Dynamic Analysis of Offshore Structures with the Effect of Soil-Structure Interaction

Dr. Anis A. Mohamad Ali*   Mr. Mohamad J. K. Essa*

Mr. Samoel M. Saleh*

Abstract

In the present study, the dynamic analysis of jacket type offshore structures under the action of sea waves is carried out. The finite element method is adopted for the solution of the problem. The effect of soil-structure interaction on the dynamic behavior of the offshore structure is taken into account due to the deformations of the soil caused by the motion of the structure, which in turn modify the response of the structure. The supporting elastic foundation is represented by Winkler type model having normal and tangential moduli of subgrade reaction. These moduli may be constant or varying linearly or nonlinearly along the embedded length of the piles that support the offshore structure. The pile tip conditions are also considered. A time domain solution is recommended. The generalized Morison's equation is used to calculate the wave forces and Airy's linear theory to describe the flow characteristics. Both free and forced vibration analyses are studied. The dynamic response has been obtained by modal analysis in conjunction with Wilson-θ method. As an example, a modified model of an actual jacket type offshore platform is analyzed under the action of wave forces.

Introduction

One of the main loading for which offshore structures are designed is caused by extreme water waves generated during intense, storms. The dominant periods of such waves are typically much longer than the fundamental periods of most fixed offshore structures and therefore static analysis is usually sufficient for obtaining the design response of these structures to extreme waves.

Due to the development of offshore oil and gas industries and moving into deeper water, a large number of platforms are being installed in the marine environment that respond more dynamically to extreme water waves. Prediction of the dynamic response of such structures in extreme sea states is therefore a primary design consideration [1].

Many offshore structures are analyzed for dynamic response by assuming that the supporting foundation is a rigid medium. However, these offshore structures are relatively flexible and rest or embedded in flexible medium, then the increased system flexibility may significantly alter the predicted dynamic response of the structure. Therefore, the foundation soil-structure interaction is important in the dynamic analysis of such systems.

In modeling the interaction between the soil and the embedded portion of the structure, different approaches may be adopted ranging from the simplified lumped spring representation to the use of three-dimensional finite element or boundary element techniques in which the foundation is taken as a continuum. In order to simplify the solution and to economize the computations, the foundation can be treated as an elastic Winkler type.

In the present work, a finite element procedure is used to predict the dynamic

* Department of Civil Engineering, University of Basrah

analysis for an offshore platform model represented as a space frame supported by long piles embedded in an elastic Winkler foundation having normal and tangential moduli of subgrade reactions. These moduli of subgrade reactions can be variable with depth along the piles. The pile tip conditions are also considered. Free vibration analysis of this model has been made on the basis of consideration the lumped mass technique. Modal analysis has been carried out of this model to determine the dynamic response. The variation of natural frequency with the soil modulus, the variation of natural frequency with the embedded pile length and the time variation of the deck displacement of the model have been obtained.

Mathematical Formulation

The offshore platform model that has been analyzed to study the free vibration characteristics is shown in Figure (1). The dynamic response of the same model due to a regular wave has also been obtained. The mathematical formulation of the platform model that is idealized as prismatic linearly elastic frame elements is briefly indicated below.

Free Vibration Analysis

The undamping free vibration equation for the framed structure is given by: -

$$[K]\{\delta\} - \omega^2 [M]\{\delta\} = \{0\} \qquad \text{------(1)}$$

where [K] is the overall stiffness matrix of the structure, [M] is the overall mass matrix of the structure, ω is the natural circular frequency and {δ} is the nodal displacement vector.

[K] and [M] have been formed by assembling the respective element matrices. The analysis has been carried out on the basis of the lumped mass formulation, in which the mass of the structure is lumped at its nodal points depending on the distribution of the mass. Moments of inertia of the member s mass are ignored.

The solution of the eigenproblem, as given in Equation (1), results in a polynomial equation of degree N in ω^2 for a structural system having N degrees of freedom. This polynomial is known as the characteristic equation of the structure. The N roots of this polynomial, which are called the eigenvalues, represent the natural frequencies that correspond to the N modes of vibration which are possible in the structural system. This solution is obtained by using the subspace iteration method [2].

Forced Vibration Analysis

Modal analysis has been made in the determination of the dynamic response. The wave forces that are acting on the members of an offshore structure are normally computed by using Morison's equation, the most widely used approach in this regard. This equation is originally developed to compute the hydrodynamic forces acting on a cylinder at right angles to the steady flow and is given as: -

$$dF = \rho \pi \frac{D^2}{4} C_m \, a \, ds + \frac{1}{2} \rho D C_d \upsilon |\upsilon| ds \qquad \text{---(2)}$$

by which, it is possible to determine the wave forces on the vertical distance ds of the cylinder due to the velocity (υ) and acceleration (a) of the water particle, and ρ is the density of water, D is the cylinder diameter and C_m and C_d are inertia and drag coefficients [3].

Various methods exist for the calculation of the hydrodynamic loads in arbitrary oriented cylinder by using Morison's equation. The method adopted here is to assume that only the components of water particle velocity and acceleration vectors normal to the member produce loads (4). For calculating the nodal forces due to the hydrodynamic loading, each element is divided equally into two parts. By applying the generalizing form of Equation (2), the wave force is calculated at each node of the frame element, then this force is assumed to act uniformly over the nearest half part of the element to each node and its effect is then converted to nodal forces.Linear wave theory has been assumed in order to calculate the wave particle characteristics (velocity and acceleration),[5]. The interaction between the fluid and the structure has not been considered.

The equations of motion of the offshore platform model due to the wave forces are: -

$$[M]\{\ddot{\delta}\} + [C]\{\dot{\delta}\} + [K]\{\delta\} = \{F(t)\} \quad \text{-------} (3)$$

It is assumed that

$$\{\delta\} = [\Phi]\{\xi\} \quad \text{-------}(4)$$

where $[\Phi]$ is the mode shapes matrix of the structure that results from the free vibration analysis and $\{\xi\}$ is a time dependent vector.

Premultiplying both sides of Equations (3) by $\{\phi_r\}^T$, the transpose of the modal shape vector corresponding to the rth natural frequency, and subsitituting Equation (4) into Equation (3) and using the orthogonality relationship, then Equation (3) is written as [6]: -

$$\ddot{\xi}_r + 2\zeta_r\omega_r\dot{\xi}_r + \omega_r^2\xi_r = \frac{f_r}{m_r} \quad \text{------------}(5)$$

where;

$$m_r = \{\phi_r\}^T[M]\{\phi_r\}$$

$$f_r = \{\phi_r\}^T\{F(t)\}$$

The equation of motion (5) is corresponding to the rth degree of freedom and then to the rth mode of vibration. Similarly N independent equations of motion corresponding to N degrees of freedom can be derived in the same manner that describe the dynamic response of the entire structural system. These uncoupled equations have been solved by using the Wilson-θ method [7].

Finally, after the modal equations are solved, the time dependent nodal displacements vector $\{\delta\}$ which is considered in the analysis will be calculated from equation (4). Once the nodal displacements are obtained, the member forces are determined using the standard procedure.

Soil-Structure Interaction

The part of the offshore structure (supported piles) that is embedded in a Winkler elastic foundation is represented by linear elastic 2-node frame finite elements of uniform cross section having one meter length and supported by continuous elastic Winkler foundation having normal and tangential moduli of subgrade reaction. The variation of soil rigidity with depth is introduced by varying these moduli of subgrade reactions.

There are several distributions of the moduli of subgrade reactions with depth, the most widely used being that developed by Plamer and Thompson [8], which is of the form: -

$$K_i = K\left(\frac{x}{L}\right)^n \quad \text{---------}(6)$$

where K_i is the normal or tangential modulus of subgrade reaction, K is the value of K_i at the tip of pile (x=L) and (n) an empirical constant depending upon the type of soil.

Several assumptions are adopted about the value of the index (n). For a cohesive soil, it is commonly taken as zero, that is the moduli are constant with depth. Davisson and Prakash [9] suggested, however, that (n=0.15) is more realistic for this type of soil in order to take into account the effect of some allowance for plastic soil behavior at the surface. These two patterns of soil reactions variation are shown in Figure (2).

The moduli of subgrade reaction can be found theoretically or experimentally. The modulus of normal subgrade reaction can be given as [10]: -

$$K_n = \frac{E_s}{d(1-\mu_s^2)} \quad \text{-----------}(7)$$

Whereas, the modulus of tangential subgrade reaction and end bearing modulus are given as [11]: -

$$K_t = \frac{E_s}{8d(1-\mu_s^2)} \quad \text{--------}(8)$$

$$K_e = \frac{E_s}{2d(1-\mu_s^2)} \quad \text{--------}(9)$$

These expressions of moduli of subgrade reactions and their distributions with depth are considered in the present study.

Case Studies

The numerical investigation of an offshore platform model as shown in Figure (1) is made. A typical cross section and an elevation of the structure are also shown. The member properties are summarized in Table (1). Four steel piles one at each main leg support the structure. These piles are driven to a depth of (60 m) below the mudline in the seabed. The whole structure, including the supported piles, is discretized into (428) frame elements and (303) nodes. The platform deck is modeled as a pyramid and has a total mass of ($11.2 * 10^6$ kg).

The fundamental sway, bending, torsion and axial mode shapes that resulted from the free vibration analysis of the model are shown in Figures (3) and (4). These four modes are considered to estimate the effect of the variation of the soil rigidity, represented by the normal soil modulus, (K_n), on the natural circular frequencies corresponding to these modes and taking into account the case of the variation of the soil reactions with depth (i.e. n=0 and n=0.15). These effects are shown in Figures (5) and (6). The natural frequencies that correspond to all modes under study are increased significantly with the soil rigidity. Moreover, the natural frequencies of the axial and bending modes are affected higher than those corresponding to sway and torsion modes by the soil rigidity. It is also noted that the natural frequency is affected by the soil rigidity when the soil reactions are constant with depth (n=0) higher than that when the soil reactions are distributed nonlinearly with depth (n=0.15).

Figures (7) and (8) show the variation of the fundamental natural frequency with the embedded pile length considering the effect of pile tip conditions. The two patterns of the soil reactions with depth are also considered. It is noted that the pile tip conditions are unimportant for long piles but are influential for short piles, especially in loose soil. The effect of the variation of soil reactions with depth is clearly shown.

For studying the dynamic response of the platform model, the following parameters have been considered.

wave height =21 m
wave period = 12 sec
wave length = 225 m
water depth = 115 m
water density = 10.25 kN/m^3
C_d = 0.8
C_m = 2.0

The assumed amount of viscous modal damping ratio (5%) for all modes of vibration considering that (2%) as a hydrodynamic damping, whereas the remaining (3%) simulates energy dissipation from sources other than hydrodynamics [12].

The variation of deck displacement over three wave periods to reach the steady state has been shown in Figure (9). It can be seen that the response of the deck displacement when the soil reactions are varied nonlinearly with depth (n=0.15) is higher than that when the soil reactions are constant (n=0).

The various loading effects as the displacement, axial force and bending moment in the deck, and the axial force and bending moment at the seabed, and their effect by the soil foundation flexibility are given in Table (2). As shown, the loading effects at the seabed are affected by the increase of the structural flexibility more than in the deck, except the deck displacement that is significantly increased with the increase of the foundation flexibility.

Conclusion

Based on an offshore platform model, free vibration characteristics and its dynamic response have been determined. The inclusion of the effect of soil-structure interaction in the dynamic analysis shows that the overall dynamic responses of the offshore structure may be very sensitive to the variation of the foundation characteristics. The boundary conditions at the bottom tip of piles affect the natural circular frequencies, especially for small piles in loose soils. The moduli of soil reactions and their distributions with depth affect significantly the natural frequencies of the structure, especially that corresponding to the bending and axial modes of vibration. The response of the parts of the structure above the ground level (seabed) are less affected by the variation of the soil characteristics than those embedded in the soil.

Notations: -

a: water particle acceleration.
C_d: drag coefficient.
C_m: inertia coefficient.
D: diameter of the frame element.
d: diameter of the pile.
E_s: elastic modulus of the soil.
{F(t)}: time varying force.
[K]: stiffness matrix.
K_n: modulus of normal soil reaction.
K_t: modulus of tangential soil reaction.
K_e: modulus of the end bearing spring.
L: pile length.
[M]: mass matrix.
υ: water particle velocity.
{δ}: nodal displacements vector.
ρ: density of water.
[Φ]: mode shapes matrix.
ζ: damping ratio.
{ξ}: time dependent vector.
ω: natural circular frequency.
μ_s: Poisson's ratio of the soil.

References: -

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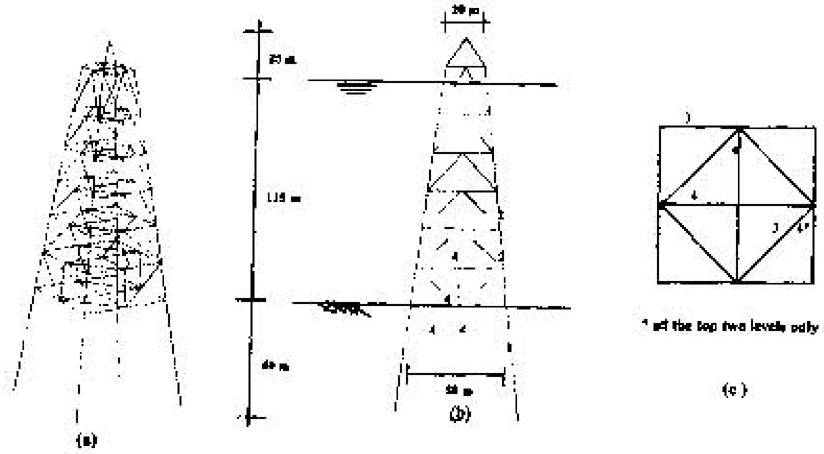

Figure (1): Sketch of Offshore Platform Model for the Case Study.
(a) Space view. (b) An elevation. (c) Typical cross section.

Table (1): Definition of member properties (tubes)

Member	1	2	3	4
Outer Diameter (m)	2.0	1.525	0.91	0.75
Thickness (mm)	55	38	25	25

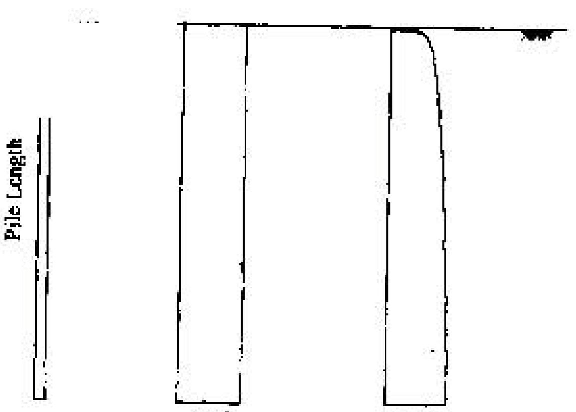

Figure (2) Patterns of Soil Reaction Variation Along the Pile Length

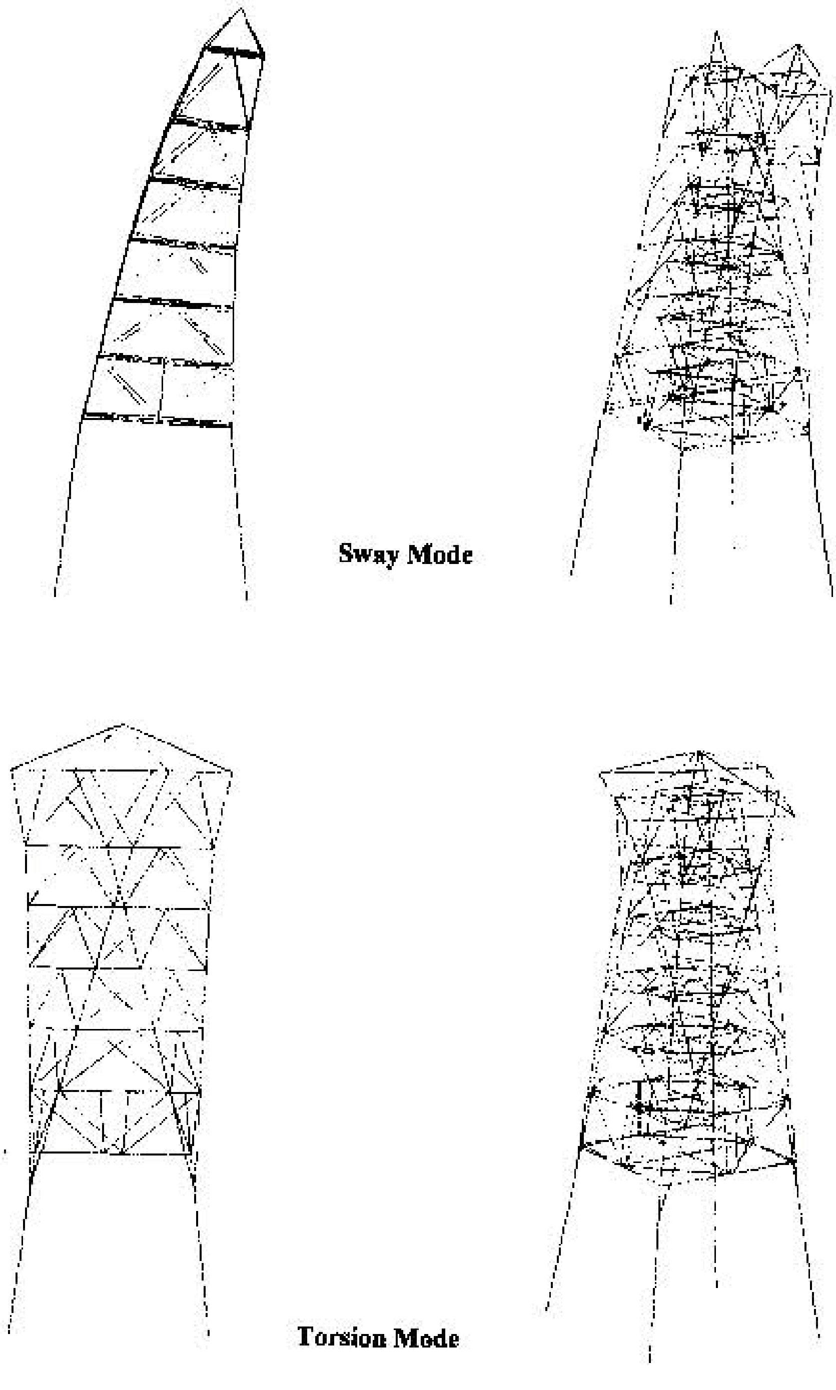

Figure (3): Patterns of the Fundamental Sway and Torsion Modes.

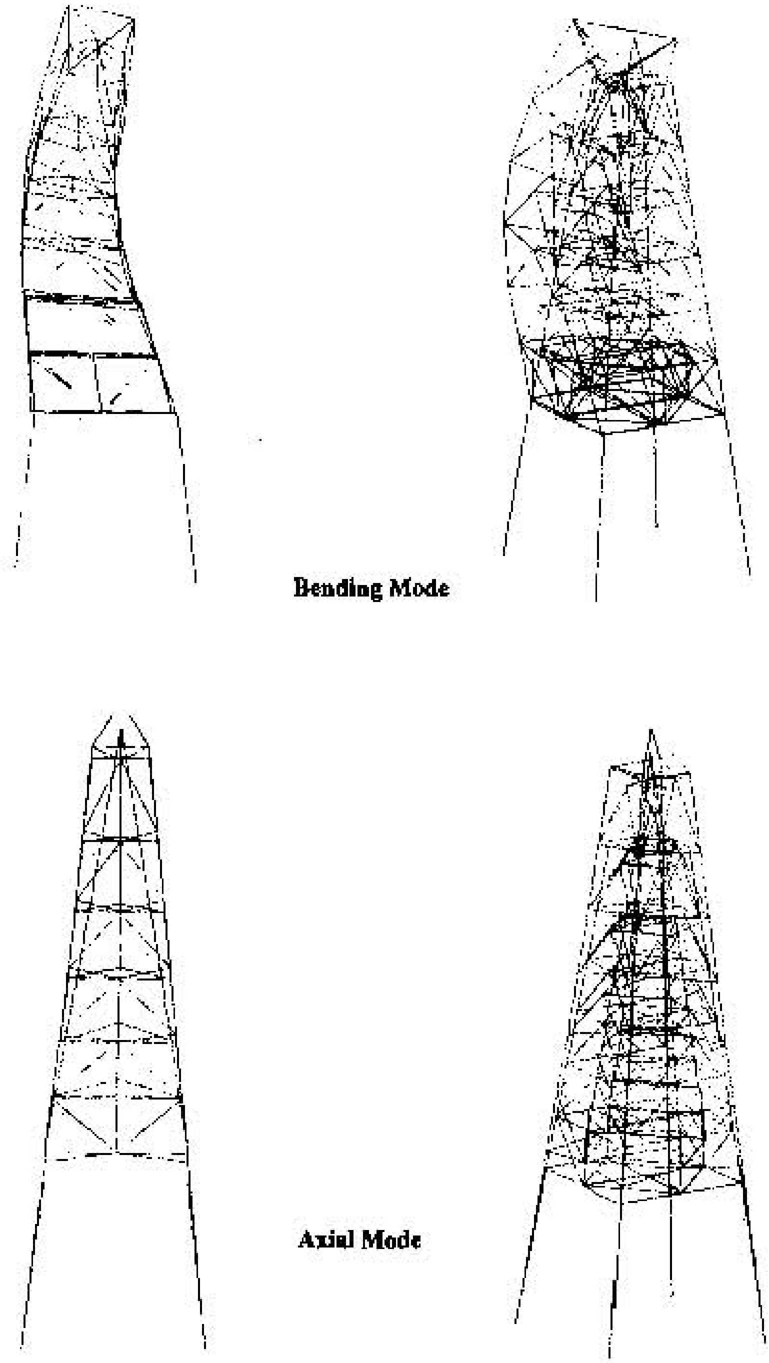

Figure (4): Patterns of the Fundamental Bending and Axial Modes.

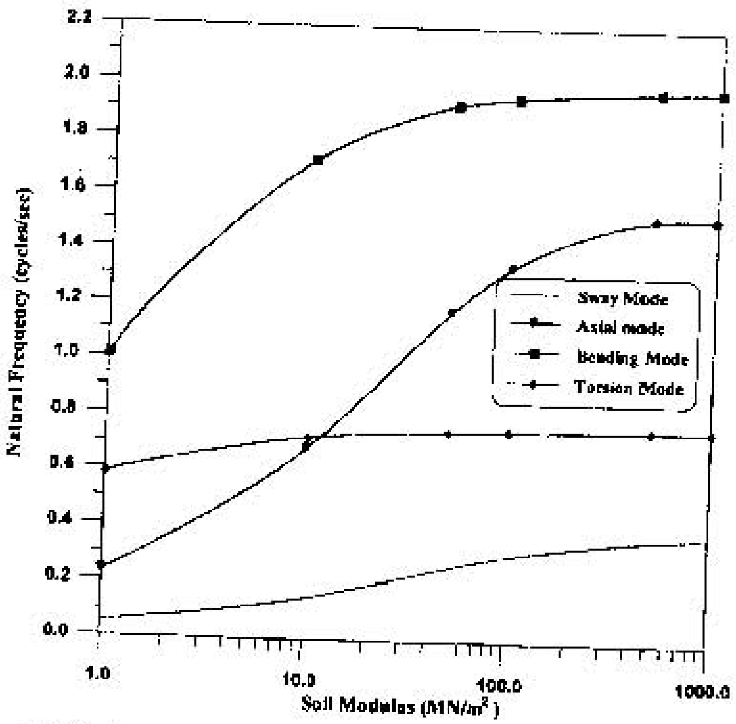

Figure (5): Variation of Natural Frequency with Soil Modulus for Different Modes of Vibration.
(Soil modulus is constant with depth, n=0)

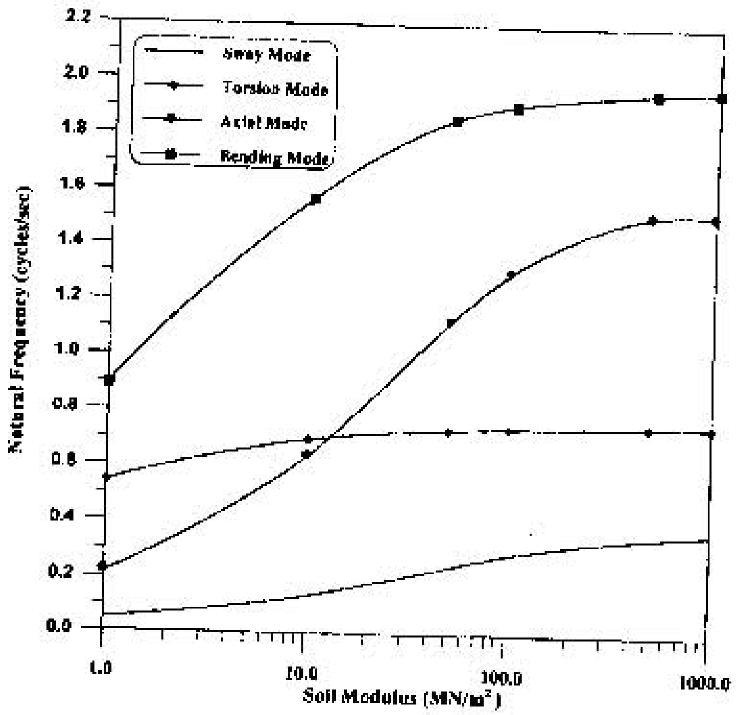

Figure (6): Variation of Natural Frequency with Soil Modulus for Different

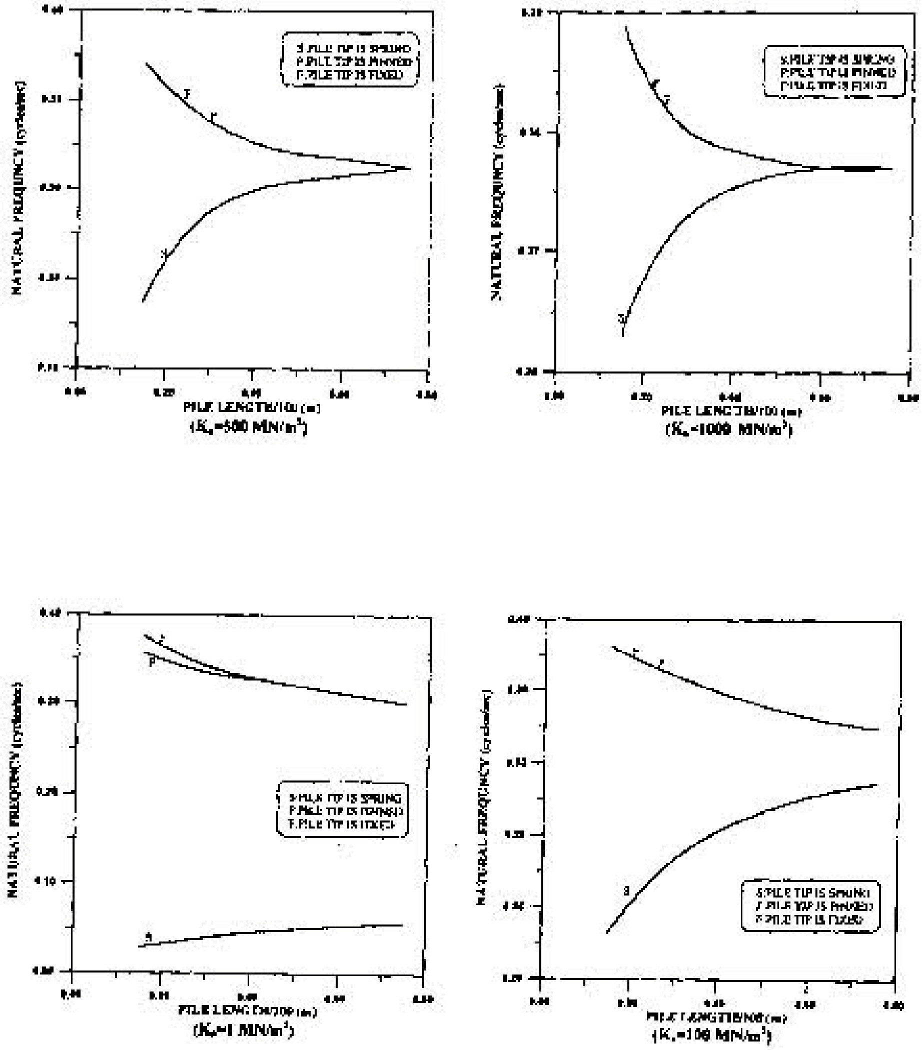

Figure (7): Variation of the Fundamental Natural Frequency with the Embedded Pile Length with Various Soil Moduli.
(Soil modulus is constant along the pile length, n=0)

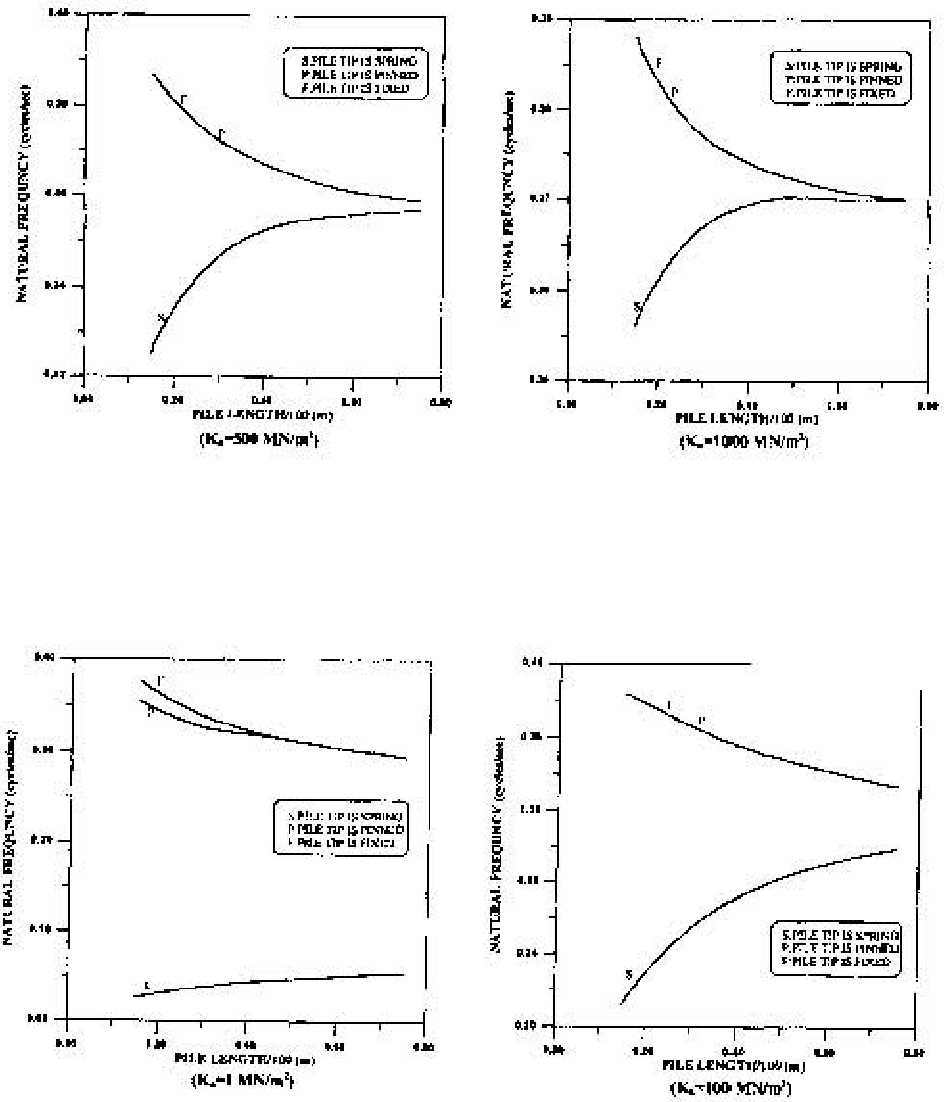

Figure (8): Variation of the Fundamental Natural Frequency with the Embedded Pile Length with Various Soil Moduli.

(Nonlinear variation of soil modulus along the pile length, n=0.15)

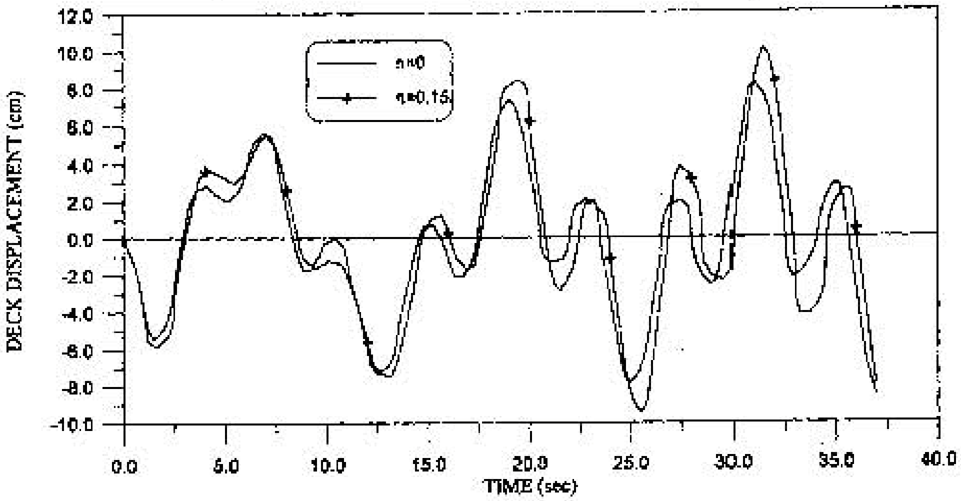

Figure (5-12): Time Variation of Deck Displacement with Clay Soil in the Seabed. (Three wave periods)

Table (2): Variation of loading effects with the soil modulus.

Soil Modulus (MN/m²)	[illegible]	Max. B.M. at the deck (kN.m)	[illegible]	Max. B.M. at the sea bed (kN.m)	[illegible]
10	[illegible]	389	[illegible]	3162	[illegible]
20	[illegible]	370	1388	1689	[illegible]
50	[illegible]	328	329	906	[illegible]
100	[illegible]	272	342	563	5762
500	[illegible]	207	[illegible]	309	[illegible]
1000	2.0	198	[illegible]	253	[illegible]