A modified fixed phase iterative recovery algorithm for restoration of gray-scale blurred images

Dr. Jassim M. Abdul-Jabbar

Department of Computer Engineering, College of Engineering, University of Basrah

Abstract

A novel iterative method for the restoration of gray-scale blurred images is presented. The method is an enhanced modification of the Fixed-Phase Iterative Algorithm (FPIA). A blurred image is enhanced by Laplace operator during the FPIA method on each iteration. This modification is originally supported theoretically by a derivation of some iterative deblurring methods that are based on the enhanced version of the blurred image instead of the blurred image itself only.

The modified fixed phase iterative algorithm (MFPIA) method is examined to restore some Gaussian-and motion-blurred gray-scale images. The restored images via this proposed method are compared with the original FPIA method. From the comparison, it is apparent that the MFPIA method is better from human visual measurements point of view with less number of iterations. In addition to that benefit the restoration by the FPIA method results in images of bad quality even with high number of iterations.

خوارزمية معدلة بتكرار الطور الثابت لاسترجاع الصور المضببة غير الملونة

د. جاسم محمد عبد الجبار

قسم هندسة الحاسبات، كلية الهندسة، جامعة البصرة

الخلاصة

قمت في هذا البحث طريقة تكرارية معدلة لاسترجاع لصور الغير الملونة المضببة مسبقاً. صممت الخوارزمية المستخدمة على اساس خوارزمية استرجاع لصور بطريقة تكرارية مع ثبات الطور. ان التعديل الذي نجري على تلك الخوارزمية التي اساساً على تعزيز الصورة المسترجعة باستخدام معامل لابلاس(Laplace Operator) حيث اصبحت عملية الاسترجاع تعتمد على نمط معزز من الصورة المضببة بدلاً من اعتماد الطريقة على لصورة المضببة فقط.

لقد تم اختبار الخوارزمية على عدد من الصور الغير الملونة المضببة بطريقتي (Gaussian & motion) و قورنت النتائج مع الخوارزمية الاصلية، فاثبتت لها الاكفأ في استرجاع لصور بنوعية جيدة وباقل عدد من المحاولات التكرارية.

1. Introduction

There are many techniques used for the restoration of blurred images. Some of them are used to reduce the blurring effect on an image by enhancing that image by an edge enhancement technique [1], while the other techniques aim to remove the blurring effect by making benefit of prior information about the original image and the blurring process [i.e. the point spread function (PSF) or the frequency response $H(u,v)$] ,(such as, inverse filter or Wiener filter[2]), or by iterative methods when there is in practice no information about the PSF (for example in remote sensing and space imaging the fluctuations in the PSF are difficult to be characterized as a random process, and there is a difficulty in statistically modeling the original image),

The degraded image can be modeled as [2,3]

$$g(n_1,n_2) = f(n_1,n_2) * h(n_1,n_2) + n_b(n_1,n_2)$$

(1)

where

$f(n_1,n_2)$ is the original image function,

$h(n_1,n_2)$ is the impulse response function for blurring,

$n_u(n_1,n_2)$ is the additive noise,

$g(n_1,n_2)$ is the degraded image, and

n_1,n_2 are the spatial domain coordinates with * denotes the convolution operator.
Supposing that there is no noise effect, Eq. (1) can be reduced to [2]

$$g(n_1,n_2) = f(n_1,n_2) * h(n_1,n_2)$$ (2)

so that the purpose of all the restoration or deblurring techniques is to separate the convolution product in order to restore $f(n_1,n_2)$ from $g(n_1,n_2)$. In the absence of the prior information about the image and the blurring process the restoration process here is referred to as the blind deconvolution process [4,5].

Practically an image can be blurred using Matlab6p1 programming language. Images can either be Gaussian or motion blurred using special instructions. The Gaussian blur can be generated by the following filter response

$$h_g(n_1,n_2) = e^{-(n_1^2+n_2^2)/2\sigma^2}$$ (3)

The number of selected pixels and the deviation sigma (σ) can be modified in order to control the Gaussian blurring degree on the image, while in motion blur, the number of selected pixels can be changed to be shifted with an angle of shifting (θ) that is called the shifting direction [6].

This paper presents a new proposed method called the modified fixed phase iterative algorithm (MFPIA) for restoration of gray-scale blurred images and gives a comparison with another known method called the fixed phase iterative recovery algorithm (FPIA) which will be described in section2 of this paper. Section3 presents the proposed method. A comparative study is given in section4. Finally some discussion and conclusions are drawn in section5.

2. The fixed phase iterative algorithm recovery of blurred images

Images can be blurred by different ways. One of the most common ways is the astigmatism of the lens. The following algorithm is based on the fact that the spectrum phase of the astigmatism system is a zero-phase spectrum [7]. In other

words, the phase spectrum of the image remains the same when the image passes through such system. If the blurred image equation is formed as in Eq. (2), then by applying the fast Fourier transform (FFT) to Eq. (2)

$$G(u,v) = F(u,v)H(u,v) \qquad (4)$$

Eq. (4) can be rewritten as

$$|G(u,v)|\exp[j\theta_{c}(u,v)]=|F(u,v)| \\ |H(u,v)|\exp[j\{\theta_{F}(u,v)+\theta_{H}(u,v)\}] \qquad (5)$$

where
$|G(u,v)|$ —magnitude of $G(u,v)$,
$|F(u,v)| =$ magnitude of $F(u,v)$,
$|H(u,v)| =$ magnitude of $H(u,v)$,
$\theta_{G}(u,v)$=phase of $G(u,v)$, $\theta_{F}(u,v)=$ phase of $F(u,v)$,
and
$\theta_{H}(u,v)=$ phase of $H(u,v)$
Obviously,
$$|G(u,v)| = |F(u,v)||H(u,v)| \qquad (6)$$
and
$$\theta_{G}(u,v)=\theta_{F}(u,v)+\theta_{H}(u,v) \qquad (7)$$

Since the astigmatism system has zero phase spectrum then $\theta_{H}(u,v)$ =0 [7], so
$$\theta_{G}(u,v)=\theta_{F}(u,v) \qquad (8)$$

i.e., the phase spectrum of blurred image is the same as that of the original image.

To implement the fixed phase-iterative recovery algorithm we need to convert the image matrix into its one-dimensional (1-D) vector representation [8]. Fig. 1 shows the flow chart of the fixed phase

iterative algorithm. In the fixed phase iterative algorithm flow chart , p is the iteration step, $f_{p+1}(n)$ is the 1-D untruncated restored sequence, $\hat{f}_{p+1}(n)$ is the truncated sequence and L is the length of the 1-D sequence $g(n)$.

From Fig. 1 there are two key sections in each step of the iterative reconstruction:- phase replacing and time domain truncation. With respect to the whole procedure, three main substeps are taken to accomplish the reconstruction process. First, by combining $|F_0(k)|$ with the preknown phase spectrum $\theta_G(k)$, the first estimated value of $F_1(k)$ can be obtained as

$$F_1(k)=|F_0(k)|\exp[j\theta_G(k)] \qquad (9)$$

After that taking the FFT^{-1} to $F_1(k)$, $f_1(n)$, the first estimated value of $f(n)$, is then obtained.

In the second substep, $f_1(n)$ which is an M-point length time domain sequence (M>2L), must be truncated into an L-point length with

$$\hat{f}_p(n)= \begin{cases} f_r(n) & 0\leq n \leq L\text{-}1 \\ 0 & L\leq n \leq M\text{-}1 \end{cases}$$
$$(10)$$

for $p=1,2,\ldots\ldots$
This substep is called "time domain truncation". Finally, by taking the FFT of $\hat{f}_1(n)$, $\hat{F}_1(k)$ will be obtained. Therefore, by replacing its phase spectrum with the preknown phase $\theta_G(k)$ and combine it with $|\hat{F}_1(k)|$, $F_2(k)$ is then obtained as

$$F_2(k) = \mid \bar{F_1}(k) \mid \exp[j\theta_c(k)] \qquad (11)$$

This substep is called "phase-spectrum replacing". Then by taking FFT^{-1} of $F_2(k)$, $f_2(n)$ can be obtained and it will come back to the second step and with a new loop begins again.

It must be noticed that M, the length of the FFT & FFT^{-1} should be greater than 2L to ensure that the recovery is done perfectly. After the whole iteration algorithm is accomplished, $\hat{f}(n)$ will be converted to $\hat{f}(n_1, n_2)$ by taking the inverse vector transform, where $\hat{f}(n_1, n_2)$ is the clear recovery image matrix. It should be noted that this algorithm requires from 20 to 50 iteration to give a good restoring image [7], which indicates the lower speed property of this algorithm. Fig. 2 shows an example on image restoration using the FPIA with 46 iterations.

3. The proposed method

In this section the proposed method will be presented. The basic idea of formulation depends on two concepts, one that is introduced by Zho Ren Feng and Zhou Hui in the fixed phase iterative recovery algorithm (FPIA) of blurred images [7] which states that the phase spectrum of the original clear image is the same as that for the blurred image. The other fact was presented in the derivation and analysis of Slepian method or Sondhi method [9], which states that the restoration of a blurred image can be implemented by using its derivative.

The proposed method combines these two concepts in a modified way in order to deblur the blurred image. That is why we call it the modified fixed phase iterative algorithm (MFPIA). The algorithm supposes that there is no noise effect $n_0(n_1, n_2) = 0$, so that again

$$g(n_1, n_2) = f(n_1, n_2) * h(n_1, n_2) \text{ for } 0 \le n_1, n_2 \le N \qquad (12)$$

The algorithm is carried out by implementing the following steps:-

1. for the first iteration, set
$$f_0(n_1, n_2) = g(n_1, n_2) \qquad (13)$$

2. suppose that p is the time of the iteration and for the sake of algorithm simplicity, $f_p(n_1, n_2)$ is converted initially to 1-D form $f_p(n)$ where $0 \le n \le N^2$ by using vector transformation.

3. convert $f_p(n)$ to its frequency domain representation $F_p(k)$ i.e. magnitude & phase using the fast Fourier transform (FFT). The length of FFT and FFT^{-1} must be greater than $2N^2$ to ensure that the recovery is done perfectly so

$$F_p(k) = FFT[f_p(n)] \text{ for } 0 \le K \le 2N^2 \qquad (14)$$

It should be noted here that for the 1st iteration, since

$f_0(n_1, n_2) = g(n_1, n_2)$, then the transformed version will be

$F_0(k) = G(k) = FFT[g(n)]$

for $0 \le k \le 2N^2$ \qquad (15)

In magnitude and phase forms

$$F_p(k) = \mid F_p(k) \mid \exp[j\theta_{F_p}(k)] \qquad (16)$$

and

$$G(k) = |G(k)| \exp[j\theta_G(k)] \quad (17)$$

4. the phase replacing process taking place here is given by

$$\theta_G(k) \longrightarrow \theta_{F_p}(k)$$

for all p values then the new sequence $F_{p+1}(k)$ is obtained as follow:

$$F_{p+1}(k) = |F_p(k)| \exp(j\theta_G(k)) \quad (18)$$

5. now applying FFT^{-1} to $F_{p+1}(k)$, $f_{p+1}(n)$ can be obtained, and since $f_{p+1}(n)$ is a $2N^2$-point length it must be truncated into an N^2-point length. The resulting truncated sequence can be obtained by the following time truncation process:

$$\hat{f}_{p+1}(n) = \begin{cases} f_{p+1}(n) & 0 \le n \le N^2-1 \\ 0 & N^2 \le n \le 2N^2-1 \end{cases} \quad (19)$$

6. using the inverse vector transform, the sequence $\hat{f}_{p+1}(n)$ can be back transformed to $\hat{f}_{p+1}(n_1, n_2)$.

7. then the derivative operation is performed by using an edge detection operator (e.g. Laplacian operator), thus $\hat{f}'_{p+1}(n_1, n_2)$ is obtained.

8. the restored image at iteration p+1 is obtained by adding the transformed sequence $\hat{f}_{p+1}(n_1, n_2)$ and its

derivative $\hat{f}'_{p+1}(n_1, n_2)$. Thus the final sequence is obtained as

$$f_{p+1}(n_1, n_2) = \hat{f}_{p+1}(n_1, n_2) + \hat{f}'_{p+1}(n_1, n_2) \quad (20)$$

9. if the restored image quality is not good the algorithm is repeated from step 3.

Fig. 3 shows the proposed method flow chart. Fig. 4 shows an example on image restoration using this method.

4. A comparative study

For comparison, a specific gray-scale image (Buffy) is selected with 256×256 spatial resolution. This image is blurred by two methods using Gaussian blur and motion blur. For Gaussian blur, $\sigma = 2$ and the number of selected pixels =3 [see Eq. (3)], while for motion blur the number of the shifted pixels =5, with a shifting angle $\theta = 0$. The results are shown in Figs. 5 & 6 respectively. From these two figures it can be seen that the MFPIA gives better results compared to the FPIA with less number of iterations.

5. Discussions & Conclusions

A new method for gray-scale image deblurring has been presented in this paper. This new method is a modification of the FPIA method it is called the modified fixed phase iterative algorithm (MPFIA). The modification is a mixed idea between the FPIA using the derivative of the blurred image that is used in the restoration of images by Slepian method. This has made

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the proposed method depend on an enhanced version of the blurred image instead of the original blurred one only.

The algorithm for the proposed method is given. The proposed method has been applied to gray-scale images that had been originally blurred by using two methods:- Gaussian blur method and Motion blur method.

The FPIA algorithm has been selected for comparison. It is evident from the results that the MFPIA method gives better performance than the FPIA method with few number of iterations. It should be noted that the FPIA method has a very little effect especially on hard blur images even with large number of iterations which is not the case with this MFPIA method.

An important advantage of this MFPIA method is that all what we need to restore a blurred image is the blurred image itself, i.e., the prior knowledge of the PSF and the original unblurred image are not required here as in some other methods such as Wiener method and Richardson-Lucy method [2,10]. This advantage makes the proposed method much more powerful in practical applications than those methods. The other advantage of the proposed method is that a final restored image of good quality can be achieved with less number of iterations as compared to other iterative methods, which means the MFPIA method is better from the speed of convergence point of view.

6. References

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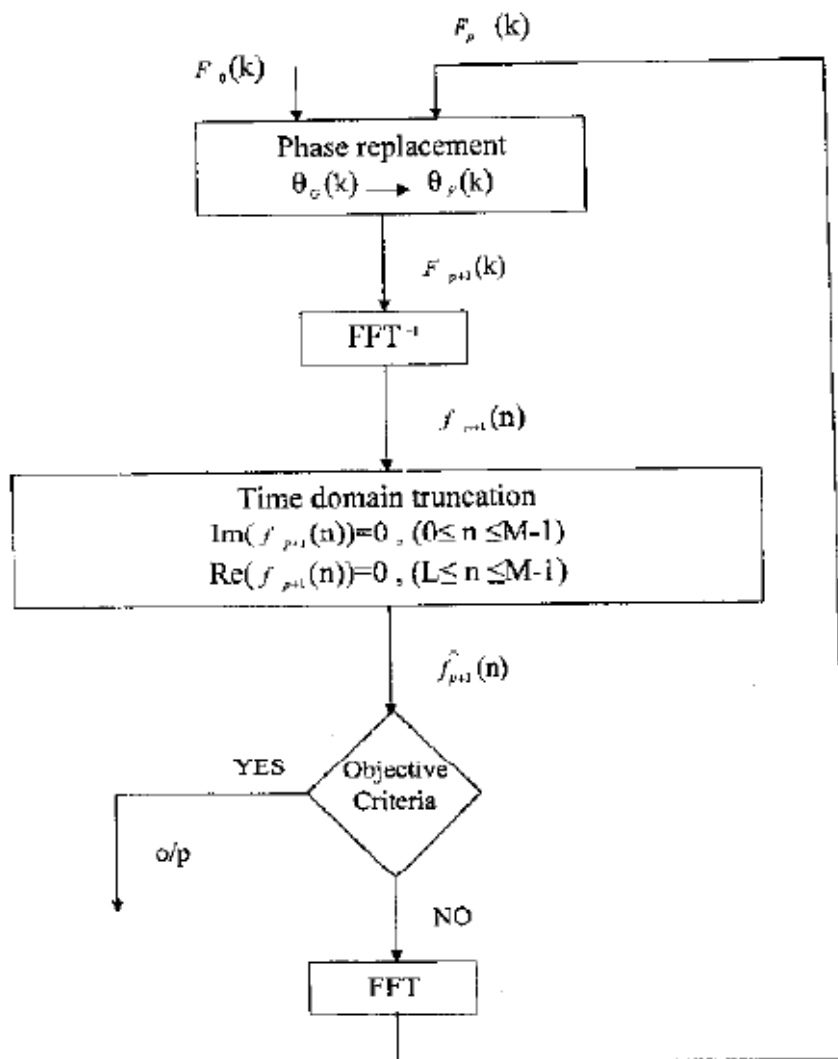http://jstarck.free.fr/pasp02.pdf
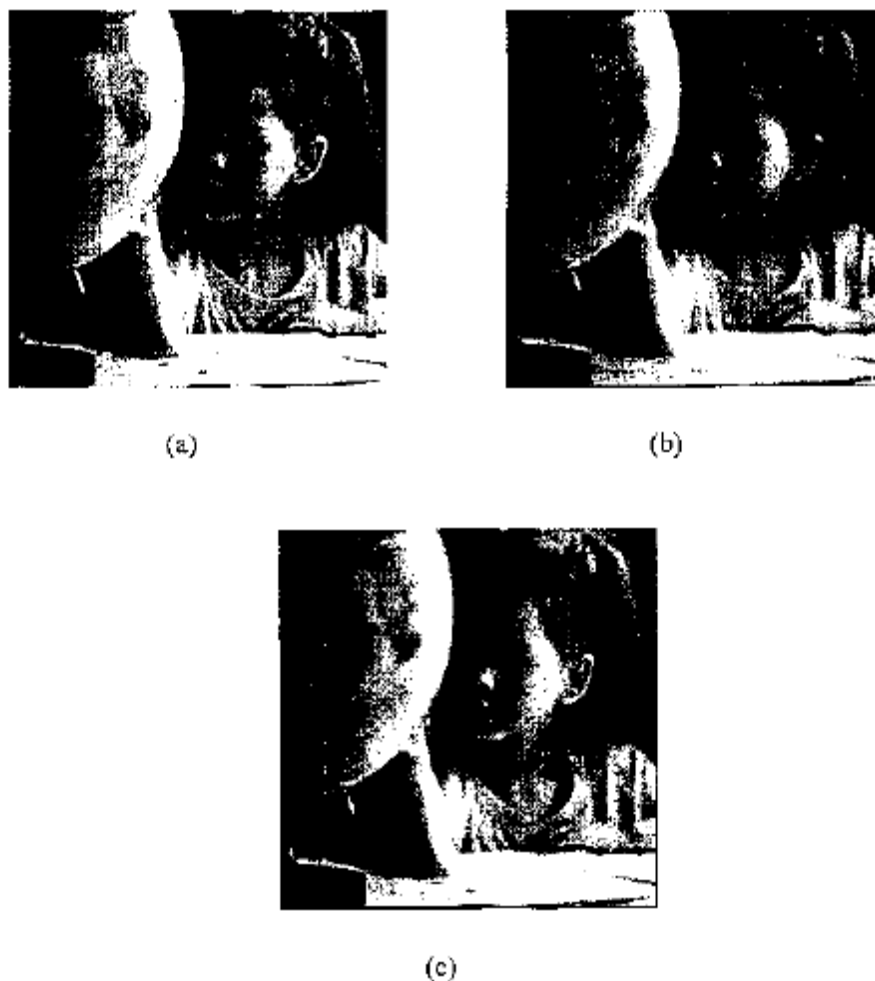
Fig. 1 The FPIA flow chart

(a)

(b)

(c)

Fig. 2 An example on FPIA method
(a) Original image
(b) Gaussian blurred image
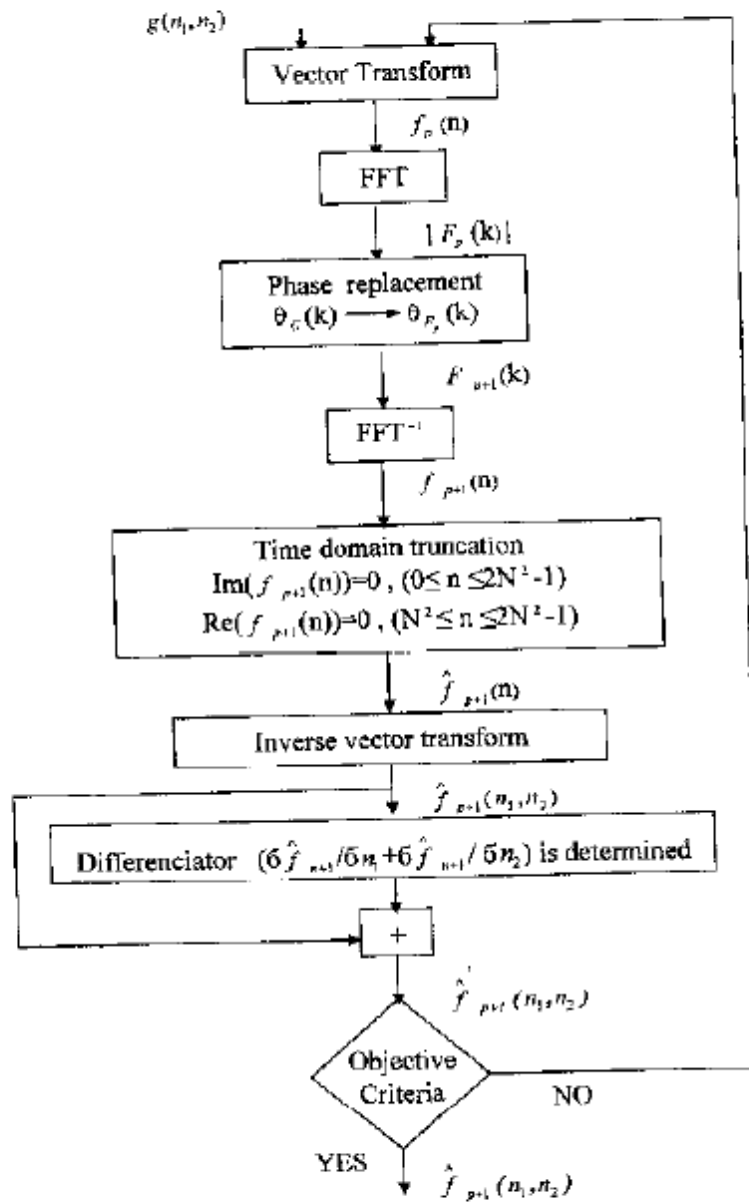(c) Restored image after 46 iterations

Fig. 3 The MFP1A method flow chart
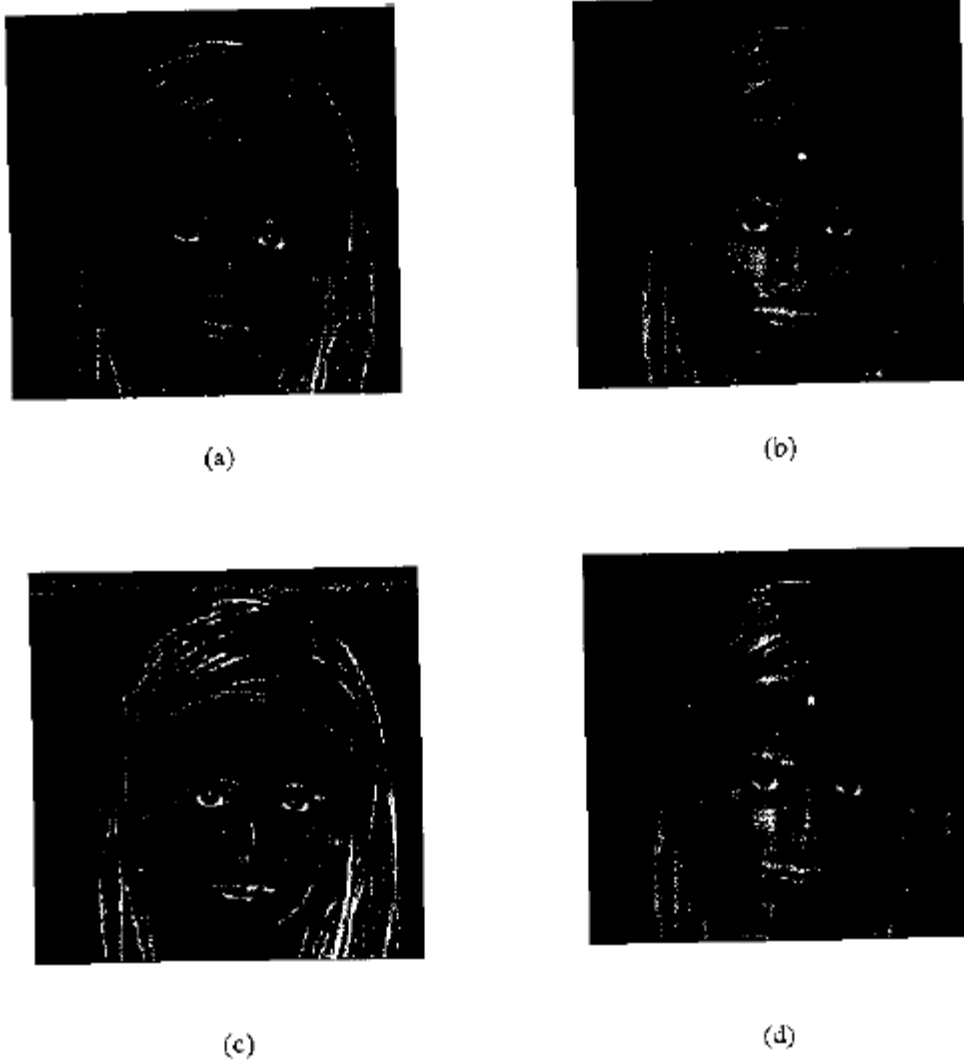
(a)

(b)

(c)

Fig. 4 An example on MFPIA method
    (a)   Original image
    (b)   Gaussian blurred image
    (c)   Restored image after 3 iterations

(a)

(b)

(c)
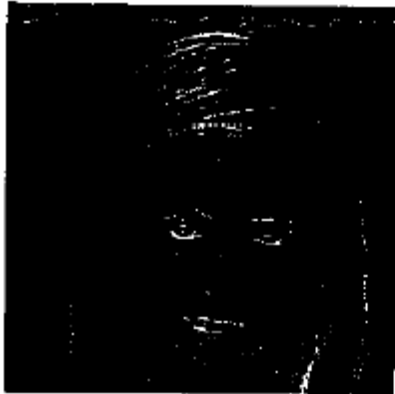
(d)

Fig. 5 Buffy image restoration
(a) Original image
(b) Gaussian blurred image
(c) Buffy image restoration using the proposed method after 2 iterations

(d) Buffy image restoration using FPIA after 50 iterations

(a)
(c)

(b)
(d)

Fig. 6 Buffy image restoration

    (a)   Original image
    (b)   Motion blurred image
    (c)   Buffy image restoration using the proposed method after 2 iterations
    (d)   Buffy image restoration using FPIA after 50 iterations