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THE STUDY OF A NEW LORENZ-LIKE MODEL

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Abstract:

This paper reports the results of a numerical study of a new Lorenz like model using number of indicators (time series, attractors, Lyapunov exponents and bifurcation diagrams) that the system is able to show most of the usual behavior of the original Lorenz system.

Keywords*:* Lorenz system, Lyapunov exponent, bifurcation diagram and Chaos.

Introduction:

The Lorenz model [1] has become almost totemistic in the field of nonlinear dynamics. Some sudden and dramatic changes in nonlinear systems may give rise to the complex behavior called chaos.

The noun chaos and the adjective chaotic are used to describe the time behavior of a system when that behavior is aperiodic (it never exactly repeats) and is apparently random or "noisy". The key word here is apparently. Underlying this apparent chaotic randomness is an order determined, in some sense by equation describing the system.

The Lorenz model has received considerable interests since its discovery [1]. This interest has grown in considerable manner since the publication of Haken famous article [2] when he rederived the what is known these days as Maxwell-Bloch model that describes the single mode laser dynamics from the Lorenz model. This discovery has opened a new era in studying the nonlinear dynamics in various types of laser classes including the single longitudinal mode, multi longitudinal mode and multi- transverse modes dynamics of laser devices. In the present work we study a new type of Lorenz model derived by Li etal [3].

A new Lorenz-like chaotic system:

The original Lorenz model reads:

$$x = a(y-x) \qquad 1.a$$
$$y = bx - cy - xy \qquad 1.b$$
$$\dot{z} = xy - dz \qquad 1.c$$

The fluid motion (represented by "x", proportional to stream function) and the temperature deviation (represented by "y" and "z"), "b" is the reduced Rayleigh number, "d" is a constant while "a" is the Prandtls number [1].

When (a, b, c, d) = (a, b, -1,d) one can get the usual Lorenz system (eq.s. 1) and when (a, b, c, d) = (a, b, c, c-a) and = (a, b, c, 0) one can get the Chen [4] and Lu etal [5] systems.

A new nonlinear three dimensional differential system derived from Lorenz system is reported by Tigan [6]. Another two new systems have been supposed lately by Yang [7] and Qing etal [8]. The following system we are studying in this article was supposed by Li etal [3]

$$x = a(y - x) \qquad 2.a$$
$$y= abx - axz \qquad 2.b$$
$$\dot{z}= xy - cz \qquad 2.c$$

a, b, and c are constants coefficients assuming: $a\neq 0$, $b>0$

In all the above mentioned systems the over dot represents the differentiation of (x, y, z) with time. It is a three dimensional autonomous system which has six terms on the right hand side of the governing equation (2). Two nonlinear terms are given xz and xy. At the steady state, and by setting $x= y= \dot{z}= 0$ one can arrive at the following three fixed points of the system (2)

$$0(0, 0, 0) \qquad =0(0,0,0)$$

$$P^+(x_o,y_o,z_o) \quad =P^+(\sqrt{cb}, \sqrt{cb}, b)$$

$$P^-(-x_o,-y_o,z_o) \quad =P^-(-\sqrt{cb}, -\sqrt{cb}, b)$$

Numerical Simulations:

To study the dynamics and the potential of the system (2) we have used Matlab with the help of the fourth order Runge-Kutta numerical technique to solve the system (2).By choosing the initial conditions. i.e. x, y, z and the constants (a, b, c) in certain combinations and making use of time series for x, y and z and producing two dimensional attractors of (x-y), (z-y), (x-z)and three dimensional attractors (x-y-z) then Lyapunov exponents L_x, L_y and L_z against time were produced based on the two dimensional attractors [9]. Finally bifurcation diagrams are produced based on Poincare sections that intercept the attractor in certain positions [10,11]. We must stress that time series, Lyapunov exponents, Poincare sections diagrams and bifurcation diagrams required special programs written for each case .

For the combination (a=5, b=4 and c=0.5, 2), results are shown in Fig.(1) while results of the combination (a=5, b=2.1, 10 and c=2) are given in Fig.(2). Bifurcation diagrams for the variation of (z) with a, b and c are given in Fig. (3). The overall behaviors for various combinations (a, b, c) are given in Fig. (4).

DISCUSSION:

According to the obtained results we can mention the following: All types of the well known Lorenz attractors are possible to be produced (especially the buttefly attractor). Each and every one of these attractors in all dimensions seen to be affected on by the variation of the system parameters viz, a, b and c as can be seen in Fig.s(1,2). When Lyapunov exponent [which means how trajectories on attractor moving away or closer from each other in combinations of these] have positive values the system is in complicated or chaotic (aperiodic) state. When it is zero or approach zero the system is in a periodic state while when it is below zero the system is stable or in a steady state. As can be seen in Fig (1) and (2) the variables x, y and z does not suffer the same variation. L_x starts higher than zero then settle down above zero while the behavior of y and z either periodic or steady. In Fig.(2) it is clear that L_x is positive while L_y is zero and L_z below zero.

With increasing a, b or c, the bifurcation diagrams of z shows a number of behaviors. The system starts with chaotic state for $3 \leq a \leq 30$ then settles down i.e. moves from chaos to periodic orbit and when $30 < a < 40$ chaos breath again then the system settles down to steady state via a reverse scenario (see Fig.3.1). Different behaviors appear when bifurcation diagram drawn against c (see Fig. 3.3).

The overall behavior of the new system is summarized in Fig.(4). Reducing any parameters on its on (~ 0) the system shows simple behaviors as can be seen in Fig.(5).

CONCLUSIONS:

The new Lorenz – like chaotic system has been studied using different tools viz time series of its three variables, attractors, Lyapunov exponents and bifurcation diagrams. Each one of the variables been affected on by the parameters appeared in the system. Although the new system of differential equations is quite different from those of the well known Lorenz system, the chaotic attracters of the two systems are resembled.

a=5,b=4,c=0.5

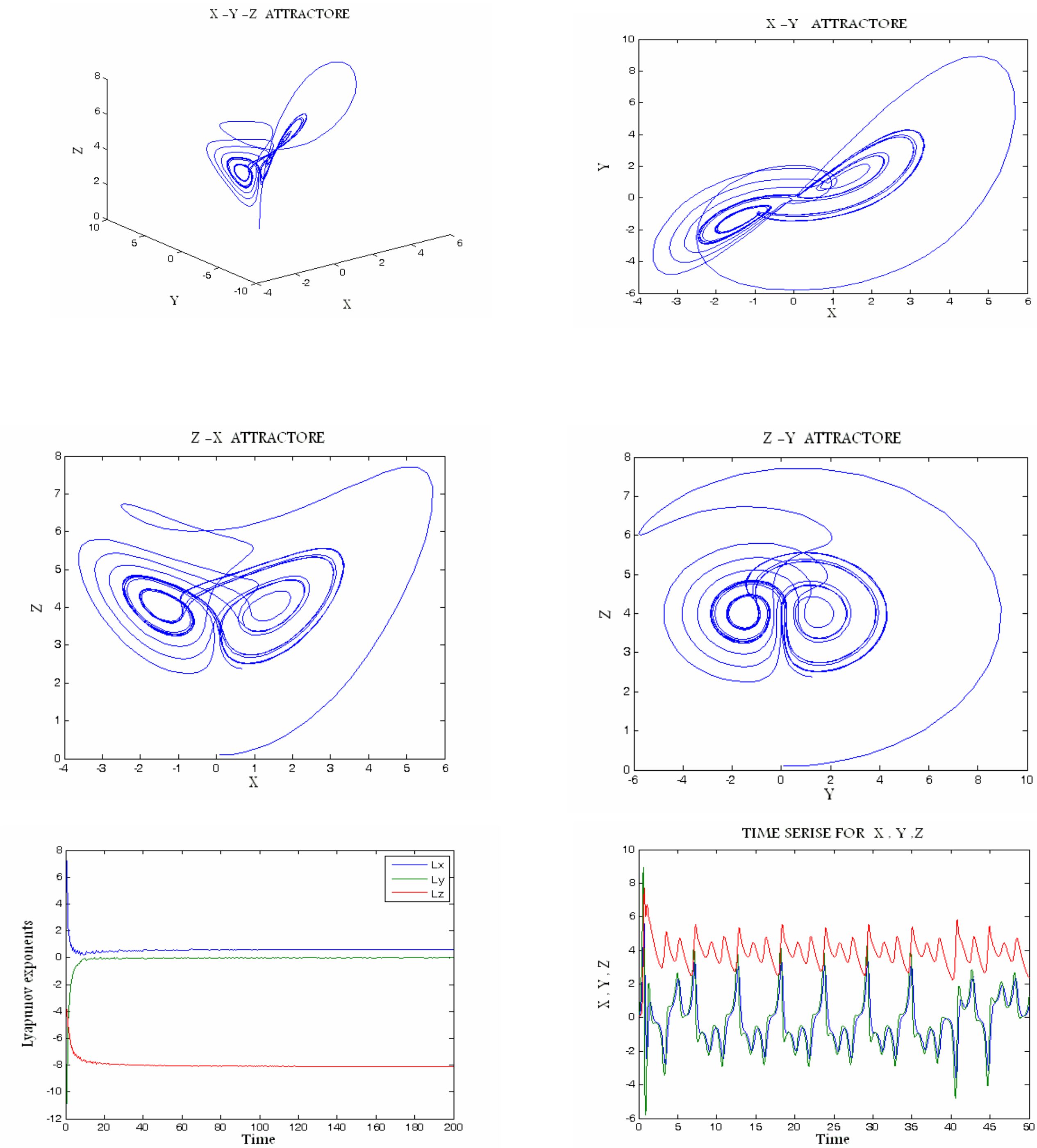


Fig (1a) : Three dimensional (a) and two dimensional (b,c,d) , Lyapunov exponents (e) and time series (f) :

cotinue

a=5,b=4,c=2

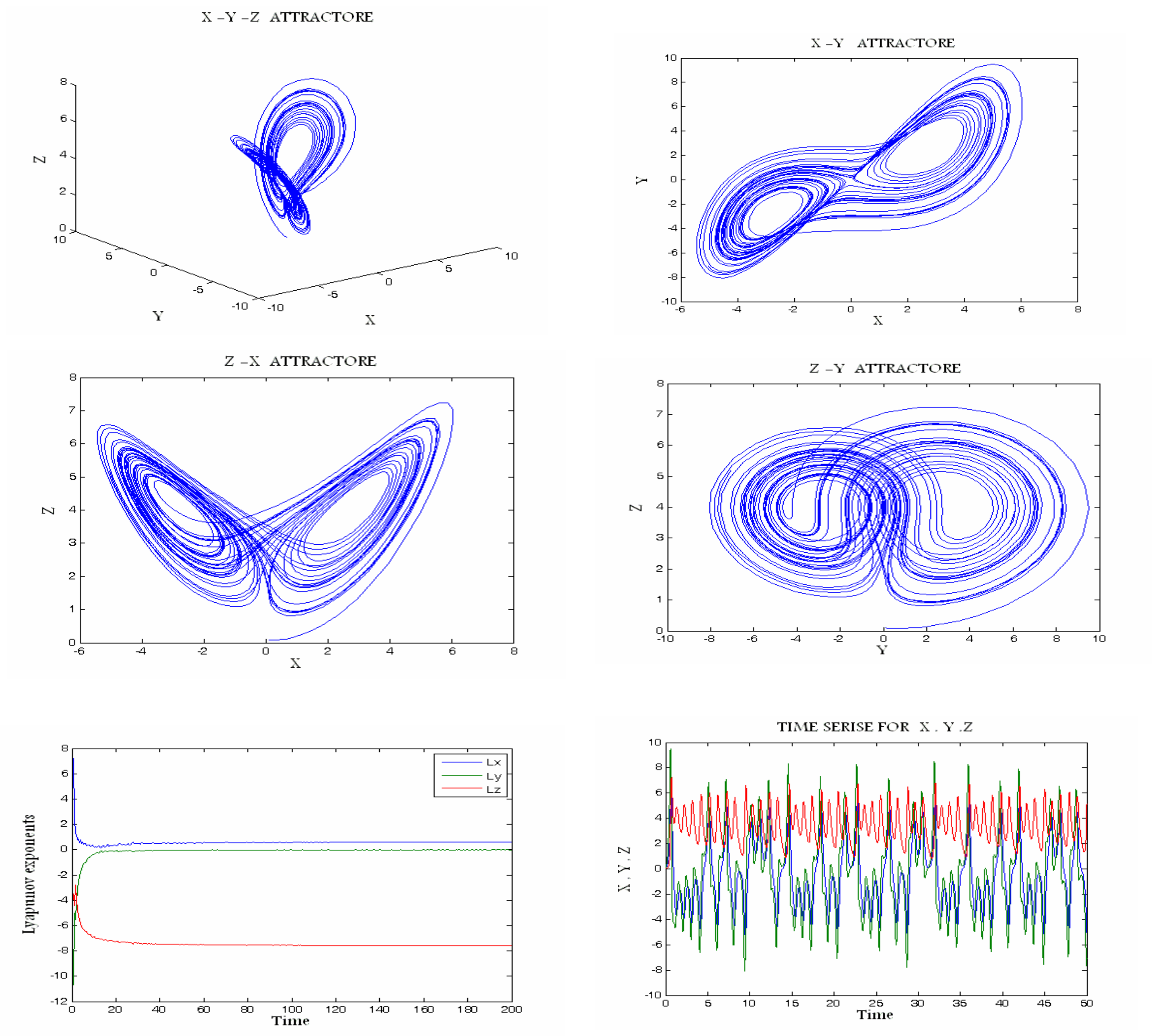


Fig : (1b)

Continued

a=5,b=2.1,c=2

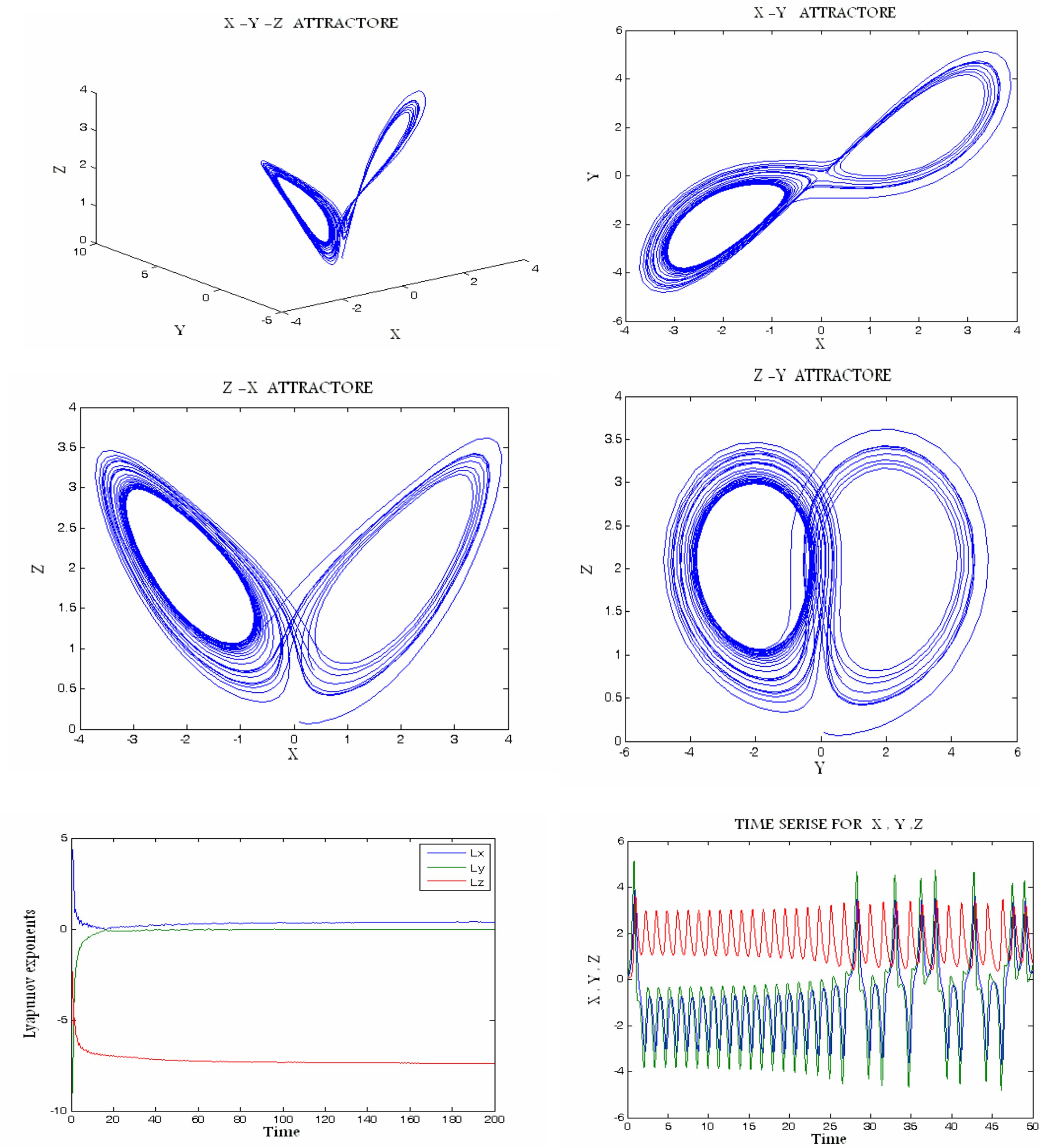


Fig : (2a) : Three dimensional (a) and two dimensional (b,c,d)attractors , Lyapunov exponents (e) and time series (f) for a=5,b=2.1,c=2

Continue

a=5,b=10,c=2

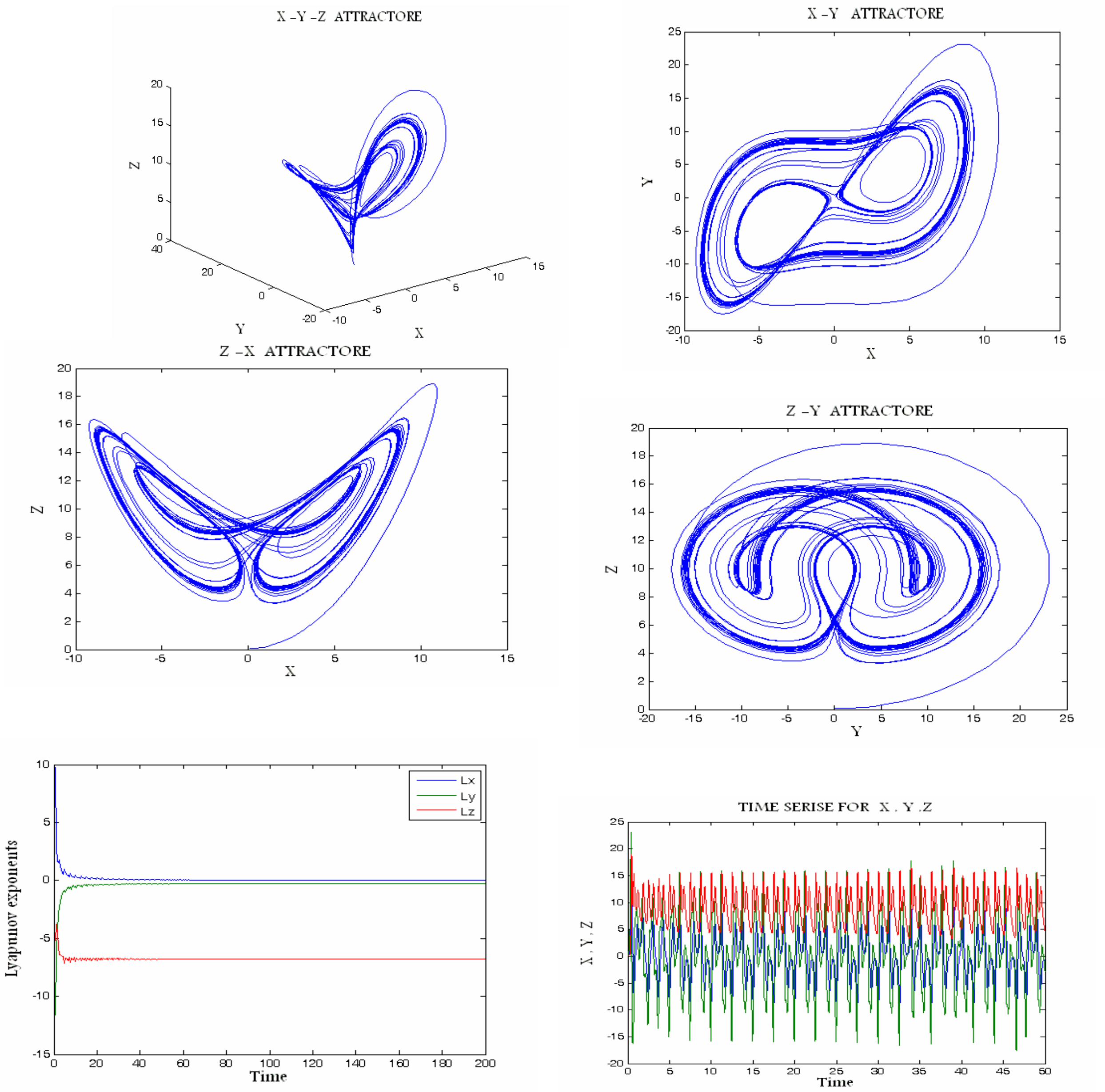
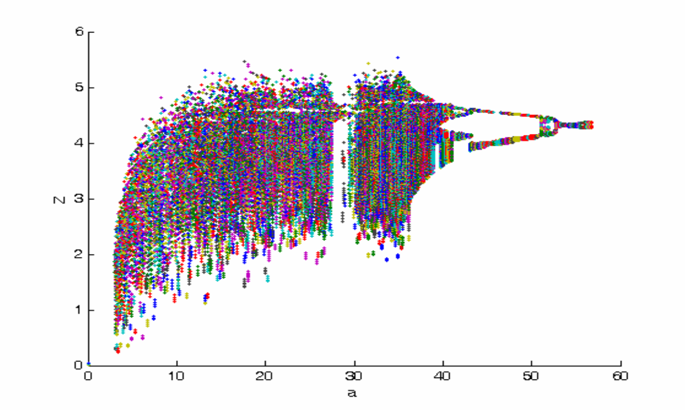


Fig:(2b)

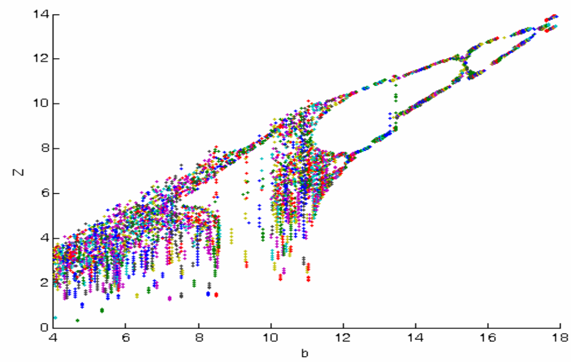
Continued

a=(0-60)
b= 4
c = 2



(1)

a=5
b=(0-20)
c = 2

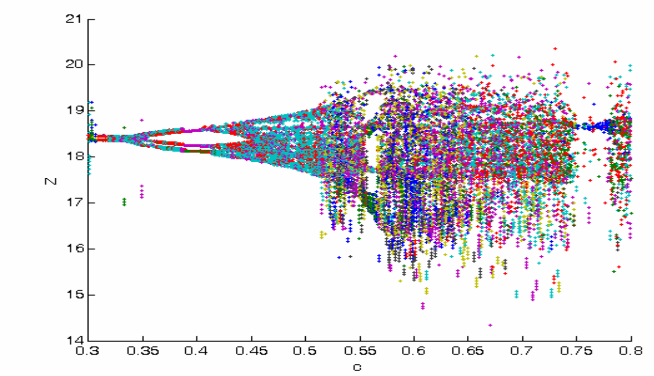


(2)

a=5

b= 4

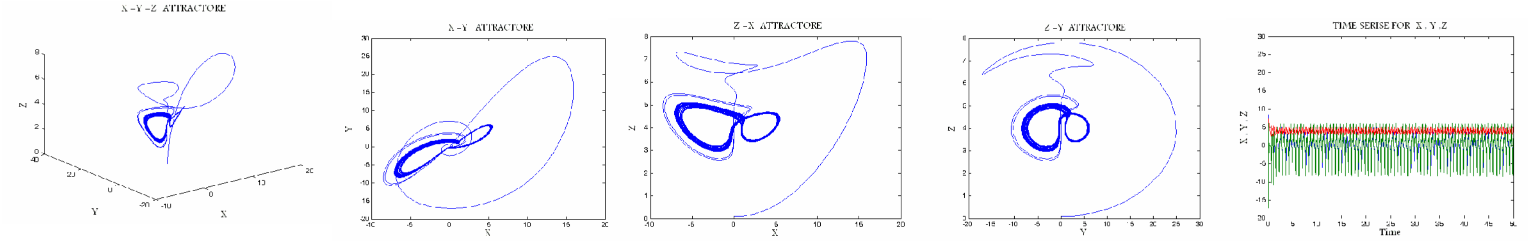
c = (0.3-0.8)



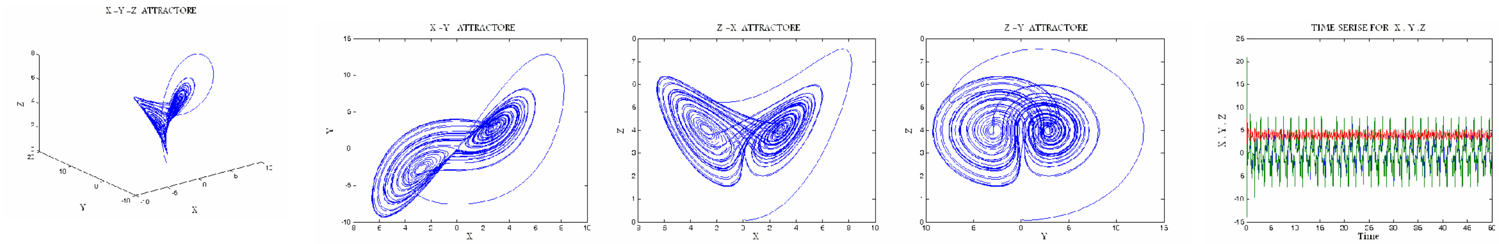
(3)

Fig (3): Bifurcation diagrams for z against 1 (a), 2 (b), 3 (c) for the parameters shown

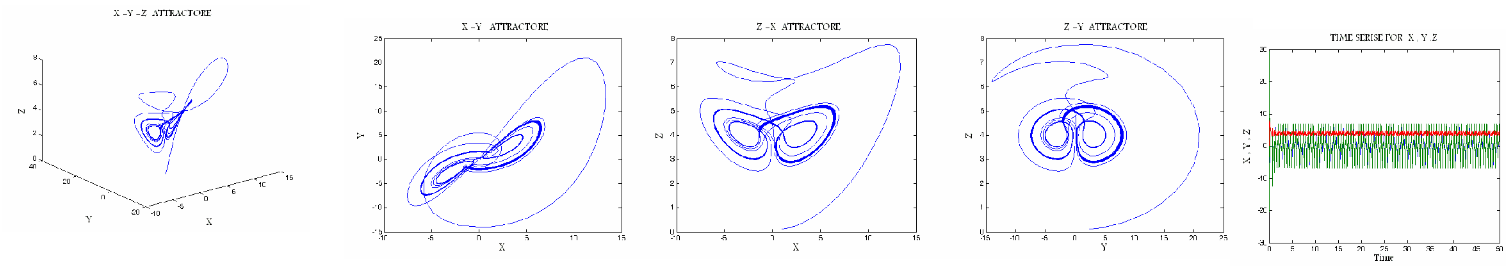
(a, b, c)



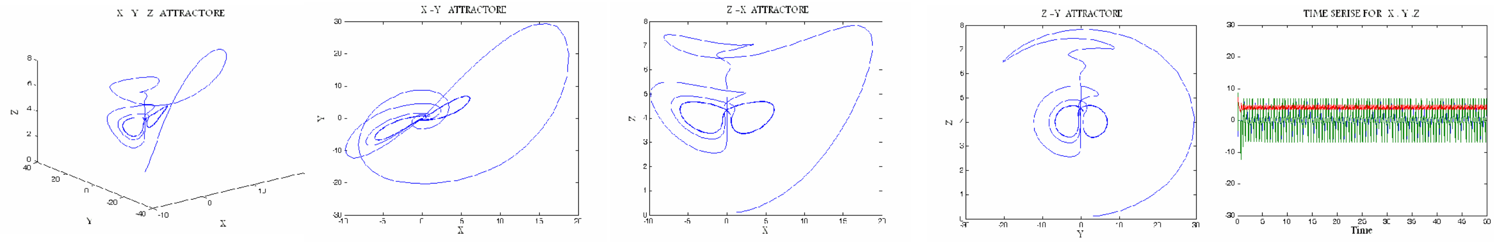
a=40, b= 4, c = 2



a=10, b= 4, c = 2



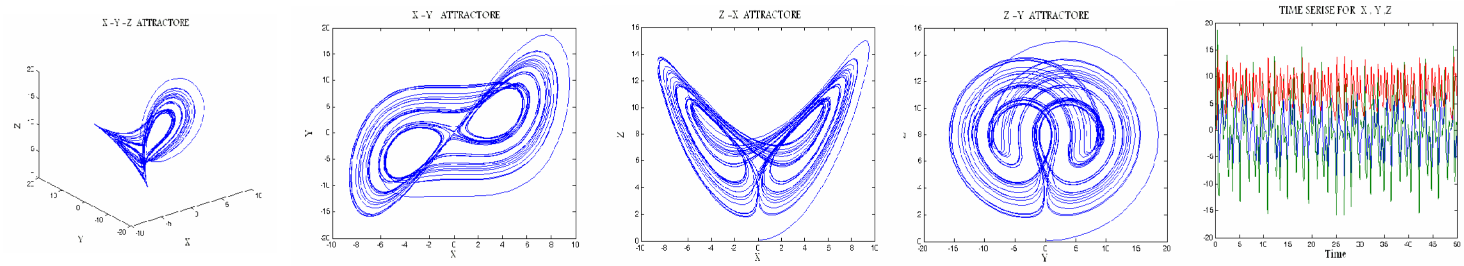
a=28, b= 4, c = 2



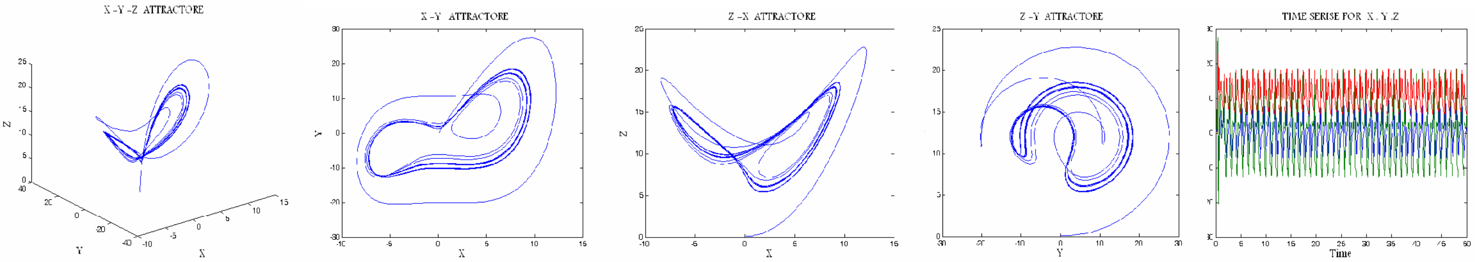
a=56, b= 4, c = 2

Fig (4a): A summary of attractors and time series of the variables x, y and z for the combination (a, b, c) shown

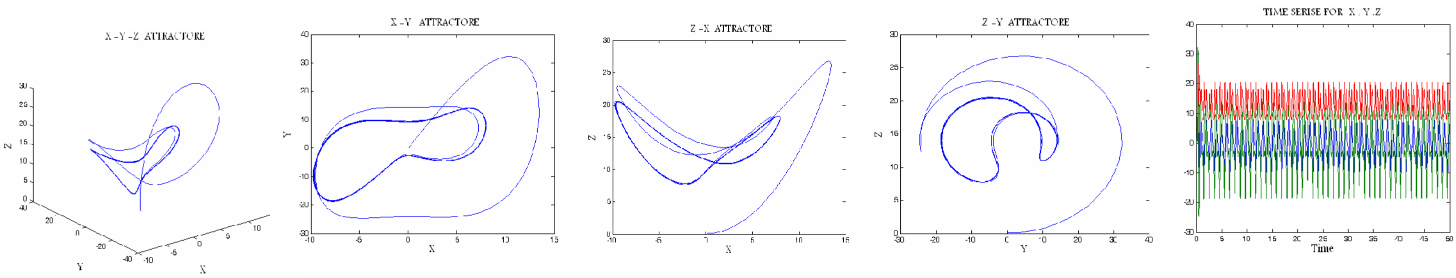
Continue



a=5, b= 8, c = 2



a=5, b= 12, c = 2



a=5, b= 14, c = 2

Fig (4b)

Continued

a=0.5,b=4,c=0.5

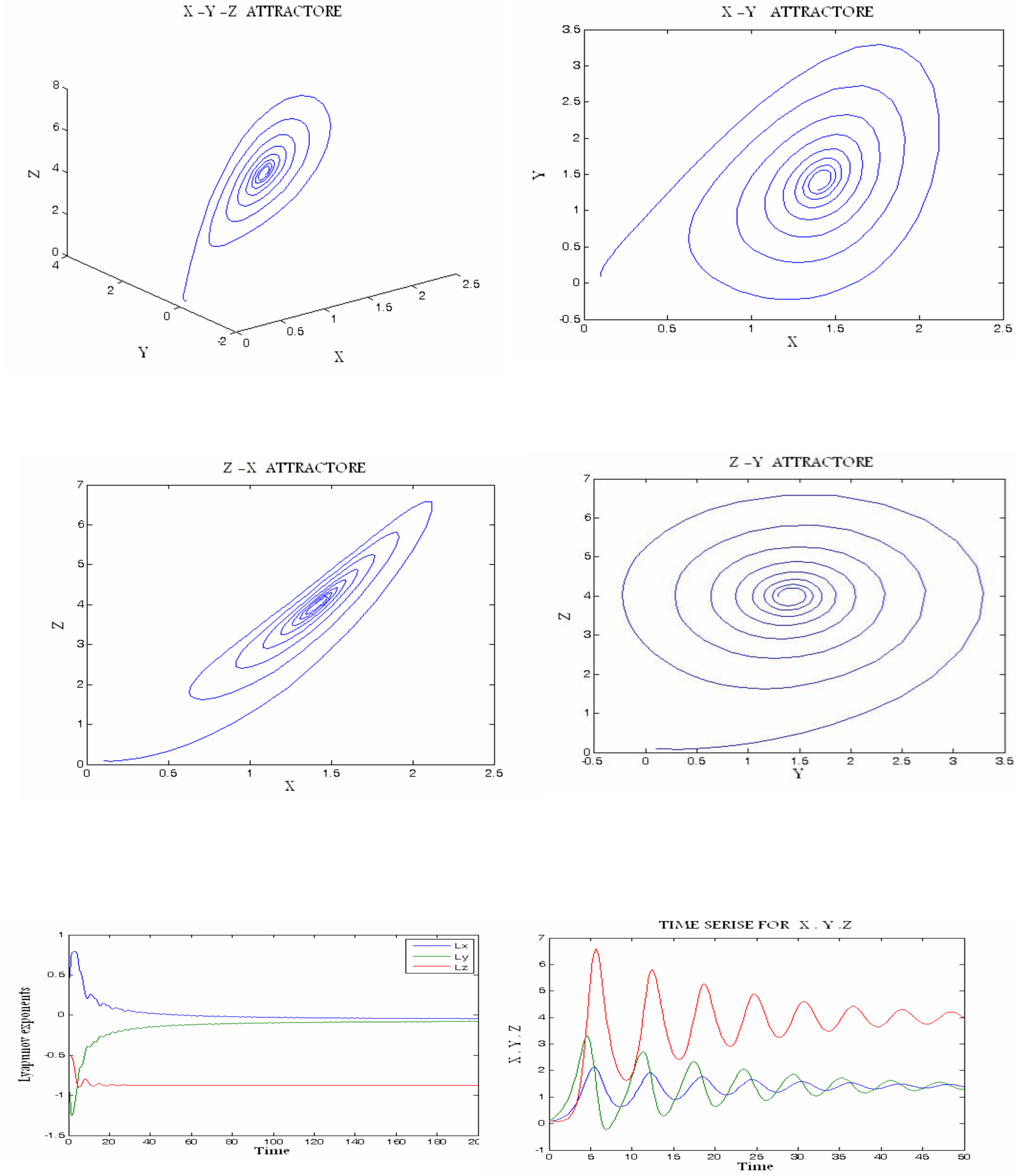


Fig (5a): Another examples of different behavior of the new system for different combinations (a, b, c)for a=0.5,b=4,c=0.5

Continue

a=5,b=4,c=0

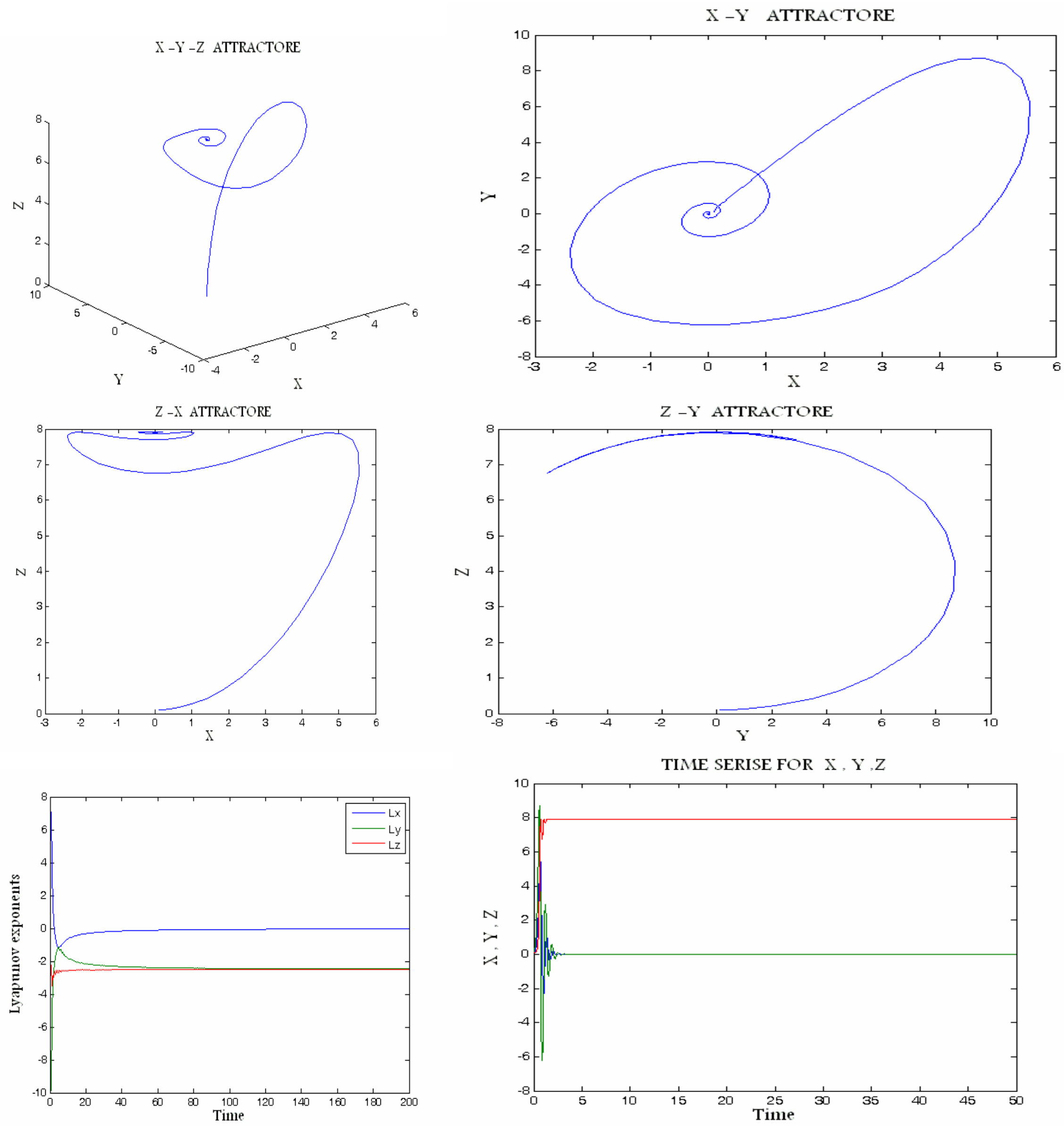
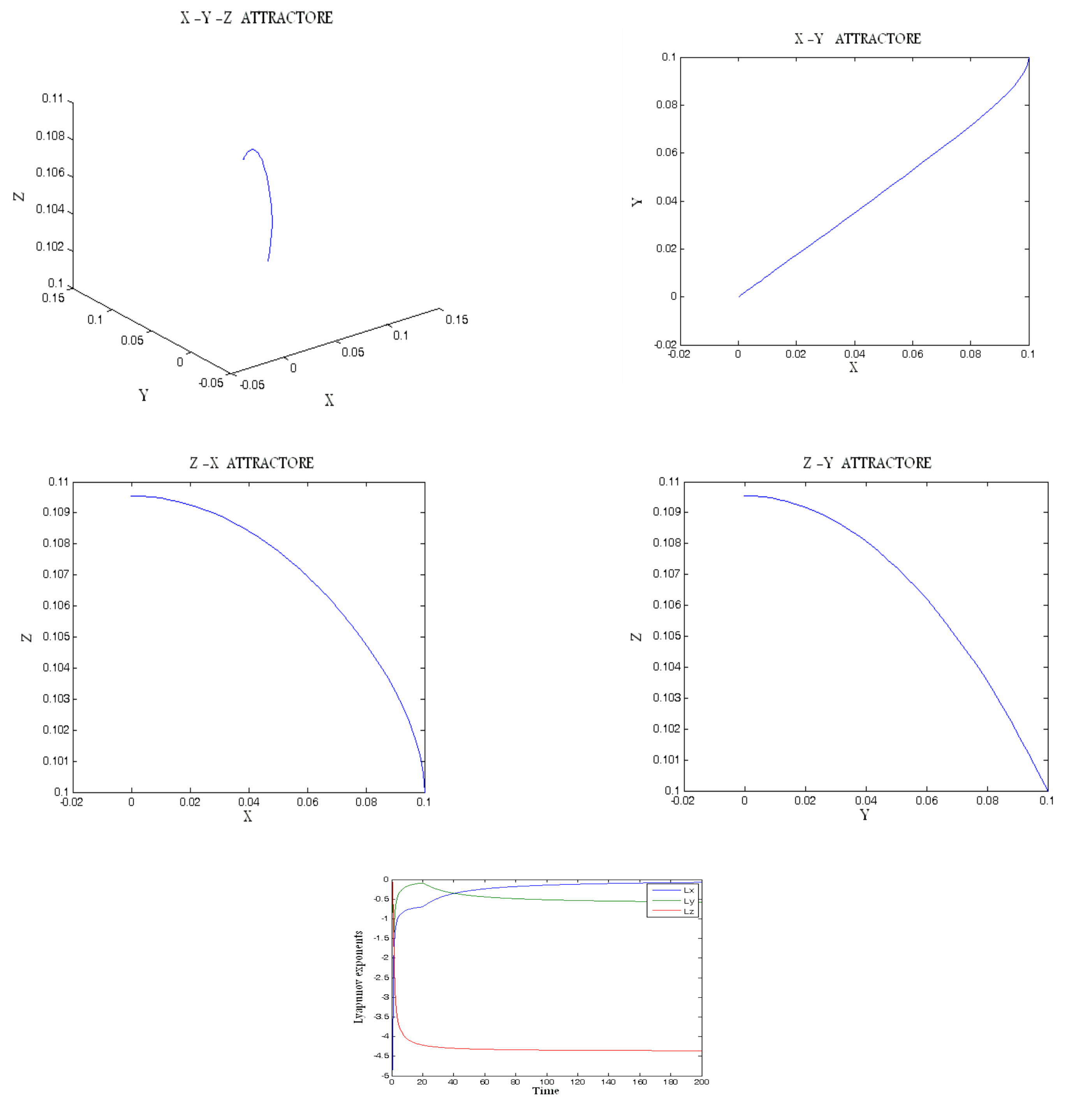


Fig (5b)

Continue



a=3,b=0,c=0

Fig (5c)

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دراسة أنموذج جديد شبيه بأنموذج لورنز

الاء منذر شري

قسم الفيزياء/كلية التربية/جامعة البصرة

الخلاصة

هذا البحث يقدم نتائج دراسة عدديه اجريت على أنموذج جديد شبيه بأنموذج لورنز التقليدي بأستعمال عدد من المؤشرات كالسلاسل الزمنية والجاذبات ودلائل ليبانوف ومخططات التفرع للحركيات التي يمكن ان يبديها النظام الجديد والشبيه بالأنموذج الاصلي