $[vj^{(1)}, vj^{(2)}, vj^{(3)}, vj^{(4)}]$ for each fuzzy basic solution $[x_{ij}^{(1)}, x_{ij}^{(2)}, x_{ij}^{(3)}, x_{ij}^{(4)}]$ such that :-

$[ui^{(1)}, ui^{(2)}, ui^{(3)}, ui^{(4)}]+[vj^{(1)}, vj^{(2)}, vj^{(3)}, vj^{(4)}]=[c_{ij}^{(1)}, c_{ij}^{(2)}, c_{ij}^{(3)}, c_{ij}^{(4)}]$.

Then from table (3-2) we have:-

$[x_{14}^{(1)}, x_{14}^{(2)}, x_{14}^{(3)}, x_{14}^{(4)}]$: $[u1^{(1)}, u1^{(2)}, u1^{(3)}, u1^{(4)}]+[v4^{(1)}, v4^{(2)}, v4^{(3)}, v4^{(4)}]=[c_{14}^{(1)}, c_{14}^{(2)}, c_{14}^{(3)}, c_{14}^{(4)}]=[0,3,4,5]$

$[x_{22}^{(1)}, x_{22}^{(2)}, x_{22}^{(3)}, x_{22}^{(4)}]$: $[u2^{(1)}, u2^{(2)}, u2^{(3)}, u2^{(4)}]+[v2^{(1)}, v2^{(2)}, v2^{(3)}, v2^{(4)}]=[c_{22}^{(1)}, c_{22}^{(2)}, c_{22}^{(3)}, c_{22}^{(4)}]=[4,7,9,12]$

$[x_{23}^{(1)}, x_{23}^{(2)}, x_{23}^{(3)}, x_{23}^{(4)}]$: $[u2^{(1)}, u2^{(2)}, u2^{(3)}, u2^{(4)}]+[v3^{(1)}, v3^{(2)}, v3^{(3)}, v3^{(4)}]=[c_{23}^{(1)}, c_{23}^{(2)}, c_{23}^{(3)}, c_{23}^{(4)}]=[2,4,6,8]$

$[x_{31}^{(1)}, x_{31}^{(2)}, x_{31}^{(3)}, x_{31}^{(4)}]$: $[u3^{(1)}, u3^{(2)}, u3^{(3)}, u3^{(4)}]+[v1^{(1)}, v1^{(2)}, v1^{(3)}, v1^{(4)}]=[c_{31}^{(1)}, c_{31}^{(2)}, c_{31}^{(3)}, c_{31}^{(4)}]=[0,0,0,0]$

$[x_{32}^{(1)}, x_{32}^{(2)}, x_{32}^{(3)}, x_{32}^{(4)}]$: $[u3^{(1)}, u3^{(2)}, u3^{(3)}, u3^{(4)}]+[v_2^{(1)}, v2^{(2)}, v2^{(3)}, v2^{(4)}]=[c_{32}^{(1)}, c_{32}^{(2)}, c_{32}^{(3)}, c_{32}^{(4)}]=[4,8,10,15]$

Now suppose $[u1^{(1)}, u1^{(2)}, u1^{(3)}, u1^{(4)}]=[0,0,0,0]$,then we have :-

$[u2^{(1)}, u2^{(2)}, u2^{(3)}, u2^{(4)}]=[-4,-1,2,7]$, $[u3^{(1)}, u3^{(2)}, u3^{(3)}, u3^{(4)}]=[-12,-2,5,18]$, $[v1^{(1)}, v1^{(2)}, v1^{(3)}, v1^{(4)}]=[-18,-5,2,12]$,

$[v_2^{(1)}, v2^{(2)}, v2^{(3)}, v2^{(4)}]=[-3,5,10,16]$, $[v3^{(1)}, v3^{(2)}, v3^{(3)}, v3^{(4)}]=[-5,2,7,12]$, $[v4^{(1)}, v4^{(2)}, v4^{(3)}, v4^{(4)}]=[0,3,4,5]$.

The next step we find $=[c_{pq}^{(1)}, c_{pq}^{(2)}, c_{pq}^{(3)}, c_{pq}^{(4)}]-[up^{(1)}, up^{(2)}, up^{(3)}, up^{(4)}]-[vq^{(1)}, vq^{(2)}, vq^{(3)}, vq^{(4)}]$

Therefore we have:-

$[\underline{c}_{12}^{(1)}, \underline{c}_{12}^{(2)}, \underline{c}_{12}^{(3)}, \underline{c}_{12}^{(4)}]=[-15,-8,9,12]$.

$[\underline{c}_{13}^{(1)}, \underline{c}_{13}^{(2)}, \underline{c}_{13}^{(3)}, \underline{c}_{13}^{(4)}]=[-12,-5,2,13]$.

$[\underline{c}_{21}^{(1)}, \underline{c}_{21}^{(2)}, \underline{c}_{21}^{(3)}, \underline{c}_{21}^{(4)}]=[-15,4,15,38]$.

$[\underline{c}_{33}^{(1)}, \underline{c}_{33}^{(2)}, \underline{c}_{33}^{(3)}, \underline{c}_{33}^{(4)}]=[-28,-8,6,25]$.

$[\underline{c}_{34}^{(1)}, \underline{c}_{34}^{(2)}, \underline{c}_{34}^{(3)}, \underline{c}_{34}^{(4)}]=[-23,-4,6,21]$.

Since $[x_{33}^{(1)}, x_{33}^{(2)}, x_{33}^{(3)}, x_{33}^{(4)}]$ has the most negative $[\underline{c}_{pq}^{(1)}, \underline{c}_{pq}^{(2)}, \underline{c}_{pq}^{(3)}, \underline{c}_{pq}^{(4)}]$,it selected as the entering variable, the Process is summarized by plus (+) and minus (-) signs in the appropriate corners in table (3-2), and$[x_{32}^{(1)}, x_{32}^{(2)}, x_{32}^{(3)}, x_{32}^{(4)}]$ is selected as the leaving variable, therefore the final result is given in table (3-3).

Table (3-3)

	FD1	FD2	FD3	FD4	Fuzzy capacity
FS1	[0,2,4,8]	[1,2,4,9]	[0,2,4,8]	[0,3,4,5] [0,2,4,6]	[0,2,4,6]
FS2	[4,8,12,16]	[4,7,9,12] [-5,-1,7,12]	[2,4,6,8] [-6,0,6,12]	[1,3,5,7] [-5,-2,3,7]	[2,4,9,13]
FS3	[0,0,0,0] [1,3,5,7]	[4,8,10,15]	[2,4,6,8] [-5,-1,3,7]	[0,5,7,9]	[2,4,6,8]
Fuzzy demand	[1,3,5,7]	[-5,-1,7,12]	[1,3,5,7]	[1,2,5,7]	[4,10,19,27]

Hence, the optimal fuzzy transportation minimum cost is:-

$$[z^{(1)}, z^{(2)}, z^{(3)}, z^{(4)}]=[21.75,48,69,91.25]\ldots\ldots\ldots\ldots\ldots\ldots(**)$$

CONCLUSIONS

In this paper we have obtained an optimal solution for a fuzzy transportation problem with trapezoidal membership function, where the initial solution obtained by using fuzzy least cost method and ranking method. And the optimal solution obtained by using fuzzy multipliers method.

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