

Longitudinal C2 form factors in sd-shell nuclei including core-polarization effect up to second order

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Abstract:

Coulomb form factors of C2 transitions in several selected sd-shell nuclei have been studied. Core-polarization effects are included through a microscopic theory that includes excitations from the core 1s and 1p orbits and also from 2s-1d shell to higher orbits with $2\hbar\omega$ excitations. The second order core-polarization leads to a minor enhancement of the calculated form factor, improving the agreement with experiment.

I.INTRODICTION

The nuclear shell model has been successful used into description of various aspects of nuclear structure, partly because it is based on a minimum number of natural assumption [1].

Shell model calculations carried out with a model space in which the nucleons are restricted to occupy few orbits are unable to reproduce the experimental data without scaling factors. Thus transition rates or electron scattering form factors in sd-shell are not explained by the simple shell model, when a few nucleons are allowed to be distributed over the sd-shell orbits out side a closed ^{16}O core [2].

Inadequacies in the shell model wave functions are revealed by the need to take into account higher configurations which are called core-polarization effects. These effects are found essential for obtaining a quantitative agreement with the experimental data [3,4].

Such a microscopic model which adopted the first order core polarization was consider recently [5] to calculate the C2 and C4 form factors of the even-even sd-shell nuclei.

The purpose of the present work is to consider the 2particle-2hole (2p-2h) excitation as a second order core-polarization through a microscopic theory for C2 transition in ^{17}O , ^{20}Ne , ^{24}Mg , ^{27}Al . In these calculations the sd-shell first order core-polarization taken from the work of Radhi et al [5], where the "universal" (USD) interaction of windenthal [6] is used.

The higher configuration are taken into account through excitations from the 1s and 1p core orbits and also from 2s-1d shell into higher shells, with $2\hbar\omega$ excitations. The modified surface delta interaction (MSDI) [7] is used in this case as a residual interaction. The single particle wave function are those of the harmonic oscillator (HO) potential with size parameter b chosen to reproduce the measured ground state root mean square charge radii of these nuclei.

II.Theory

The core polarization effect on the form factors is based on a microscopic theory, which combines shell-model waves functions and configuration with higher energy as particle-hole perturbation expansion. The reduced matrix element of the electron scattering operator \hat{T}_Λ^ξ is expressed as a sum of the sd-model space (sd) contribution and the core-polarization (cp) contribution, as follows

$$\langle \Gamma_f \parallel \hat{T}_\Lambda^\xi \parallel \Gamma_i \rangle = \langle \Gamma_f \parallel \hat{T}_\Lambda^\xi \parallel \Gamma_i \rangle_{sd} + \langle \Gamma_f \parallel \delta \hat{T}_\Lambda^\xi \parallel \Gamma_i \rangle_{cp} \quad \dots(1)$$

where the states $|\Gamma_i\rangle$ and $|\Gamma_f\rangle$ are described by the model space wave functions. The Greek symbols are used to denote quantum numbers in coordinate space and isospace, i.e $\Gamma_i \equiv J_i T_i$, $\Gamma_f \equiv J_f T_f$ and $\Lambda \equiv J T$. The notation ξ represents longitudinal (L) or the transverse (electric E or magnetic M). The sd-shell model space element is given by [9],

$$\langle \Gamma_f \parallel \hat{T}_\Lambda^\xi \parallel \Gamma_i \rangle_{sd} = \sum_{\alpha_f \alpha_i} \chi^{\Lambda}_{\Gamma_f \Gamma_i}(\alpha_f, \alpha_i) \langle \alpha_f \parallel \hat{T}_\Lambda^\xi \parallel \alpha_i \rangle \quad \dots(2)$$

where $\chi^{\Lambda}_{\Gamma_f \Gamma_i}(\alpha_f, \alpha_i)$ are the structure factor (one body density matrix element), given by,

$$\chi^{\Lambda}_{\Gamma_f \Gamma_i}(\alpha_f, \alpha_i) = \frac{\langle \Gamma_f \parallel [a^+(\alpha_f) \otimes \tilde{a}(\alpha_i)]^\Lambda \parallel \Gamma_i \rangle}{\sqrt{2\Lambda + 1}} \quad \dots(3)$$

α_f and α_i label single-particle states for the sd-shell model space.

Similarly, core-polarization matrix element is written as:

$$\langle \Gamma_f \parallel \delta \hat{T}_\Lambda^\xi \parallel \Gamma_i \rangle_{cp} = \sum_{\alpha_f \alpha_i} \chi^{\Lambda}_{\Gamma_f \Gamma_i}(\alpha_f, \alpha_i) \langle \alpha_f \parallel \delta T_\Lambda^\xi \parallel \alpha_i \rangle \quad \dots(4)$$

Up to the second order perturbation theory, the single-particle matrix element for the higher-energy configuration is given by [9]

$$\begin{aligned} \langle \alpha_f \parallel \delta T_\Lambda^\xi \parallel \alpha_i \rangle = & \langle \Gamma_f \parallel \hat{T}_\Lambda^\xi \frac{Q}{E - H_o} V_{res} \parallel \Gamma_i \rangle + \langle \Gamma_f \parallel V_{res} \frac{Q}{E - H_o} \hat{T}_\Lambda^\xi \parallel \Gamma_i \rangle \\ & + \langle \Gamma_f \parallel V_{res} \frac{Q}{E - H_o} \hat{T}_\Lambda^\xi \frac{Q}{E - H_o} V_{res} \parallel \Gamma_i \rangle \\ & + \langle \Gamma_f \parallel \hat{T}_\Lambda^\xi \frac{Q}{E - H_o} V_{res} \frac{Q}{E - H_o} V_{res} \parallel \Gamma_i \rangle \\ & + \langle \Gamma_f \parallel V_{res} \frac{Q}{E - H_o} V_{res} \frac{Q}{E - H_o} \hat{T}_\Lambda^\xi \parallel \Gamma_i \rangle \end{aligned} \quad \dots(5)$$

The operator Q is the projection operator on the space outside the model space and E is the true energy of the system. For the residual interaction V_{res} we adopt the MSDI [8]. The strength of MSDI are denoted by A_T, B, C where T indicates to the isospin (0 or 1). These parameters are set equal to $A_0 = A_1 = B = 0.45$ MeV and $C = 0$ for both nuclei ^{20}Ne , ^{24}Mg and set equal $A_0 = A_1 = B = 20/A$ and $C = 0$ for both nuclei ^{17}O , ^{27}Al . This choice of parameters is used in ref [5,17]. For the calculation of the C2 form factor of sd-shell nuclei .

The longitudinal inelastic electron scattering form factor for a given multipolarity Λ and momentum transfer q is expressed as [10],

$$|F_\Lambda(q)|^2 = \frac{1}{2j_i + 1} \left(\frac{4\pi}{Z^2} \right) \left| \langle \Gamma_f \parallel \hat{T}_\Lambda^\xi \parallel \Gamma_i \rangle \right|^2 |F_{f.s} F_{c.m}|^2 \quad \dots(6)$$

Where $F_{f.s} = e^{-0.43q^2/4}$ is the finite nucleon-size correction and $F_{c.m} = q^2 b^2 / 4A$ is the center of mass correction. Where A is the mass number and b is the harmonic oscillator size parameter.

III.Results and Discussion

The longitudinal C2 form factors of few sd-shell nuclei have been calculated by using the radial wave function for harmonic oscillator (HO) with size parameter b which was chosen to reproduce the measured ground state root mean square (rms) charge radii and these were displayed in table(1).

The nucleus ^{17}O : The longitudinal electron scattering form factors for $\frac{1}{2}^+ \frac{1}{2}$ state in ^{17}O which calculated by using the core-polarization effects including the first and second order contributions are shown in figure(1). Calculations that carried out up to second order core polarization result agree very well with the experimental data in the first maximum and has a good agreement at $q \geq 1.7 \text{ fm}^{-1}$. While, the only first order calculation overestimates the experimental data in the second maximum. To compare our calculation with second order calculation of ref. [12], when two particle-two hole excitation from 1s and 1p shell core orbits to higher allowed orbits with $2\hbar\omega$ excitation, our calculation results describe the data better than the ref. [12], result in both region of first and second maximum as shown in figure(5). The form factors for the model space are not found, we believe this can be understood on the basis that in this nucleus the nucleon outside the core is a neutron where it has no charge which implies no Coulomb form factor, but when the effects of core-polarization are taken into account the form factor for both first and second order contribution taken place.

The nucleus ^{20}Ne : Figure(2) shows the calculations for the isoscalar C2 transition($J_i^\pi = 0^+, T_i = 0$) to ($J_f^\pi = 2^+, T_f = 0$) at $E_x = 1.63 \text{ MeV}$. First order core-polarization effect enhances C2 form factors at the first and second maximum and the contribution of the second order bring the calculated values very close to the experimental data. The data are much better reproduced in the $q > 1 \text{ fm}^{-1}$, especially after inclusion of second order core-polarization effect.

The nucleus ^{24}Mg : The calculations for the isoscalar C2 transition from the ground state ($J_i^\pi = 0^+, T_i = 0$) to the excited state ($J_f^\pi = 2^+, T_f = 0$) at $E_x = 1.37 \text{ MeV}$ are shown in figure(3). The inclusion of the core-polarization to the second order gives a good agreement with experimental data. This enhancement in the inclusion of second order effect over the first-order predication is much better as the enhancement obtained by ref. [12], as shown in figure(6).

The nucleus ^{27}Al : Figure(4) shows the calculation for the C2 transition from the ground state ($J_i^\pi = 5/2^+, T_i = 1/2$) to the excited state ($J_f^\pi = 1/2^+, T_f = 1/2$) at $E_x = 0.884 \text{ MeV}$. The calculated longitudinal C2 electron scattering form factor including the core polarization effect and the model space for ^{27}Al indicate that the second-order contribution improve the agreement up to $q > 1.5 \text{ fm}^{-1}$ and somehow closer to the experimental data in the second maximum, then the result start to overestimates the experimental data at $q > 2.5 \text{ fm}^{-1}$.

The results of the present work show that in general the effects of second order core polarization improve the agreement with experiment. The core polarization calculations presented

in the present work succeeded in describing the electron scattering data at the both open-shell nuclei ^{17}O , ^{27}Al and closed even-even($N=Z$) for ^{20}Ne and ^{24}Mg sd-shell nuclei. Also in comparison with result of ref. [12], the effect of higher excited configuration is found to be essential in the moment transfer dependent of form factors.

Table(1): The value of the size parameter b and the excitation energy E_x [11].

<i>Nucleus</i>	J_i^π	$b(\text{fm})$	$E_x(\text{MeV})$
^{17}O	$5/2^+$	1.763	0.87
^{20}Ne	0^+	1.869	1.63
^{24}Mg	0^+	1.813	1.37
^{27}Al	$5/2^+$	1.804	0.884

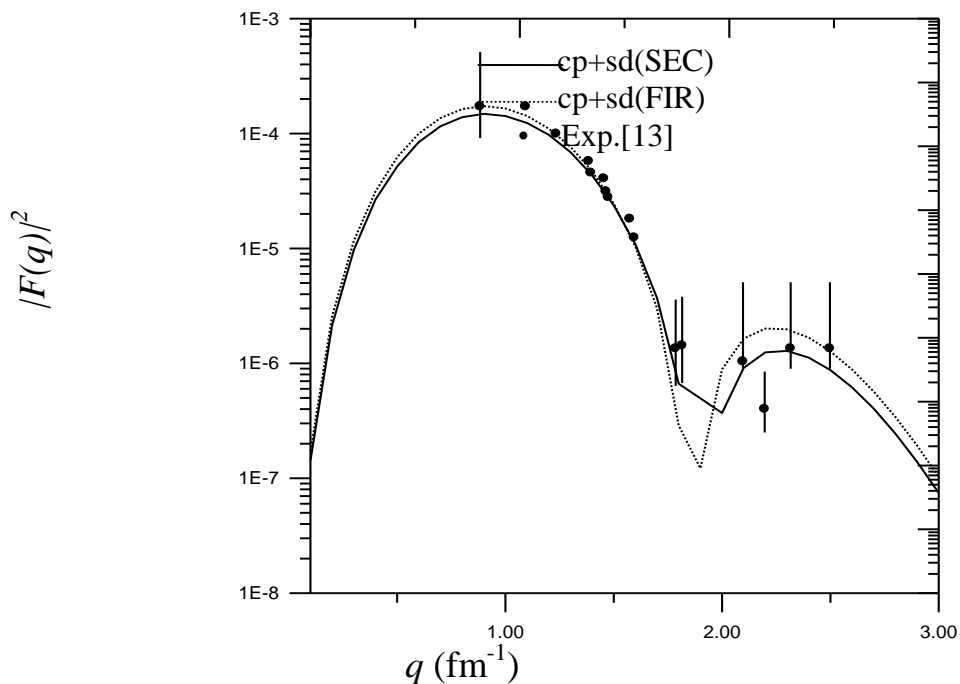
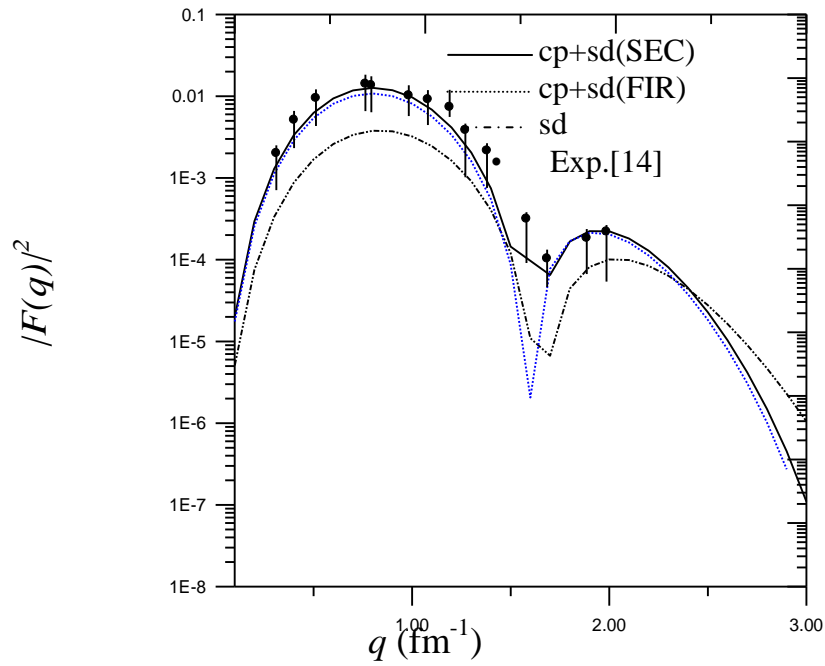


Fig.(1)The Coulomb form factor of quadrupole transition to the $1/2^+ 1/2$ state in ^{17}O . The experimental data from ref.[13].



Fig(2) The Coulomb form factor of quadrupole transition to the $2^+ 0$ state in ^{20}Ne . The experimental data from ref.[14].

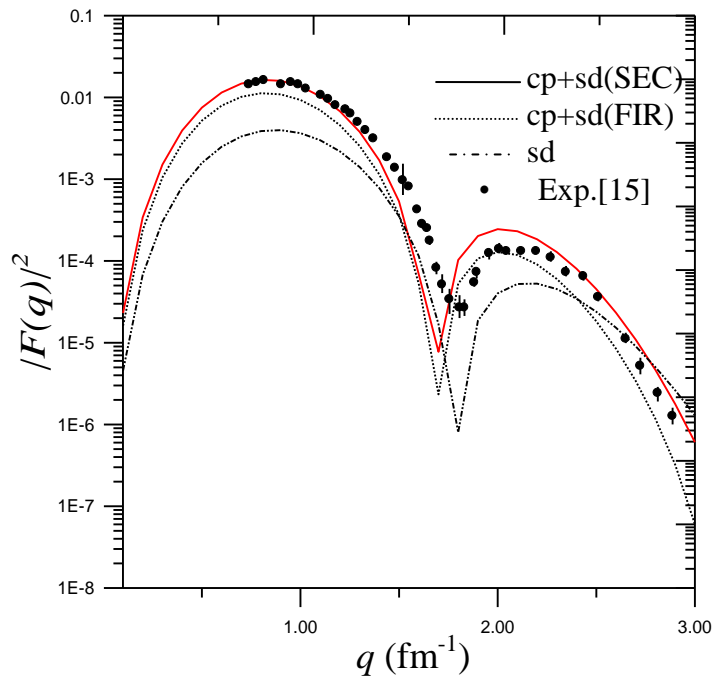


Fig.(3) The Coulomb form factor of quadrupole transition to the 2^+ state in ^{24}Mg . The experimental data from ref.[15].

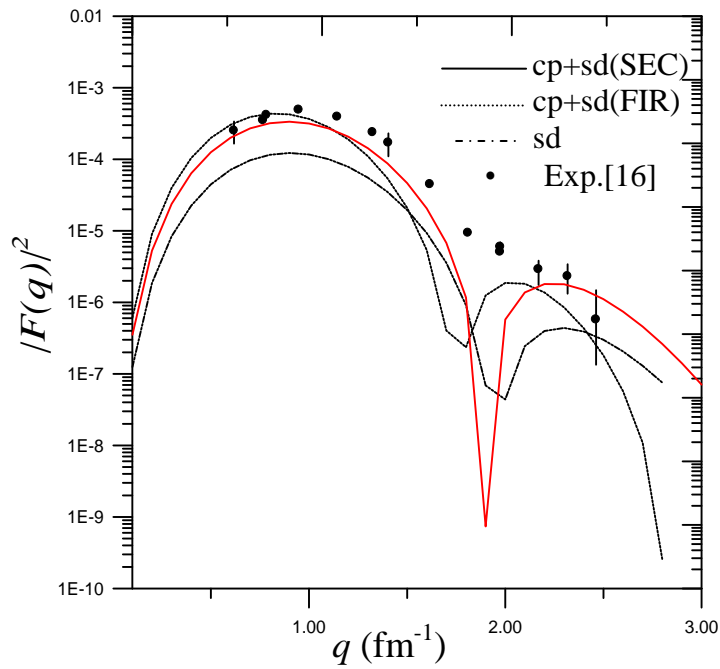


Fig.(4) The Coulomb form factor of quadrouple trastion to the $1/2^+ 1/2$ state ^{27}Al . The experimental data from ref. [16].

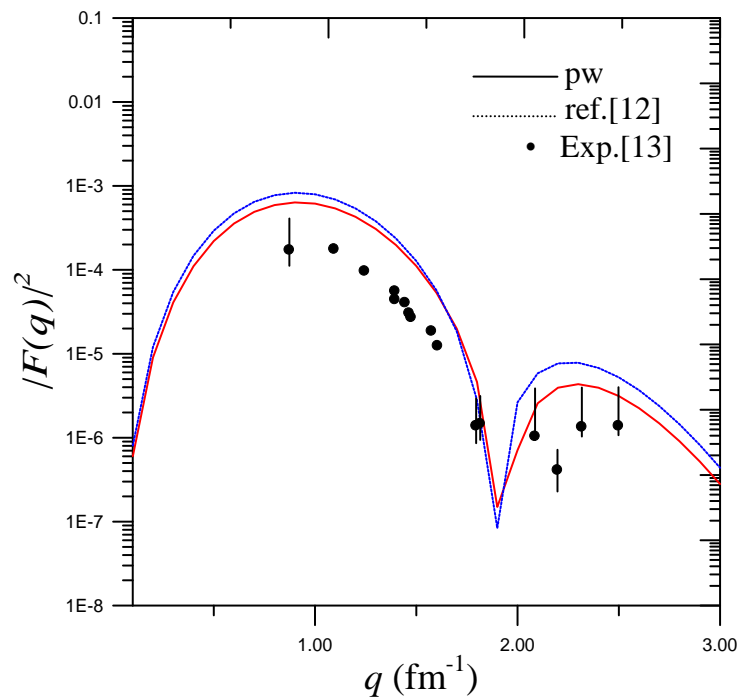


Fig.(5) The Coulomb form factor of quadrouple transition to the $1/2^+$ state in ^{17}O . The experimental data from ref.[13] .

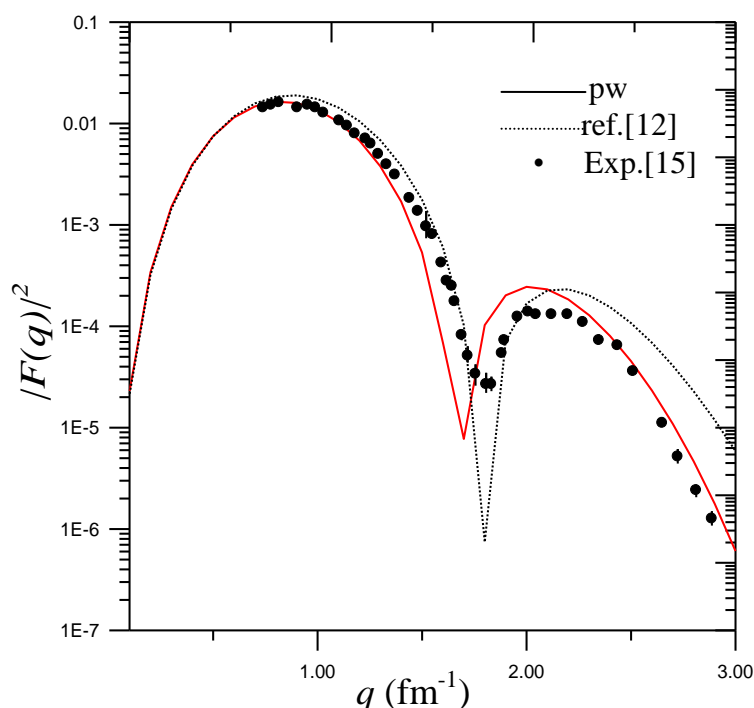


Fig.(6) The Coulomb form factor of quadrupole transition to the 2^+ state in ^{24}Mg . The experimental data from ref.[15] .

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