The transformations induced on hypersurfaces of almost Hermitian manifolds

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Abstract

In this paper the authors study the transformations on hypersurfaces of almost Hermitian manifolds. It's known that the almost contact metric structure is defined on hypersurfaces of almost Hermitian manifolds. The holomorphic projective transformations of almost Hermitian manifolds induce the transformations of the almost contact metric structure. These transformations were called as the contact projective transformations. The main result of this paper is that each holomorphic almost geodesic is not invariant under contact projective transformations.

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1. Introduction.

The theory of geodesic transformations is old area of investigation in Riemannian geometry. T. Levi-Civita [2], H.Weyl [5], T.Y.Thomas [6] was founders of this theory. Geodesic transformations of Riemannia manifolds with supplementary structure were studied by V.F. Kirichenko, N.N. Dondukova [7], [8], E.A. Suleymanova [9] etc. It is proved that Riemannian manifolds with supplementary structure have trivial geodesic transformation very often. Hence it is necessary to study generalization of geodesic transformations.

In this paper we constructed example of such transformation for hypersurface of almost Hermitian manifold. It is called contact projective transformation. The contact projective transformation of almost contact metric structure is induced by holomorphic projective transformation of almost Hermitian manifold. We have proved that holomorphic almost geodesic is not invariant under contact projective transformation in general case. The example of curve to be invariant under contact projective transformations was constructed.

2. Definition of contact projective transformation on hypersurface.

Let (M,J,g) be an almost Hermitian manifold with dimension $2n>2$; $X(M)$ be the module of smooth vector fields. Consider oriented hypersurface N of the almost Hermitian manifold. The almost Hermitian structure defines an almost contact metric structure on N [1]. Consider $X_N(M)=\{T_p(M): p\in N\}$. It is called a restriction of $X(M)$. The field of unit normal ν is defined in each point of hypersurface N such that any positive base of $T_p(N)$ and ν are base of $T_p(M)$. The set of vectors ν is module **N** such that $\mathbf{N}\perp X(N)$ and $X_N(M)=X(N)\oplus\mathbf{N}$.

Denote $\zeta(X)=g(\nu,X)$, $\xi=J(\nu)$, $\eta(X)=g(\xi,X)$, $\Pi=id-\zeta\otimes\nu-\eta\otimes\xi$, $\Psi=J\circ\Pi$, $\Phi=\Psi|_{X(N)}$. It is known [1] that (Φ,g,η,ξ) is the almost contact structure of N. Let us recall the next definitions.

Definition 2.1 [1]. Let Φ be (1, 1) tensor field; g be Riemannian metric; ξ be a vector field; η be a covector field. Four tensor fields $\{\Phi, g, \xi, \eta\}$ on a smooth manifold M^{2n+1} are called an almost contact metric structure if the following conditions hold:

1) $\Phi(\xi)=0$; 2) $\eta \circ \Phi = 0$; 3) $\Phi^2 = -id + \xi \otimes \eta$;

4) $\eta(\xi)=1$; 5) $g(\Phi X, \Phi Y) = g(X,Y) - \eta(X)\eta(Y)$.

Definition 2.2 [2]. Let $\gamma : I \to M$ be a curve on almost Hermitian manifold M, μ_p be the tangent vector of γ in any point $p \in \gamma$, $\tilde{\mu}_q$ be the image of μ_p under the parallel translation for any $q \in \gamma$. The γ is called the holomorphic almost geodesic if $\tilde{\mu}_q \in L\left(\mu_q, (J\mu)_q\right)$.

Definition 2.3 [3]. A mapping $(J,g) \to (J,\tilde{g})$ is called *a* holomorphic projective transformation of almost Hermitian structure (J,g) if almost Hermitian structures (J,g) and $(J,\tilde{g})$ have common holomorphic almost geodesics.

Consider a holomorphic projective transformation $(J,g) \to (J,\tilde{g})$. Let (Φ, g, η, ξ) be the almost contact metric structure on hypersurface N of the almost Hermitian manifold (M,J,g), $(\tilde{\Phi}, \tilde{g}, \tilde{\eta}, \tilde{\xi})$ be the almost contact metric structure on hypersurface N of the almost Hermitian manifold $(M,J,\tilde{g})$. By f denote the tensor of holomorphic projective deformation. It is well known that f is given by $\tilde{g}(X,Y) = g(X, fY)$, $X,Y \in X(M)$.

Theorem 2.4. the following equalities are holds:

$$\tilde{\xi} = \pm\sqrt{\left(\zeta\left(f^{-1}\nu\right)\right)^{-1}}\, f^{-1}(\xi) \tag{1}$$

$$\tilde{\eta} = \pm\sqrt{\left(\zeta\left(f^{-1}\nu\right)\right)^{-1}}\, \eta \tag{2}$$

$$\tilde{\Phi} = \Phi - \eta \otimes \nu + \left(\zeta\left(f^{-1}\nu\right)\right)^{-1} \eta \otimes f^{-1}\nu \tag{3}$$

Proof. Let $\tilde{\nu}$ be the field of unit normal for hypersurface N of the almost Hermitian manifold $(M,J,\tilde{g})$. Consider vector field $f\tilde{\nu}$. Clearly, $g(f\tilde{\nu}, X) = \tilde{g}(\tilde{\nu}, X) = 0$, $\forall X \in X(N)$. Therefore, $f\tilde{\nu} \in \mathbf{N}$ and $f\tilde{\nu} = \lambda\nu$, where λ is some smooth function on M. We have

$\tilde{\zeta}(\nu) = \tilde{g}(\nu, \tilde{\nu}) = g(\nu, f\tilde{\nu}) = g(\nu, \lambda\nu) = \lambda$. Therefore,

$$f\tilde{\nu} = \tilde{\zeta}(\nu)\nu \tag{4}$$

Note that $\tilde{\zeta}(X) = \tilde{g}(\tilde{\nu}, X) = g(f\tilde{\nu}, X) = \tilde{\zeta}(\nu)\zeta(X)$, $\forall X \in X(M)$. In particular, if $X = \tilde{\nu}$, then $\zeta(\nu)\zeta(\tilde{\nu}) = 1$. Using (4), we get $1 = \tilde{\zeta}(\nu)\zeta(\tilde{\nu}) = \tilde{\zeta}(\nu)\zeta\left(\tilde{\zeta}(\nu) f^{-1}\nu\right)$,

i.e. $\tilde{\zeta}(\nu)^2 = \left(\zeta\left(f^{-1}\nu\right)\right)^{-1}$. Using (4), we get

$$f\tilde{\nu} = \pm\sqrt{\left(\zeta\left(f^{-1}\nu\right)\right)^{-1}}\, \nu \tag{5}$$

We will denote $\pm\sqrt{\left(\zeta\left(f^{-1}\nu\right)\right)^{-1}} = \lambda$ bellow. Since $\tilde{\zeta}(X) = \tilde{\zeta}(\nu)\zeta(X)$, $\forall X \in X(M)$, we have

$$\tilde{\zeta} = \lambda \zeta \tag{6}$$

It is known [3] that $J \circ f = f \circ J$. Therefore, $J \circ f^{-1} = f^{-1} \circ J$. Using (5), we get $\tilde{\xi} = J(\tilde{\nu}) = = J\left(\lambda f^{-1} \nu\right) = \lambda f^{-1} J(\nu)$. Therefore,

$$\tilde{\xi} = \lambda f^{-1}(\xi) \equiv \pm \sqrt{\left(\zeta\left(f^{-1}\nu\right)\right)^{-1}} f^{-1}(\xi) \tag{7}$$

Using (4), (7), we get $\tilde{\eta}(X) = \tilde{g}\left(\tilde{\xi}, X\right) = \tilde{g}\left(\tilde{\zeta}(\nu) f^{-1}(\xi), X\right) = \tilde{\zeta}(\nu)\eta(X)$. Therefore,

$$\tilde{\eta} = \lambda \eta \equiv \pm \sqrt{\left(\zeta\left(f^{-1}\nu\right)\right)^{-1}} \eta \tag{8}$$

Finally, we prove (3) .We have $\tilde{\Psi} = J \circ \tilde{\Pi} = J \circ \left(id - \tilde{\zeta} \otimes \tilde{\nu} - \tilde{\eta} \otimes \tilde{\xi}\right) = = J \circ \left(id - \lambda^2 \zeta \otimes f^{-1}\nu - \lambda^2 \eta \otimes f^{-1}\xi\right)$. Therefore, $\tilde{\Psi} = \Psi + J \circ (\zeta \otimes \nu + \eta \otimes \xi - \lambda^2 \zeta \otimes f^{-1}\nu - -\lambda^2 \eta \otimes f^{-1}\xi = \Psi + \zeta \otimes \xi - \eta \otimes \nu - \lambda^2 \zeta \otimes f^{-1}\xi + \lambda^2 \eta \otimes f^{-1}\nu$. Therefore,

$$\tilde{\Psi} = \Psi + \zeta \otimes \xi - \eta \otimes \nu - \lambda^2 \zeta \otimes f^{-1}\xi + \lambda^2 \eta \otimes f^{-1}\nu$$

Since $\zeta\big|_{X(N)} = 0$, we obtain (3). □

Denote $\Pi = id - \zeta \otimes \nu - \eta \otimes \xi$, $\tilde{\Pi} = id - \tilde{\zeta} \otimes \tilde{\nu} - \tilde{\eta} \otimes \tilde{\xi}$, $\operatorname{Im}\Pi = L$, $\operatorname{Im}\tilde{\Pi} = \tilde{L}$.

Lemma 2.5. $L = \tilde{L}$.

Proof. Since $\eta(X) = \zeta(X) = 0$ for all $X \in L$, we get $\tilde{\Pi}(X) = (id - \tilde{\eta} \otimes \tilde{\xi} - \tilde{\zeta} \otimes \tilde{\nu})(X) = = \left(id - \lambda^2 \eta \otimes f^{-1}\xi - \lambda^2 \zeta \otimes f^{-1}\nu\right)(X) = X$ i.e. $L \subset \tilde{L}$. Conversely, we have $\Pi(X) = \left(id - \eta \otimes \xi - \zeta \otimes \nu\right)(X) = \left(id - \lambda^{-1}\tilde{\zeta} \otimes \nu - \lambda^{-1}\tilde{\eta} \otimes \xi\right)(X) = X$ for all $X \in \tilde{L}$ i.e. $\tilde{L} \subset L$. □

We get that module $X_N(M) = \mathbf{N} \oplus \mathbf{M} \oplus L = \tilde{\mathbf{N}} \oplus \tilde{\mathbf{M}} \oplus L$, where $\mathbf{N} = L(\nu)$, $\tilde{\mathbf{N}} = L(\tilde{\nu})$, $\mathbf{M} = L(\xi)$, $\tilde{\mathbf{M}} = L\left(\tilde{\xi}\right)$, $L = \operatorname{Im}\Pi = \operatorname{Im}\tilde{\Pi}$.

Definition 2.6. The mapping $\phi : (N, \Phi, g, \eta, \xi) \to \left(N, \tilde{\Phi}, \tilde{g}, \tilde{\eta}, \tilde{\xi}\right)$ is called a contact projective transformation.

3. Special occurrence of contact projective transformation.

Dente by $\tilde{\nu}_p \parallel \nu_p$ to the parallels vectors for each point $p \in N$. Therefore, we have $\tilde{\nu}_p = c(p)\nu_p$, $c(p) = const$ or $\tilde{\nu} = c\nu$, $c \in C^\infty(N)$. Using (5) and definition for tensor of holomorphic projective deformation, we get $1 = \tilde{g}(\tilde{\nu}, \tilde{\nu}) = g\left(f\tilde{\nu}, c\nu\right) = g\left(\lambda \nu, c\nu\right) = c\lambda$ i.e.

$$\tilde{\nu} = \lambda^{-1}\nu \tag{9}$$

Using (5) and (9), we get $f(\nu) = \lambda^2 \nu$. Conversely, let ν be a eigenvector of f i.e. $f_p \nu_p = c\nu_p$, $c = const$. Since f is invertible operator, we get $\nu_p = cf^{-1}\nu$. Using (5), we obtain $\tilde{\nu} = \lambda f^{-1}\nu$ i.e. $\tilde{\nu}_p \parallel \nu_p$ for all $p \in N$. These results can be summarized as follows.

Proposition 3.1. $\tilde{\nu}_p \parallel \nu_p$ for all $p \in N$ if and only if ν is eigenvector of f for all $p \in N$. □

Proposition 3.2. If $\tilde{\nu}_p \parallel \nu_p$ for all $p \in N$ then ξ is eigenvector of f for all $p \in N$.

Proof. Since $\xi = J(\nu)$, we get $\tilde{\xi} = J(\tilde{\nu}) = J(\lambda^{-1}\nu) = \lambda^{-1}\xi$. Therefore, using (1), we obtain $f(\lambda^{-1}\xi) = \lambda\xi$ i.e. $f(\xi) = \lambda^2\xi$. □

Proposition 3.3. If $\tilde{\nu}_p \square \nu_p$ for all $p \in N$ then $\tilde{\Phi} = \Phi$.

Proof. We have $\tilde{\Psi} = J \circ \tilde{\Pi} = J \circ (id - \tilde{\zeta} \otimes \tilde{\nu} - \tilde{\eta} \otimes \tilde{\xi}) = J \circ (id - \zeta \otimes \nu - \eta \otimes \xi) = \Psi$. Since $\Phi = \Psi|_{X(N)}$, we obtain $\tilde{\Phi} = \Phi$. □

Proposition 3.4. If $\tilde{\nu}_p \square \nu_p$ for all $p \in N$ then distributions $\mathbf{N}, \mathbf{M}, L$ is invariant under f.

Proof. Using proposition 1 and proposition 2, we get that $\mathbf{N}, \mathbf{M}$ is invariant under f. Since $\tilde{L} = L$, we obtain $g(fX, \xi) = \tilde{g}(X, \xi) = \tilde{g}(X, \lambda\tilde{\xi}) = 0$ for all $X \in L$. Similarly, we get $g(fX, \nu) = 0$. Therefore, $fX \in L$ for all $X \in L$. □

Since $X(N)$ is invariant under f, we can define the restriction f on $X(N)$. Denote it by same letter f.

Definition 3.5. The section f on $X(N)$ is called the tensor of contact projective deformation.

Proposition 3.6. Let ∇^M be the Riemannian connection of the almost Hermitian manifold (M, J, g); $\tilde{\nabla}^M$ be the Riemannian connection of the almost Hermitian manifold $(M, J, \tilde{g})$; ∇ be the Riemannian connection of the almost contact manifold (N, Φ, g, η, ξ); $\tilde{\nabla}$ be the Riemannian connection of the almost contact manifold $(N, \tilde{\Phi}, \tilde{g}, \tilde{\eta}, \tilde{\xi})$; T be the tensor of affine deformation for connections $\nabla, \tilde{\nabla}$. We have

$$T(X,Y) = \psi(X)Y + \psi(Y)X - \psi(\Phi X)\Phi Y - \psi(\Phi Y)\Phi X - \eta(X)\psi(\nu)\Phi Y - \eta(Y)\psi(\nu)\Phi X,$$

where ψ is some differential form on manifold N.

Proof. By [4,page 21 – 23] we have $\nabla_X^M Y = \nabla_X Y + h(X,Y)\nu$ and $\tilde{\nabla}_X^M Y = \tilde{\nabla}_X Y + \tilde{h}(X,Y)\tilde{\nu}$. Therefore, $T(X,Y) = \tilde{\nabla}_X Y - \nabla_X Y = T^M(X,Y) - \tilde{h}(X,Y)\tilde{\nu} - h(X,Y)\nu$, where T^M is the tensor of affine deformation for connections $\nabla^M, \tilde{\nabla}^M$. We have [3]

$$T^M(X,Y) = \psi(X)Y + \psi(Y)X - \psi(JX)JY - \psi(JY)JX - \tilde{h}(X,Y)\tilde{\nu} + h(X,Y)\nu \tag{10}$$

where ψ is some differential form on M, $X, Y \in X(N)$. Let us remember that $\pi_1 = \zeta \otimes \nu$, $\Pi_1 = id - \zeta \otimes \nu$ are projectors on $\mathbf{N}$ and $X(N)$ respectively. Applying Π_1 to (10), we get

$$T(X,Y) = \psi(X)Y + \psi(Y)X - \psi(JX)\Pi_1 JY - \psi(JY)\Pi_1 JX \tag{11}$$

We have $JX = J(X - \eta(X)\xi + \eta(X)\xi)$ for all $X \in X(N)$. Since $X - \eta(X)\xi \in L$, we get $J(X - \eta(X)\xi) = \Phi X$. Since $\xi = J(\nu)$, we get $\Pi_1 JX = \Phi X$. Finally, using (11), we obtain required formula. □

Corollary 3.7. Under the conditions of proposition 3.6, we have

$$\tilde{\nabla}_X(\Phi)Y = \nabla_X(\Phi)Y + \psi(\Phi Y)X - \psi(\Phi Y)\Phi X - \psi(Y)\Phi X + \psi(\Phi Y)\Phi^2 X + \eta(Y)\psi(\nu)\Phi^2 X$$

□

Theorem 3.8. If $\tilde{\nu}_p \square \nu_p$ for all $p \in N$ then induced contact projective transformation is trivial, i.e. $\psi = 0$.

Proof. Since $g(X,\Phi Y)+g(\Phi X,Y)=0$ and $\tilde{g}(X,\Phi Y)+\tilde{g}(\Phi X,Y)=0$, we get $g(\nabla_X(\Phi)Y,Y)=\tilde{g}(\tilde{\nabla}_X(\Phi)Y,Y)=0$. Let Y be a eigenvector of f with eigenvalue ϖ. Therefore, $0=\tilde{g}(\tilde{\nabla}_X(\Phi)Y,Y)=g(\tilde{\nabla}_X(\Phi)Y, fY)=\varpi g(\tilde{\nabla}_X(\Phi)Y,Y)$. Using corollary 3.7, we get

$$g(\psi(\Phi Y)X,Y)-g(\psi(\Phi^2Y)\Phi X,Y)-g(\eta(\Phi Y)\psi(\nu)\Phi X,Y)-g(\psi(Y)\Phi X,Y)+$$
$$+g(\psi(\Phi Y)\Phi^2X,Y)+g(\eta(Y)\psi(\nu)\Phi^2X,Y)=0 \qquad (12)$$

for all $X \in X(N)$, for all eigenvector Y of operator f. Since g is metric of fixed sign, we get that operator f has base of eigenvectors for all point of manifold N. Therefore, (12) is true for all $Y \in X(N)$. Since g is nonsingular metric, we obtain

$$\psi(\Phi Y)X-\psi(\Phi^2Y)\Phi X-\psi(Y)\Phi X+\psi(\Phi Y)\Phi^2X+\eta(Y)\psi(\nu)\Phi^2X=0$$

Since there exists linearly independent system $(X,\Phi X,\Phi^2X)$ for all point of N, we get

$$\psi\circ\Phi=0;\quad \psi\circ\Phi^2+\psi=0;\quad \psi\circ\Phi+\psi(\nu)\eta=0;$$

Therefore, $\psi(Y)=0$ for all $Y \in X(N)$ and $\psi(\nu)=0$. Using proposition 3.6, we get that contact projective transformation is trivial. □

4. General occurrence of contact projective transformation.

Let $\nu \not\square \tilde{\nu}$, i.e. $\mathbf{N} \neq \tilde{\mathbf{N}}$. Operator of contact projective deformation $\hat{f}$ is defined on manifold N. In this case $\hat{f}$ is not section of operator f on the manifold N.

Lemma 4.1. $\hat{f}=\Pi_1\circ f$, where $\Pi_1 = id-\zeta\otimes\nu$.

Proof. By definition we have $\tilde{g}(X,Y)=g(X,\hat{f}Y)$ for all $X,Y \in X(N)$ and $\tilde{g}^M(X,Y)=g^M(X,fY)$ for all $X,Y\in X(M)$. Therefore, $\tilde{g}(X,Y)=\tilde{g}^M(X,Y)$ for all $X,Y\in X(N)$. On the other hand we have $g^M(X,fY)=g^M(X,\Pi_1 fY+\pi_1 fY)=g(X,\Pi_1 fY)$. Therefore, $\hat{f}(Y)=\Pi_1\circ f(Y)$ for all $Y\in X(N)$. Here f denote the section of operator f on $X(N)$. □

Lemma 4.2. The tensor of affine deformation T for connections $\nabla,\tilde{\nabla}$ is given by formula

$$T(X,Y)=\psi(X)Y+\psi(Y)X-\psi(\Phi X)\Phi Y+\eta(X)\psi(\nu)\Phi Y-\psi(\Phi Y)\Phi X+\eta(Y)\psi(\nu)\Phi X+$$
$$+\frac{\tilde{h}(X,Y)}{\lambda}(\nu-\lambda^2 f^{-1}\nu) \qquad (13)$$

Proof. Using (5), (10), we get

$$T(X,Y)=\psi(X)Y+\psi(Y)X-\psi(JX)JY-\psi(JY)JX+h(X,Y)\nu-\tilde{h}(X,Y)\lambda f^{-1}\nu \qquad (14)$$

Appling Π_1 to both sides of (14), we get required formula. □

By ϑ we denote vector field $\nu-\lambda^2 f^{-1}\nu$. We have $\vartheta\in X(N)$.

Lemma 4.3. $\vartheta \in L$.
Proof. We have $\Pi(\vartheta) = (id - \zeta \otimes \nu - \eta \otimes \xi)(\nu - \lambda^2 f^{-1}\nu) = \nu - \lambda^2 f^{-1}\nu - \nu + \lambda^2 \zeta(f^{-1}\nu)\nu +$ $+\lambda^2 \eta(f^{-1}\nu)\xi$. Since $\zeta(f^{-1}\nu) = \lambda^{-2}$ and $\eta(f^{-1}\nu) = \lambda^{-2}\tilde{\eta}(\tilde{\nu}) = 0$, we get $\Pi(\vartheta) = \vartheta$. □

Definition 4.4. Let $\gamma : I \to N$ be a curve on almost contact metric manifold N; μ_p be the tangent vector of γ in any point $p \in \gamma$; $\tilde{\mu}_q$ be the image of μ_p under the parallel translation for any $q \in \gamma$. The γ is called the holomorphic almost geodesic if $\tilde{\mu}_q \in L\left(\mu_q, (\Phi\mu)_q\right)$.

Let γ be a holomorphic almost geodesic on almost contact metric manifold (N, Φ, g, η, ξ). We will prove that γ is not holomorphic almost geodesic for the almost contact structure $(\tilde{\Phi}, \tilde{g}, \tilde{\eta}, \tilde{\xi})$. Let μ be the field of tangent vectors for γ. Since γ is a holomorphic almost geodesic, we get $\nabla_\mu \mu = a(t)\mu + b(t)\Phi\mu$. Here a, b are some functions on manifold N. We will calculate $\tilde{\nabla}_\mu \mu = \nabla_\mu \mu + T(\mu, \mu)$. Using (3), (13), we obtain

$$\tilde{\nabla}_\mu \mu = \left(a(t) + 2\psi(\mu)\right)\mu + \left(b(t) - 2\psi(\tilde{\Phi}\mu) + 2\psi(f^{-1}\nu)\lambda^2\eta(\mu)\right)\tilde{\Phi}\mu + \left(b(t)\eta(\mu) - 2\psi(\tilde{\Phi}\mu)\eta(\mu) + 2\psi(f^{-1}\nu)\lambda^2\eta^2(\mu) + \lambda^{-2}\tilde{h}(\mu,\mu)\right)\vartheta \quad (15)$$

We see that γ is not invariance under contact projective transformation in general case. We will change the definition of holomorphic almost geodesic such that obtained curve is invariance under contact projective transformation.

Definition 4.5. Let $\gamma : I \to N$ be a curve on almost contact metric manifold N; μ_p be the tangent vector of γ in any point $p \in \gamma$; $\tilde{\mu}_q$ be the image of μ_p under the parallel translation for any $q \in \gamma$. The γ is called the contact almost geodesic if $\tilde{\mu}_q \in L\left(\mu_q, (\Phi\mu)_q, \vartheta_q\right)$.

Theorem 4.6. A contact almost geodesic on a hypersurface of an almost Hermitian manifold is invariance under contact projective transformations.
Proof. Let $\tilde{f}$ be the tensor of holomorphic projective deformation from almost contact structure $(\tilde{\Phi}, \tilde{g}, \tilde{\eta}, \tilde{\xi})$ to almost contact structure (Φ, g, η, ξ). Obviously, $\tilde{f} = f^{-1}$, where f is the tensor of holomorphic projective deformation from almost contact structure (Φ, g, η, ξ) to almost contact structure $(\tilde{\Phi}, \tilde{g}, \tilde{\eta}, \tilde{\xi})$. We denote $\tilde{\vartheta} = \tilde{\nu} - \tilde{\lambda}^2 \tilde{f}^{-1}\tilde{\nu}$, where $\tilde{\lambda} = \zeta(\tilde{\nu})$. Using (4), we get $\tilde{\lambda} = \lambda^{-1}$. Therefore,

$$\tilde{\vartheta} = -\lambda\vartheta \quad (16)$$

Using (15), (16), we get

$$\tilde{\nabla}_\mu \mu = \left(a(t) + 2\psi(\mu)\right)\mu + \left(b(t) - 2\psi(\tilde{\Phi}\mu) + 2\psi(f^{-1}\nu)\lambda^2\eta(\mu)\right)\tilde{\Phi}\mu - -\lambda\left(b(t)\eta(\mu) - 2\psi(\tilde{\Phi}\mu)\eta(\mu) + 2\psi(f^{-1}\nu)\lambda^2\eta^2(\mu) + \lambda^{-2}\tilde{h}(\mu,\mu)\right)\tilde{\vartheta}$$

This concludes the proof. □

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