Pitch Mode Downwash Transient Response at High Speed and High Altitude

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ABSTRACT

In this research a study of the downwash effects for high altitudes and high speeds using a Transient Response approach, the longitudinal equation of motion including the effect of downwash in high altitude and speed were solved. It was found that the downwash in at high altitude and low-speed more effected because it's increases in pitch rate and rate of change of downwash velocity due to decreases in lift ,therefore requires greater control surface area and largest deflection angles. It can be adopted in this research to real requirements in the design of flying objects.

الاستجابة الزمنية للانجراف السفلي في الطور الطولي للسرعات والارتفاعات العالية

الخلاصة

تم في هذا البحث دراسة تأثيرات الانجراف السفلي في الارتفاعات والسرعات العالية باستخدام طريقة الاستجابة الزمنية ،تم تحليل معادلات الحركه الطوليه والمتضمنه تأثير الانجراف السفلي للارتفاعات والسرعات العاليه. حيث تبين إن ثاثير الانجراف السفلي اكبر في الارتفاعات العاليه والسرعات البطيئه وذالك بسب زيادة معدل الخطوه ومعدل التغير في سرعة الانجراف السفلي بسبب قلة الرفع، لذا يتطلب اسطح سيطره ذات مساحة اكبر و زوايا انحراف اكبركذلك . ويمكن اعتماد هذا البحث للمتطلبات الحقيقية في تصميم الأجسام الطائرة.

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List of Symbols

Symbol	Definition	unit
C_{x_u}	The change in the force in the X direction due to a change in forward velocity.	-
$C_{x_\propto}$	The change in the force in the X direction due to a change in angle of attack " $\propto$ "	-
$C_{x_{\dot\alpha}}$	The change in the force in the X direction due to rate of change in " $\propto$ " .	-
C_{x_q}	The change in the force in the X direction due to a pitching velocity.	-
C_{m_u}	The change in the pitching moment due to a change in forward velocity.	-
$C_{m_\propto}$	The change in the pitching moment due to a change in angle of attack " $\propto$ "	-
$C_{m_{\dot\alpha}}$	The effect of rate of change in " $\propto$ " on the pitching moment coefficient .	-
C_{m_q}	The effect on the pitching moment due to a pitch rate.	-
$C_{m_{\delta e}}$	The elevator effectiveness.	-
C_{z_q}	The change in the Z force due to a pitching velocity.	-
C_{z_u}	The change in the normal force due to a change in forward velocity (u) .	-
$C_{z_\propto}$	The variation of the Z force with angle of attack.	-
$C_{z_{\dot\alpha}}$	The effect of the rate of change of angle of attack " $\propto$ " on the Z force.	-
C_w	Gravity coefficient	-
m	Mass of flying body.	slug
S	Wing area of flying body.	sq.ft
U	Forward velocity of flying body.	ft/sec
I_y	Moment of inertia about pitch plane OY.	slug ft^2
q	Dynamic pressure.	Ib/sq ft
c	Mean aerodynamic chord.	ft
k	Rough allowance.	-
$\bar{\alpha}$	The variation in the angle of attack from equilibrium.	
θ	Angle of pitch	rad
ϵ	Angle of downwash at tailplane	rad
δe	Angular displacement of elevator	rad
l_t	Distance of aerodynamic center of tailplane aft of c.g. of aircraft	ft
α	Incidence of mean aerodynamic chord of wing	rad
s	Root of characteristic equation	-----
C_L	Flying body Lift Coefficient	------

C_D	Flying body Drag Coefficient	------
A,B,	Constant	------
$S \cdot M$	Static Margin	ft

INTRODUCTION

Principle characteristics of flying body stability depend on the number of design criteria [1]. The most important one, which is considered in this work, is the criteria for longitudinal dynamic stability and response. Several flying qualities parameters such as short and phugoid modes, frequency response largely influenced by aerodynamic parameters changes .downwash is one of these aerodynamic parameter which in the process of longitudinal dynamic stability and control,[2].

The longitudinal dynamic stability requires a complete analysis of aircraft motion following displacement from equilibrium; this will involve a detailed investigation of the A/C equation of motion [3].

The equation of motion in constructed by many parameters such as airplane geometry and aerodynamic forces such as lift ,drag.,[4].There are many factors which influence the amount of aerodynamic lift such as the shape, size, inclination, and flow conditions of the air passing the object. For a three dimensional wing, there is an additional effect on lift, called downwash [5].

The wing tip vortices produce a downwash of air behind the wing which is very strong near the wing tips and decreases toward the wing root. The local angle of attack of the wing is increased by the flow induced by the downwash [6], giving an additional, downstream-facing, and component to the aerodynamic force acting over the entire wing. The downstream component of the force is called induced drag because it faces downstream and has been "induced" by the action of the tip vortices. The lift near the wing tips is defined to be perpendicular to the local flow. The local flow is at a greater angle of attack than the free stream flow because of the induced flow. Resolving the tip lift back to the free stream reference produces a reduction in the lift coefficient of the entire wing, [7].

MATHEMATIC ANALYSIS

Since the aircraft is free to translate in three dimensional spaces as well as to rotate about its center of gravity. Hence six equations of motion are required for solving the six degrees of freedom. The linearized equations of aircraft longitudinal motion after using small disturbance theory and assuming the aircraft as a rigid body are, [7].

$$\left(\frac{m\,U}{Sq}\,\bar{\dot{u}} - C_{x_u}\bar{u}\right) + \left(-\frac{c}{2U}C_{x_{\dot\alpha}}\,\bar{\dot{\alpha}} - C_{x_\alpha}\,\bar{\alpha}\right) + \left[-\frac{c}{2U}C_{x_q}\dot{\theta} - C_w(\cos\theta)\theta\right] = 0 \quad \ldots\ (1)$$

$$-\left(C_{z_u}\bar{u}\right) + \left[\left(\frac{mU}{Sq} - \frac{c}{2U}C_{z_{\dot\alpha}}\right)\bar{\dot{\alpha}} - C_{z_\alpha}\bar{\alpha}\right] + \left[\left(-\frac{mU}{Sq} - \frac{c}{2U}C_{z_q}\right)\dot{\theta} - C_w(\sin\theta)\theta\right]$$
$$= C_{z_{\delta e}}\delta e(s) \quad \ldots\ldots\ldots\ldots\ (2)$$

$$-\left(C_{m_u}\bar{u}\right) + \left(-\frac{c}{2U}C_{m_{\dot\alpha}}\bar{\dot{\alpha}} - C_{m_\alpha}\bar{\alpha}\right) + \left(\frac{I_y}{Sqc}\ddot{\theta} - \frac{c}{2U}C_{m_q}\dot{\theta}\right)$$
$$= C_{m_{\delta e}}\delta e(s) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (3)$$

Where [7].

$C_{x_u} = -2C_D$, $C_w = -\frac{mg}{Sq}$, $C_{z_u} = -2C_L$, $C_{z_{\dot\alpha}} = 2\left(\frac{dC_m}{di_t}\right)^t_{\delta,\alpha}\left(\frac{d\epsilon}{d\alpha}\right)$, $C_{z_\propto} = -\frac{\partial C_L}{\partial\alpha} - C_D$, $C_{z_q} = 2k\left(\frac{dC_m}{di}\right)^t_{\delta,\alpha}$, $C_{m_{\dot\alpha}} = 2\left(\frac{dC_m}{di}\right)^t_{\delta,\alpha}\left(\frac{d\epsilon}{d\alpha}\right)\frac{l_t}{c}$, $C_{m_q} = 2k\left(\frac{dC_m}{di}\right)^t_{\delta,\alpha}\left(\frac{l_t}{c}\right)$

$C_{m_\propto} = (S\cdot M)_\delta\left(\frac{dC_L}{d\propto}\right)^{\propto}_{\delta}$,

Rewrite equations 1, 2 and 3 in Laplace transformation form with neglecting $C_{x_{\dot\alpha}}$, C_{x_q} *and* C_{m_u} *yields*, [7].

$$\left(\frac{mU}{Sq}s - C_{x_u}\right)\bar{u}(s) - C_{x_\propto}\bar{\propto}(s) - C_w(\cos\theta)\theta(s) = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (4)$$

$$-C_{z_u}\bar{u}(s) + \left[\left(\frac{mU}{Sq} - \frac{c}{2U}C_{z_{\dot\alpha}}\right)s - C_{z_\propto}\right]\bar{\propto}(s) + \left[\left(-\frac{mU}{Sq} - \frac{c}{2U}C_{z_q}\right)s - C_w(\sin\theta)\right]\theta(s) = C_{z_{\delta e}}\delta e(s) \quad \ldots(5)$$

$$\left(-\frac{c}{2U}C_{m_{\dot\alpha}}s - C_{m_\propto}\right)\bar{\propto}(s) + \left(\frac{I_y}{Sqc}s^2 - \frac{c}{2U}C_{m_q}\right)\theta(s) = C_{m_{\delta e}}\delta e(s) \quad \ldots. \quad (6)$$

The transfer function for δe input to θ output using determination:-

$$\frac{\theta(s)}{\delta e(s)} = \frac{\begin{vmatrix} \frac{mU}{Sq}s - C_{x_u} & -C_{x_\propto} & 0 \\ C_{z_u} & \left(\frac{mU}{Sq} - \frac{c}{2U}C_{z_{\dot\alpha}}\right)s - C_{z_\propto} & C_{z_{\delta e}}\delta e \\ 0 & \left(-\frac{c}{2U}C_{m_{\dot\alpha}}s - C_{m_{\dot\propto}}\right) & C_{m_{\delta e}}\delta e \end{vmatrix}}{\begin{vmatrix} \frac{mU}{Sq}s - C_{x_u} & C_{x_\propto} & -C_w(\sin\theta) \\ -C_{z_u} & \left(\frac{mU}{Sq} - \frac{c}{2U}C_{z_{\dot\alpha}}\right)s - C_{z_\propto} & \left(-\frac{mU}{Sq} - \frac{c}{2U}C_{z_q}\right)s - C_w(\sin\theta) \\ 0 & \left(-\frac{c}{2U}C_{m_{\dot\alpha}}s - C_{m_{\dot\propto}}\right) & \frac{I_y}{Sqc}s^2 - \frac{c}{2U}C_{m_q} \end{vmatrix}}$$

r ……(7)

In other form

$$\frac{\theta(s)}{\delta e(s)} = \frac{AS^2 + BS + C}{DS^4 + ES^3 + FS^2 + G} \quad \ldots\ldots(8)$$

Where A, B, C, D, E, F, and G are constant determine from flying-body data.
For a unit step input multiply equation (8) by 1/S

$$\frac{\theta}{\delta e}(s) = \frac{AS^2 + BS + C}{s(DS^4 + ES^3 + FS^2 + G)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (9)$$

By using factories method equ. (9) becomes :

$$\frac{\theta}{\delta e}(s) = \frac{AS^2 + BS + C}{s(s+p_1)(s+p_2)(s+p_3)(s+p_4)} \quad \ldots\ldots\ldots (10)$$

Where p_1, p_2, p_3, p_4 imaginary root of equation
by Laplace inverse of equation above yield

$$\frac{\theta}{\delta e}(t) = \alpha_1 + \alpha_2 e^{-p_1 t} + \alpha_3 e^{-p_2 t} + \alpha_4 e^{-p_3 t} + \alpha_5 e^{-p_4 t} \quad \ldots\ldots\ldots (11)$$

as its mention that 〖p_(1,) p〗_(2),p_(3), p_(4) are imaginary roots p_(1), p_(2 =) g_1±h_1 j and

p_(3), p_(4 =) g_2±h_2 j Simplified equation (11) yields:

$$\frac{\theta}{\delta e}(t) = \alpha_1 + (\alpha_2 + \propto_3)\, e^{-g_1 t}\cos(h_1 t) - i(\alpha_2 - \propto_3)\, e^{-g_1 t}\sin(h_1 t) + (\alpha_4 + \propto_5)\, e^{-g_2 t}\cos(h_2 t) + i(\alpha_4 - \propto_5)\, e^{-g_2 t}\sin(h_2 t) \quad \ldots.(12)$$

Therefore:

$$\frac{\theta}{\delta e}(t) = \alpha_1 + 2e^{-g_1 t}(A_1\cos(h_1 t) + A_2\sin(h_1 t)) + 2\, e^{-g_2 t}(A_3\cos(h_2 t) - A_4\sin(h_2 t)) \quad \ldots\ldots (13)$$

by solving all these equations with their aerodynamic coefficients and constants for downwash varying from 0.3 to 0.6 and varying time from (0 -40) sec. The flying body transfer function will be of the form:

RESULTS AND DISCUSSION

The downwash increases at constant altitude and constant speed the static and dynamic stability decreases with no change in control characteristic equation, the decreases in stability due change in flying body characteristic equation and Specifically in E- parameter which present the pitching moment derivatives, Cm_α (static stability) and Cm_μ(dynamic stability),(Fig(2), (3) and (4),Table(1),(2)and(3)).

For constant downwash at different altitude and constant speed the only dynamic stability will be effected(decreasing) with enlarge the control characteristic equation that mean bigger control surface and more deflection angle required and also the flying-body motion characteristic equation enlarge except E-parameter which remain constant and that mean the static stability will not effected but the dynamic stability only effected due to large change in characteristic equation parameters A,B,C and D which present the rate of change of downward velocity and rate of pitch rate due to lift decreases (Fig(5), (6) and (7),Table(1),(2)and(3)).

For constant downwash at different speed and constant altitude the behavior will be entirely different from above because the control surface characteristic equation will decrease it that's mean less control required and also the flying body characteristic equation parameters decreases except the E-parameter which remain constant (no static stability change) and that gives less pitch rate and less rate of downward velocity change and that goes to reason of lift increase (Fig(8), (9) and (10),Table(1),(2)and(3)).

CONCLUSIONS

1. Downwash affects the static stability and the dynamic stability, (Table1,2 and 3),Fig(2)).
2. Downwash gradient becomes more powerful at high altitude and high speed and have great effect of destabilizing the overall flying body motion not only the tail efficiency, (Fig(3)to Fig (5) and Fig(8) to Fig(10)).
3. High altitude downwash required powerful control and more control deflection angle, (Fig(5)to Fig (8)).
4. High speed downwash more to be considered for dynamic stability and its effect more negative than the high altitude , (Table1,2 and 3),Fig(5) and Fig(8)).
5. Constant downwash for high altitude and high speed effect only the dynamic stability, (Table1,2 and 3), (Fig(3)to Fig (5) and Fig(8) to Fig(10)).

REFERENCES

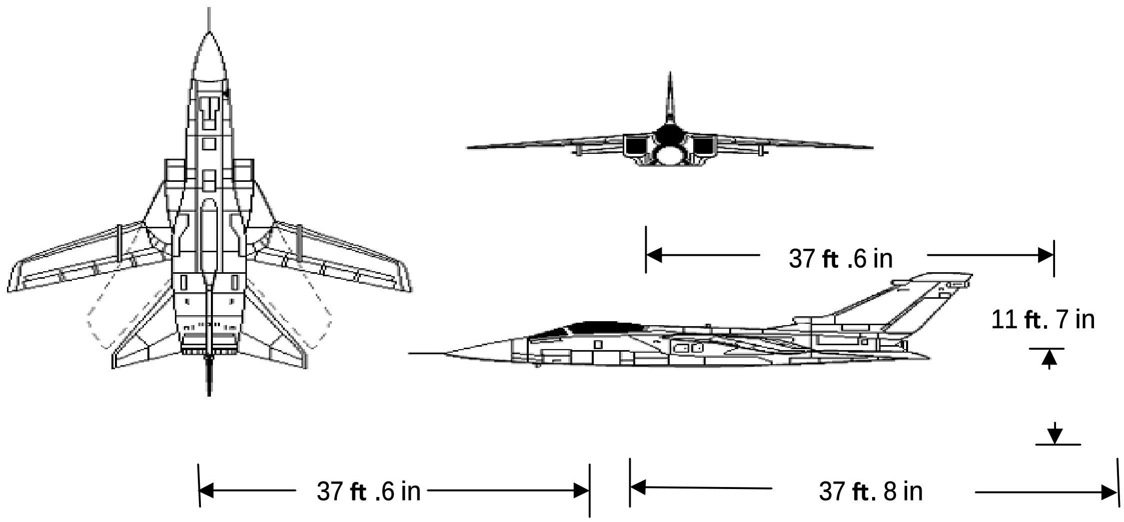
[1]Bandu N.Pamadi.," Performance ,Stability ,dynamic and Control" Amazon 2004.
[2]Jan Roskam,Chuan Eduard Lan., " Airplane Aerodynamic and Performance" Amazon 2004.
[3]Malconlm , J.Abzug., "Airplane Stability and control" 2002.
[4]Thomas R.Yechout,Steven L.Morris., "Introduction to Flight Mehanics" 2003.
[5]Cook, M.V. " Flight Dynamic Principle" Amazon, 2007.
[6]Steven A Brandt. "Introduction to Aeronautics" 2004.
[7]John H.blacklock. "Automatic Control of Aircraft and Missiles " 1991

Table (1) Downwash=0.1 Data for Different Altitude and Speed

			Transfer Function	Short period	Phugoid Period
Downwash = 0.1	Altitude	20000ft	$\frac{28.70091x^2-19.15103x-0.67148}{9.58697x^5+15.22372x^4+41.06668x^3+0.69135x^2+0.96588x}$	$\frac{0.786593413496+14.5242536391i}{x+0.00407778137534-0.153967194975i}$	$\frac{2.54583847017-2.23303799039i}{x+0.78990196405-1.90342885966i}$
		40000ft	$\frac{-134.62860x^2-41.47753x-0.67148}{97.35384x^5+71.38907x^4+178.95830x^3+1.26661x^2+0.96588x}$	$\frac{1.47678266143+152.213658636i}{x+0.00247648252632-0.0736053664041i}$	$\frac{32.3634234114-17.6527920323i}{x+0.364170945043-1.30252584342i}$
		60000ft	$\frac{-918.23380x^2-108.32300x-0.67148}{1733.70700x^5+486.83110x^4+1171.21300x^3+2.98892x^2+0.96x}$	$\frac{-46.0938281812+2752.53911538i}{x+0.00110795396173-0.0287266804737i}$	$\frac{648.730582416-209.338798671i}{x+0.139293826731-0.809140055438i}$
	Speed (ft/sec)	400 ft/sec	$\frac{302.91440x^2-62.21630x-0.67148}{492.85380x^5+240.91770x^4+402.65610x^3+1.89991x^2+0.966x}$	$\frac{7.474215553-770.581379909i}{x+0.00165104375513+0.0490702444897i}$	$\frac{163.841822477-89.3632875872i}{x+0.242759874858-0.868356339436i}$
		600	$\frac{-134.62860x^2-41.47753x-0.67148}{97.35384x^5+71.38907x^4+178.95830x^3+1.26661x^2+0.96588x}$	$\frac{1.47678266143+152.213658636i}{x+0.00247648252632-0.0736053664041i}$	$\frac{32.3634234114-17.6527920323i}{x+0.364170945043-1.30252584342i}$
		800	$\frac{-75.72860x^2-31.10815x-9.67148}{30.80336x^5+30.11982x^4+100.66400x^3+0.94995x^2+0.96588x}$	$\frac{144.28+39.11i}{x+0.003302-0.04981i}$	$\frac{9.93605236975-5.2627293328i}{x+0.4856029651-1.73668908932i}$

Table (2) Downwash=0.367 Data for Different Altitude and Speed

			Transfer Function	Short period	Phugoid Period
Downwash = 0.1	Altitude	20000ft	$\dfrac{28.70091x^2-19.15103x-0.67148}{9.58697x^5+15.22372x^4+41.06668x^3+0.69135x^2+0.96588x}$	$\dfrac{0.786593413496+14.5242536391i}{x+0.00407778137534-0.153967194975i}$	$\dfrac{2.54583847017-2.23303799039i}{x+0.78990196405-1.90342885966i}$
		40000ft	$\dfrac{-134.62860x^2-41.47753x-0.67148}{97.35384x^5+71.38907x^4+178.95830x^3+1.26661x^2+0.96588x}$	$\dfrac{1.47678266143+152.213658636i}{x+0.00247648252632-0.0736053664041i}$	$\dfrac{32.3634234114-17.6527920323i}{x+0.364170945043-1.30252584342i}$
		60000ft	$\dfrac{-918.23380x^2-108.32300x-0.67148}{1733.70700x^5+486.83110x^4+1171.21300x^3+2.98892x^2+0.96x}$	$\dfrac{-46.0938281812+2752.53911538i}{x+0.00110795396173-0.0287266804737i}$	$\dfrac{648.730582416-209.338798671i}{x+0.139293826731-0.809140055438i}$
	Speed (ft/sec)	400 ft/sec	$\dfrac{302.91440x^2-62.21630x-0.67148}{492.85380x^5+240.91770x^4+402.65610x^3+1.89991x^2+0.966x}$	$\dfrac{7.474215553-770.581379909i}{x+0.00165104375513+0.0490702444897i}$	$\dfrac{163.841822477-89.3632875872i}{x+0.242759874858-0.868356339436i}$
		600	$\dfrac{-134.62860x^2-41.47753x-0.67148}{97.35384x^5+71.38907x^4+178.95830x^3+1.26661x^2+0.96588x}$	$\dfrac{1.47678266143+152.213658636i}{x+0.00247648252632-0.0736053664041i}$	$\dfrac{32.3634234114-17.6527920323i}{x+0.364170945043-1.30252584342i}$
		800	$\dfrac{-75.72860x^2-31.10815x-9.67148}{30.80336x^5+30.11982x^4+100.66400x^3+0.94995x^2+0.96588x}$	$\dfrac{144.28+39.11i}{x+0.003302-0.04981i}$	$\dfrac{9.93605236975-5.2627293328i}{x+0.4856029651-1.73668908932i}$



$$m = 5800\ slugs, S = 2400\ sq\ ft, I_y = 2.62x10^6, \frac{l_t}{C} = 2.89$$

Figure(1) Flying body case study views

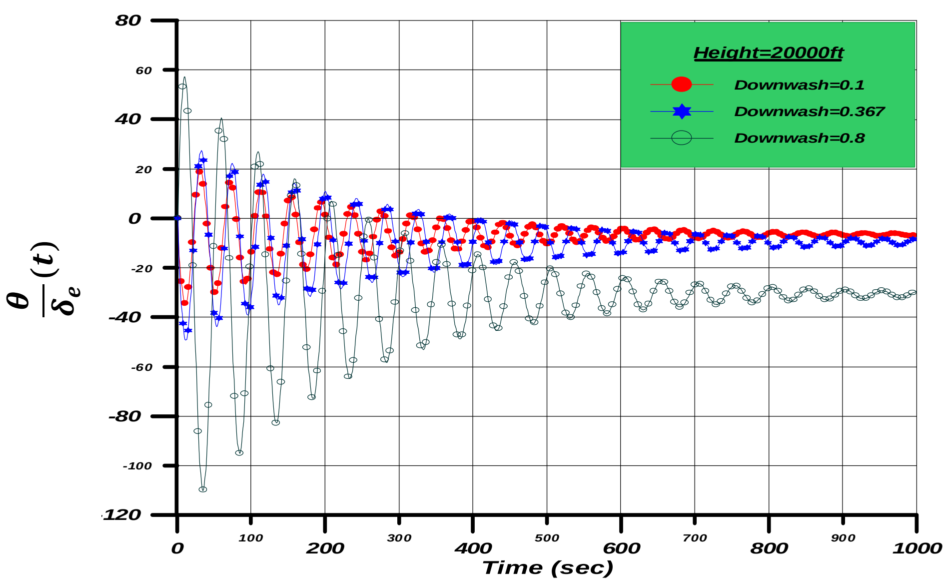


Figure (2) Variation of Downwash with Fix Height(20000ft) and Fixed Speed (600 ft/sec)

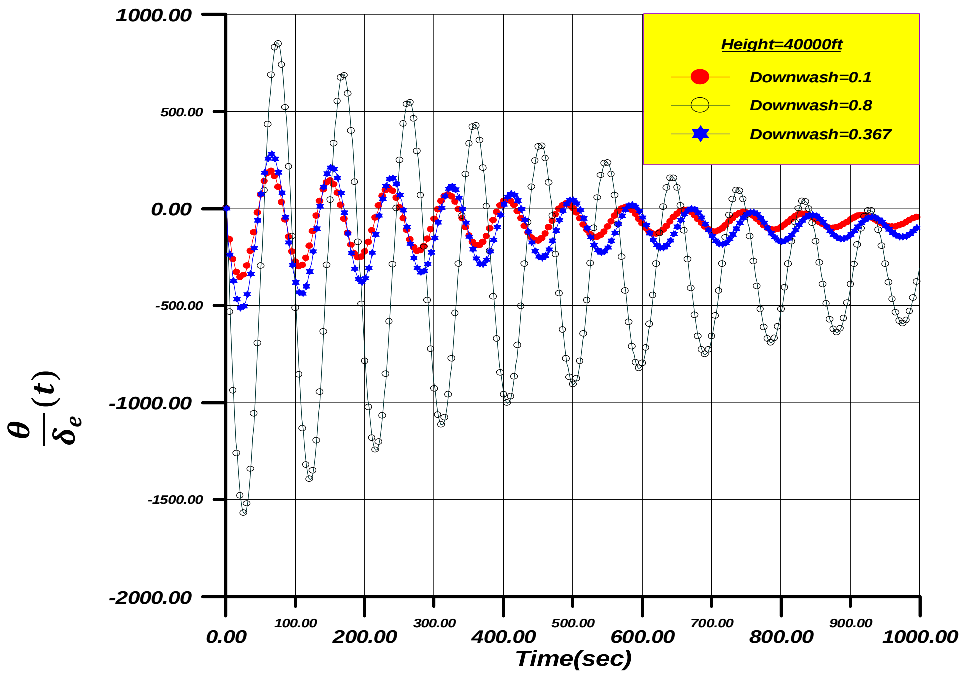


Figure (3) Variation of Downwash with Fix Height(40000ft) and Fixed Speed (600 ft/sec)

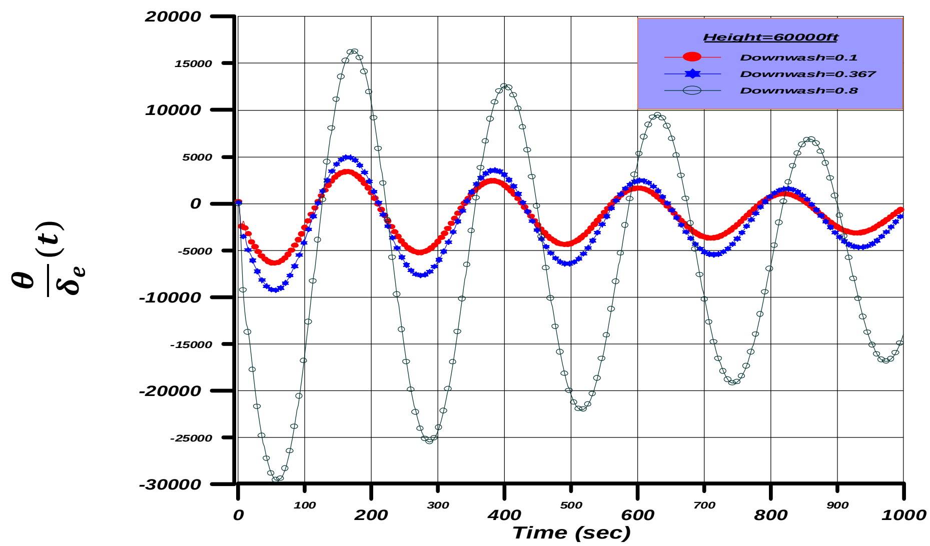


Figure (4) Variation of Downwash with Fix Height(60000ft) and Fixed Speed (600 ft/sec)

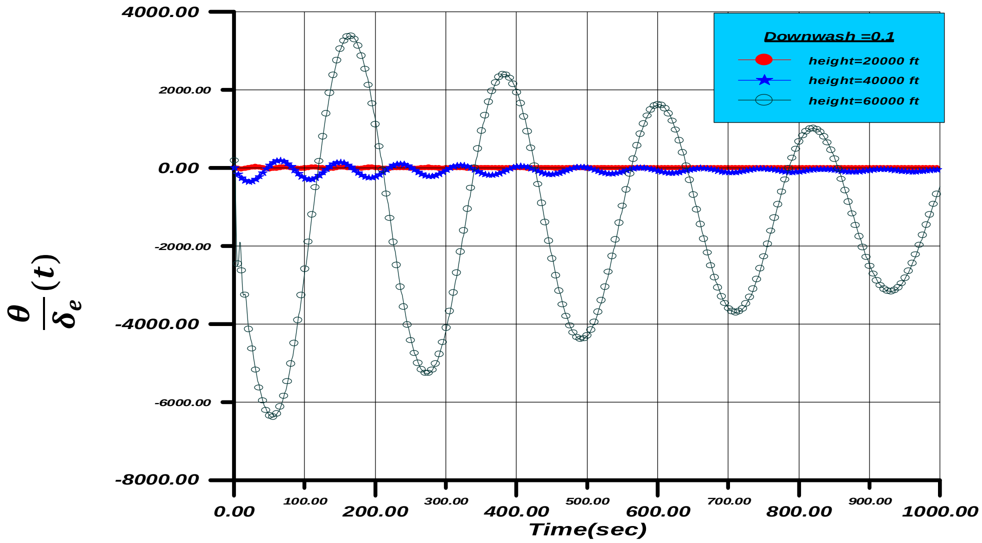


Figure (5) Variation of Height with Fixed Downwash (0.1) and Fixed Speed (600 ft/sec)

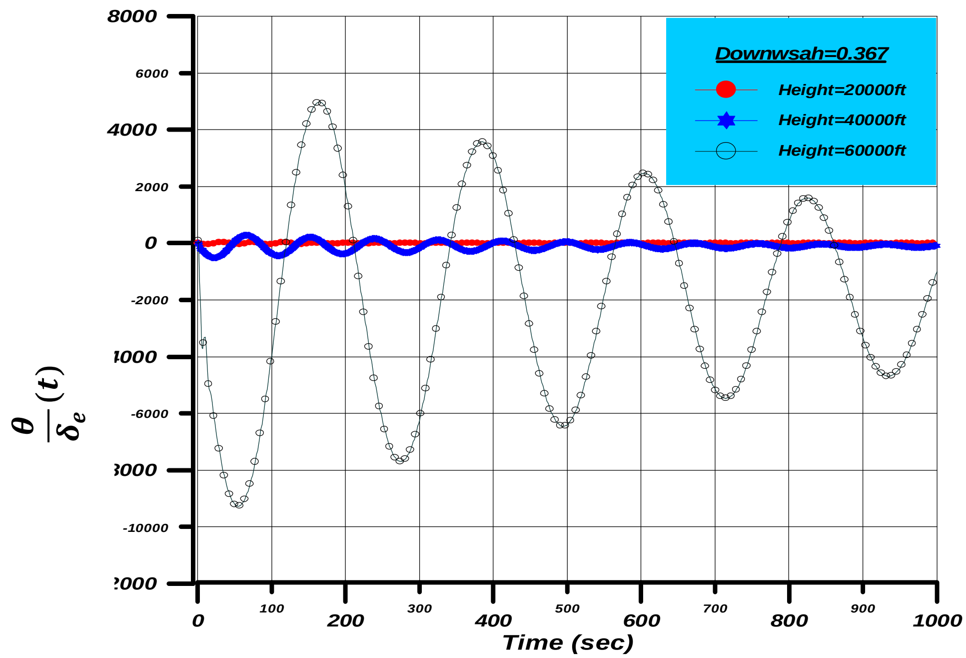


Figure (6) Variation of Height with Fixed Downwash (0.0.367) and Fixed Speed (600 ft/sec)

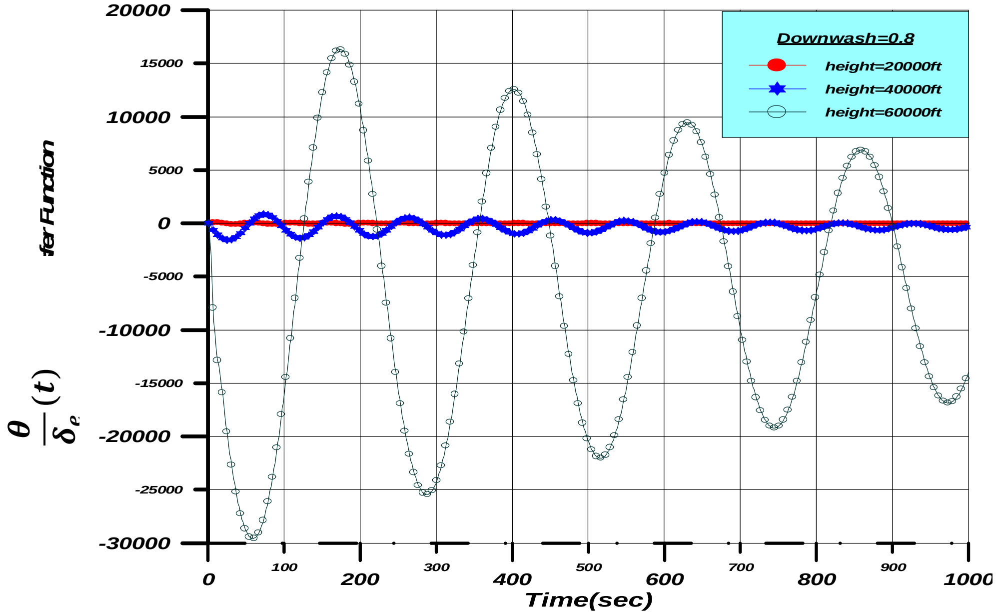
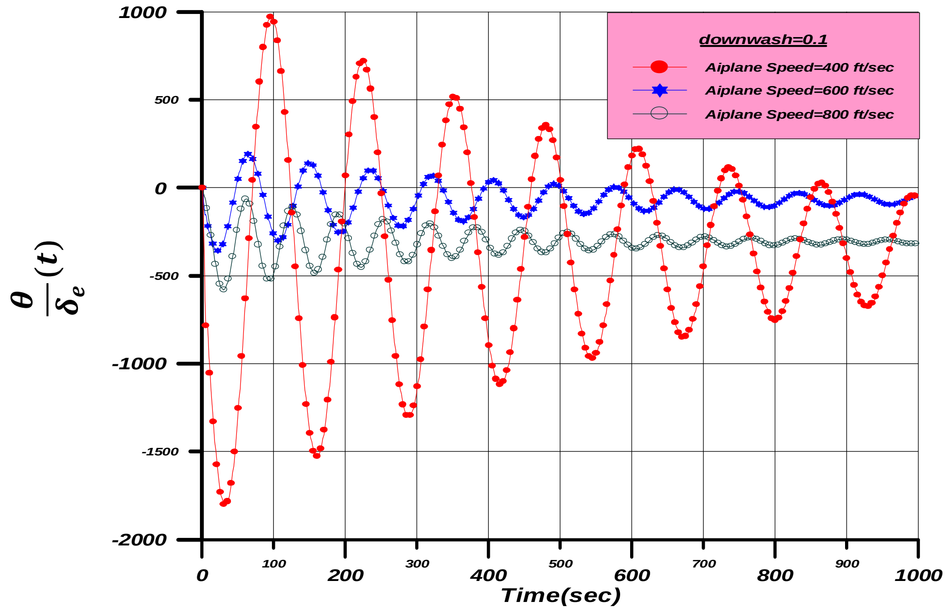
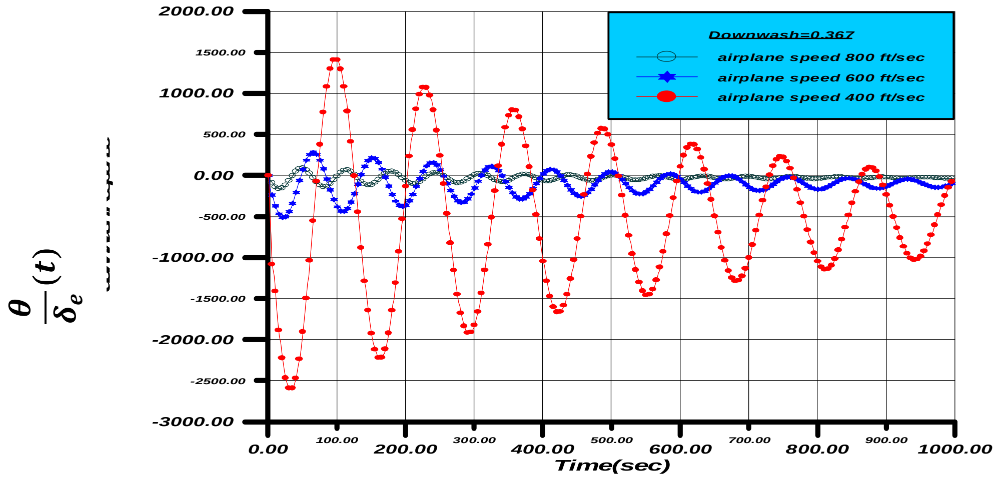


Figure (7) Variation of Height with Fixed Downwash (0.8) and Fixed Speed (600 ft/sec)



Figure(8) Variation of Speed with Fixed Downwash (0.1)and Fix Height(40000ft)



Figure(9) Variation of Speed with Fixed Downwash (0.367) and Fix Height(40000ft)

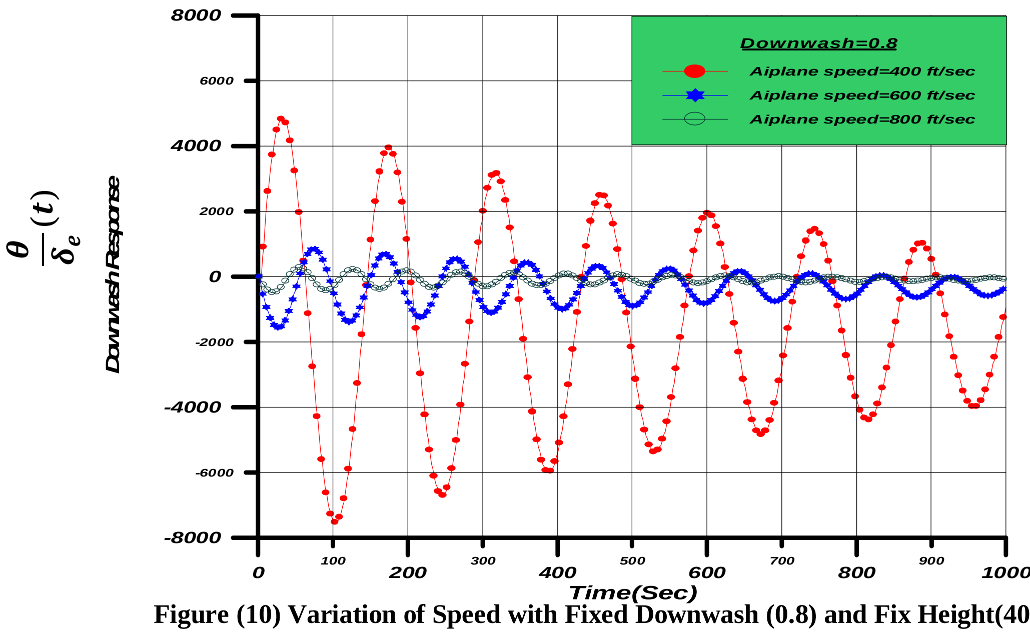


Figure (10) Variation of Speed with Fixed Downwash (0.8) and Fix Height(40000ft)