

Nuclear structure of $^{74,76}\text{Ge}$ isotopes which have gamma unstable O(6) dynamical symmetry using interacting boson model

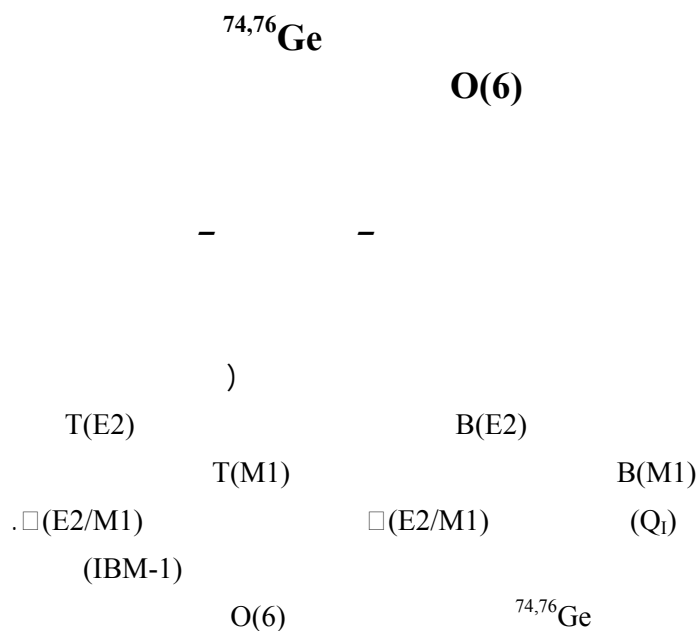
A.A.W. Redah

Physics Department, Science College, Thi-Qar University

Abstract

In this work we studied the nuclear properties (energy levels , gamma transitions , quadrupole electric transitions B(E2) and its reduced matrix elements T(E2) , dipole magnetic transitions B(M1) and its reduced matrix elements T(M1) , electric quadrupole moments for ground state (Q_0) , finally classified these transitions to its delta mixing ratio $\delta(E2/M1)$ and reduced mixing ratio $\delta(E2/M1)$.

All of these structure we calculated using IBM-1 programs for germanium isotopes ($A=74,76$) which have gamma unstable of the O(6) dynamical symmetry. Our results show a good agreement with the available experimental data.



Introduction

The interacting boson model was introduced in an attempt to describe in a unified way collective properties of nuclei, this model is rooted in the spherical shell model which is the fundamental model for describing properties of nuclei⁽¹⁾(Bonatsos,1988).

In the simplest version of the interacting boson model (IBM-1), it is assumed that low-lying collective states in even-even nuclei away from closed shells are dominated by excitation of the valence protons and the valence neutrons (particles outside the major closed shell) while the closed shell core is inert. Furthermore, it is assumed that the particle configurations which are most important in shaping the properties of the low-lying states are those in which identical particles are coupled together forming pairs of angular momentum 0 and 2^(1,2)(Bonatsos,1988; Arima et al.,1987).

In addition, these proton (neutron) pairs are treated as bosons, the boson with angular momentum (L=0) are denoted by s (s) and are called s-bosons, while bosons with (L=2)⁽³⁾(Casten et al.,1988) are denoted by d (d) and are called d-bosons. The number of valence proton (neutron) pairs, N (N) is counted from nearest magic number.

Interacting boson model was applied by two types of structures, the first is phenomenological structure which contains linear algebra and group theory, the second type is geometrical collective properties. In the present work, we used the (IBM-1) which deals with phenomenological structure underlying U(6)⁽⁴⁾(Arima et al.,1981) group basis leads to a simple Hamiltonian which is capable of describing the three specific limits of collective structure: vibrational U(5), rotational SU(3) and gamma unstable O(6).

Theoretical part

The hamiltonian operator according to IBM-1 can be written by creation and annihilation operators as follows^(2,5)(Arima et al.;Abrahams et al.1981):

$$\hat{H} = \epsilon \hat{n}_d + a_0 (\hat{P} \hat{P}) + a_1 (\hat{L} \hat{L}) + a_2 (\hat{Q} \hat{Q}) + a_3 (\hat{T}_3 \hat{T}_3) + a_4 (\hat{T}_4 + \hat{T}_4) \dots \dots \dots (1)$$

Where $\epsilon = \epsilon_s + \epsilon_d$ is the boson energy

a_0, a_1, a_2, a_3, a_4 are the phenomenological parameters.

For O(6) dynamical symmetry, the hamiltonian operator becomes:

$$\hat{H} = a_0(\hat{P}.\hat{P}) + a_1(\hat{L}.\hat{L}) + a_3(\hat{T}_3.\hat{T}_3)..... (2)$$

The electric quadrupole transition operator in the IBM-1 can be written as:

$$\hat{T}^{(E2)} = \alpha_2 \left[\hat{d}^+ \times \hat{s} + \hat{s}^+ \times \hat{d} \right]_{\mu}^{(2)} + \beta_2 \left[\hat{d}^+ \times \hat{d} \right]_{\mu}^{(2)} (3)$$

Where α_2 is the effective charge of boson.

β_2 is the effective charge of one particle in d-boson.

While the electric quadrupole transition probability can calculated by:

$$B(E2; I_i \rightarrow I_f) = \frac{1}{2I_i + 1} \left| \langle I_f | \hat{T}^{(E2)} | I_i \rangle \right|^2 (4)$$

Where I is angular momentum.

And the electric quadrupole moment (Q_1) is:

$$Q_1 = \sqrt{\frac{16\pi}{5}} \begin{pmatrix} I & 2 & I \\ -I & 0 & I \end{pmatrix} \langle I_f | \hat{T}^{(E2)} | I_i \rangle (5)$$

The magnetic dipole transition operator in the IBM-1 can be written as:

$$\begin{aligned} \hat{T}^{(M1)} = & \beta_1 \left[\hat{d}^+ \times \hat{d} \right]_{\mu}^{(1)} + \alpha'_1 \left[\left(\hat{d}^+ \times \hat{s} + \hat{s}^+ \times \hat{d} \right)_{\mu}^{(2)} \times \left(\hat{d}^+ \times \hat{d} \right)_{\mu}^{(1)} \right] + \\ & \gamma'_1 \left[\left(\hat{d}^+ \times \hat{d} \right)_{\mu}^{(0)} \times \left(\hat{d}^+ \times \hat{d} \right)_{\mu}^{(1)} \right] + \delta'_1 \left[\left(\hat{d}^+ \times \hat{d} \right)_{\mu}^{(2)} \times \left(\hat{d}^+ \times \hat{d} \right)_{\mu}^{(1)} \right] + \\ & \eta'_1 \left[\left(\hat{s}^+ \times \hat{s} \right)_{\mu}^{(0)} \times \left(\hat{d}^+ \times \hat{d} \right)_{\mu}^{(1)} \right] (6) \end{aligned}$$

Where $\beta_1, \alpha'_1, \gamma'_1, \delta'_1, \eta'_1$ are Linear Coefficients.

While the magnetic dipole transition probability can calculated by:

$$B(M1; I_i \rightarrow I_f) = \frac{1}{2I_i + 1} \left| \langle I_f | \hat{T}^{(M1)} | I_i \rangle \right|^2 (7)$$

The mixing ratios $\delta(E2/M1)$ and reduce mixing ratios $\square(E2/M1)$ in IBM-1 can be found from⁽⁶⁾(Pfeifer,1998):

$$\delta(E2/M1; I_i \rightarrow I_f) = \sqrt{\frac{3}{100}} q \frac{\langle I_f | \hat{T}^{(E2)} | I_i \rangle}{\langle I_f | \hat{T}^{(M1)} | I_i \rangle} (8)$$

$$\Delta(E2/M1) = \frac{\delta(E2/M1)}{0.835 \times E_\gamma} \dots\dots\dots (9)$$

Where $q = E_\gamma / \hbar c$

Results and Discussion

In present work we had studied the nuclear properties of germanium Ge (A=74,76) isotopes where it belong to gamma unstable dynamical symmetry O(6) by using interacting boson model. At the first we classified all germanium isotopes to its dynamical symmetries by comparing the experimental energy levels of these isotopes with the ideal chart of U(5), SU(3) and O(6) in IBM-1, another test used also, which is the energy ratios E(4)/E(2) , E(6)/E(2) and E(8)/E(2) comparing with ideal values for the three dynamical symmetries, then choosing the isotopes which deal with gamma unstable O(6), as shown in table (1).

Table (1): theoretical and experimental data for branching ratios comparison with ideal values for IBM-1.

Dynamical Symmetry	E(4 ₁ ⁺)/E(2 ₁ ⁺)		E(6 ₁ ⁺)/E(2 ₁ ⁺)		E(8 ₁ ⁺)/E(2 ₁ ⁺)	
U(5)	2		3		4	
O(6)	2.5		4.5		7	
SU(3)	3.33		7		12	
Isotope	Exp. ⁽⁷⁾	IBM-1	Exp. ⁽⁷⁾	IBM-1	Exp. ⁽⁷⁾	IBM-1
Ge-74	2.4563	2.6560	——	4.9679	——	7.9357
Ge-76	2.5053	2.6663	——	4.9989	——	7.9977

Table (2) listed the two isotopes were used in this work according to their atomic mass number, total number of boson (N) and corresponding hamiltonian parameters, reduced matrix element parameters for the electric quadrupole transitions (α_2, β_2) and magnetic dipole transitions ($\beta_1, \alpha'_1, \gamma'_1, \delta'_1, \eta'_1$). The total number of bosons in ⁷⁴Ge is [2□(particle)+3□(hole)=6] and in ⁷⁶Ge is [2□(particle)+3□(hole)=5].

These parameters were used in (IBS1.For, IBMT.For and M5.For) programs of interacting boson model, which help us to calculate the energy levels, electric moments (Q_I) for ground state, electric quadrupole transitions probability $B(E2)$ and magnetic dipole transitions probability $B(M1)$ comparing with experimental data for Ge-74,76 were shown in table (3,4). Figure (1) explains the agreement between the available experimental data and theoretical calculation of IBM-1 for energy levels with the angular momentums of the levels. Figure (2) shows which transitions or spin sequences has a high values of electric quadrupole transitions probability $B(E2)$ for Ge-74,76. In figure (3) we found the same behavior of magnetic transitions $B(M1)$ for Ge-74, while in Ge-76 there are high and low probabilities in the some spin sequences. The electric quadrupole moments (Q_I) is one of many methods to predict the deformations from the spherical shape of nuclear structure. In the present work, we were used IBM-1 analysis, that's mean when $Q_I=0$, the nuclei or levels has a spherical shape while $Q_I>0$ for prolate and $Q_I<0$ for oblate, therefore figure (4) shows the two isotopes in this work has a prolate shape and the levels which has odd angular momentums has a low deformation and the spherical shape for 3_1 level.

Table (2): total number of boson and the parameters of energy levels, electric transitions and magnetic transitions used in programs of IBM-1.

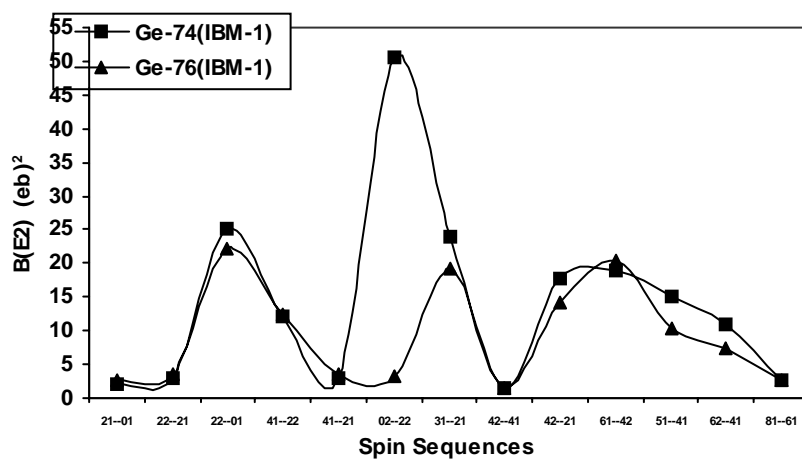
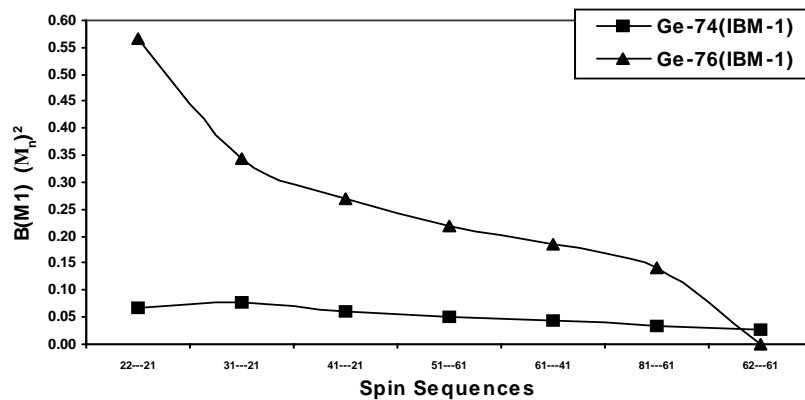
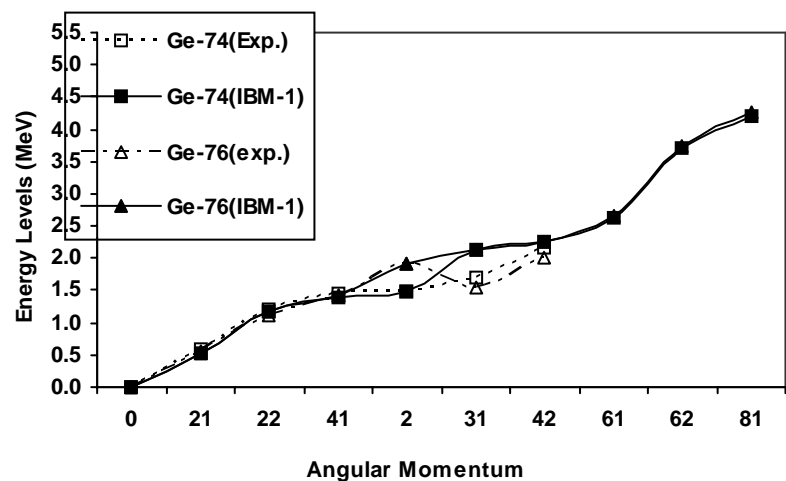
Isotope	N	$\hat{p} \cdot \hat{p}$ (MeV)	$\hat{I} \cdot \hat{I}$ (MeV)	$\hat{T}_3 \cdot \hat{T}_3$ (MeV)	a_2 (eb)	β_2 (eb)	β'_1 (μ_N)	α'_1 (μ_N)	γ'_1 (μ_N)	δ'_1 (μ_N)	η'_1 (μ_N)
Ge-74	6	0.2125	0.0380	0.2150	0.4274	5.1200	0.1720	0.0156	0.3896	0.3583	0.1294
Ge-76	5	0.4300	0.0390	0.2130	0.5464	5.5702	0.2513	0.4832	0.0165	0.2744	-0.3723

Table (3): energy levels, electric moments for ground state, electric quadrupole transitions probability and magnetic dipole transitions probability comparison with experimental data for Ge-74 isotope.

I^π	Energy Levels (MeV)		Q_1	$I_i^+ - I_f^+$	$B(E2) (eb)^2$	$B(M1) (\mu_N)^2$
	Exp. ^(8,9,10,11)	IBM-1				
0_1^+	0.0000	0.0000		—	—	—
2_1^+	0.5959	0.5290	$10^{+2} \times 0.1612$	$2_1^+ - 0_1^+$	$10^{+1} \times 0.2192$	0.0000
2_2^+	1.2043	1.1740		$2_2^+ - 2_1^+$	$10^{+1} \times 0.2871$	$10^{-1} \times 0.6737$
4_1^+	1.4637	1.4050	$10^{+2} \times 0.2740$	$2_2^+ - 0_1^+$	$10^{+2} \times 0.2522$	0.0000
0_2^+	1.4826	1.4875		$4_1^+ - 2_2^+$	$10^{+2} \times 0.1213$	—
3_1^+	1.6972	2.1330	$10^{-6} \times 0.4133$	$4_1^+ - 2_1^+$	$10^{+1} \times 0.2871$	$10^{-1} \times 0.6151$
0_3^+	1.9130	1.9350		$0_2^+ - 2_2^+$	$10^{+2} \times 0.5044$	—
4_2^+	2.1658	2.2650		$0_2^+ - 2_1^+$	$10^{-11} \times 0.1802$	0.0000
2_3^+	2.1980	2.0165		$3_1^+ - 4_1^+$	0.8351	$10^{-1} \times 0.7908$
0_4^+	2.2270	2.5500		$3_1^+ - 2_2^+$	$10^{+1} \times 0.2088$	$10^{-1} \times 0.3828$
6_1^+	2.5720	2.6280	$10^{+2} \times 0.4025$	$3_1^+ - 2_1^+$	$10^{+2} \times 0.2402$	$10^{-1} \times 0.7908$
2_4^+	2.6460	2.6615		$0_3^+ - 2_2^+$	$10^{+1} \times 0.2923$	0.0000
4_3^+	—	2.8925		$0_3^+ - 2_1^+$	$10^{+2} \times 0.3363$ Exp. ⁽¹¹⁾ = 30.00 ± 3.2	—
2_5^+	—	3.0790		$4_2^+ - 4_1^+$	$10^{+1} \times 0.1392$	$10^{-1} \times 0.3743$
2_6^+	—	3.1090		$4_2^+ - 2_2^+$	$10^{+1} \times 0.1531$	$10^{-2} \times 0.1264$
0_5^+	—	3.1875		$4_2^+ - 2_1^+$	$10^{+2} \times 0.1761$	$10^{-1} \times 0.3743$
4_4^+	—	3.3400		$2_3^+ - 2_2^+$	0.0000	$10^{-14} \times 0.6658$
5_1^+	—	3.5050	$10^{+2} \times 0.1832$	$2_3^+ - 4_1^+$	$10^{-13} \times 0.4306$	$10^{-14} \times 0.5555$
3_2^+	—	3.6205		$2_4^+ - 4_1^+$	0.0000	$10^{-2} \times 0.3617$
6_2^+	—	3.7030		$0_4^+ - 2_2^+$	$10^{-11} \times 0.1322$	—
2_7^+	—	3.7240		$0_4^+ - 2_1^+$	$10^{-11} \times 0.4689$	—
4_5^+	—	3.7525		$6_1^+ - 4_2^+$	$10^{+2} \times 0.1899$	0.0000
4_6^+	—	3.9550		$6_1^+ - 4_1^+$	$10^{+1} \times 0.2923$	$10^{-1} \times 0.4258$
6_3^+	—	4.1155		$2_4^+ - 4_2^+$	0.0000	$10^{-1} \times 0.6755$
8_1^+	—	4.1980		$2_4^+ - 2_3^+$	$10^{+1} \times 0.1409$	—
2_8^+	—	4.3990		$8_1^+ - 6_1^+$	$10^{+1} \times 0.2591$	$10^{-1} \times 0.3256$
5_2^+	—	4.7950		$4_3^+ - 2_4^+$	$10^{+2} \times 0.1544$	0.0000
7_1^+	—	5.2240		$2_5^+ - 0_4^+$	0.4384	—
8_2^+	—	5.4880		$5_1^+ - 6_1^+$	0.6123	$10^{-1} \times 0.5033$
3_3^+	—	6.0030		$5_1^+ - 4_1^+$	$10^{+2} \times 0.1517$	$10^{-1} \times 0.5033$
10_1^+	—	6.1150		$6_2^+ - 6_1^+$	0.8243	$10^{-1} \times 0.2591$
7_2^+	—	6.7290		$6_2^+ - 4_1^+$	$10^{+2} \times 0.1084$	$10^{-1} \times 0.2591$

Table (4): energy levels, electric moments for ground state, electric quadrupole transitions probability and magnetic dipole transitions probability comparison with experimental data for Ge-76 isotope.

I^π	Energy Levels (MeV)		Q_1	$I_i^+ - I_r^+$	$B(E2) (eb)^2$	$B(M1) (\mu_N)^2$
	Exp. ^(8,9,10,11)	IBM-1				
0_1^+	0.0000	0.0000		—	—	—
2_1^+	0.5628	0.5322	$10^{+2} \times 0.1601$	$2_2^+ - 2_1^+$	$10^{+1} \times 0.3412$	0.5647
2_2^+	1.1090	1.1712		$2_2^+ - 0_1^+$	$10^{+2} \times 0.2216$	0.0000
4_1^+	1.4100	1.4190	$10^{+2} \times 0.2782$	$4_1^+ - 2_2^+$	$10^{+2} \times 0.1251$	—
3_1^+	1.5394	2.1294	$10^{-6} \times 0.3110$	$4_1^+ - 2_1^+$	$10^{+1} \times 0.3412$	0.2683
0_2^+	1.9111	1.9170		$3_1^+ - 4_1^+$	0.9383	0.3450
4_2^+	2.0199	2.2710		$3_1^+ - 2_1^+$	$10^{+2} \times 0.1935$	0.3450
0_3^+	—	2.5800		$0_2^+ - 2_1^+$	$10^{+2} \times 0.2709$ Exp. ⁽¹¹⁾ = 27.00 ± 2.0	—
6_1^+	—	2.6604	$10^{+2} \times 0.4185$	$4_2^+ - 4_1^+$	$10^{+1} \times 0.1564$	$10^{-15} \times 0.3137$
2_3^+	—	3.0882		$4_2^+ - 2_2^+$	$10^{+1} \times 0.1720$	$10^{-1} \times 0.1035$
2_4^+	—	3.1122		$4_2^+ - 2_1^+$	$10^{+2} \times 0.1419$	$10^{-15} \times 0.3137$
4_3^+	2.0140	3.3360		$6_1^+ - 4_2^+$	$10^{+2} \times 0.2053$	0.0000
5_1^+	—	3.5130	$10^{+2} \times 0.1950$	$6_1^+ - 4_1^+$	$10^{+1} \times 0.3284$	0.1858
6_2^+	—	3.7254		$2_3^+ - 0_2^+$	0.9554	—
2_5^+	2.5030	3.7512		$2_3^+ - 3_1^+$	$10^{+1} \times 0.1194$	0.0000
4_4^+	2.7330	3.9990		$2_3^+ - 4_1^+$	$10^{+1} \times 0.3870$	0.3644
8_1^+	—	4.2564		$2_3^+ - 2_2^+$	$10^{+2} \times 0.1238$	$10^{-15} \times 0.9923$
0_4^+	—	4.3000		$2_4^+ - 4_1^+$	0.0000	$10^{-1} \times 0.1941$
2_6^+	—	4.3662		$4_3^+ - 3_1^+$	—	0.2124
0_5^+	—	4.4970		$4_3^+ - 2_4^+$	0.0000	0.3548
4_5^+	—	4.6140		$4_3^+ - 4_1^+$	$10^{+1} \times 0.1626$	0.2024
3_2^+	—	4.7094		$4_3^+ - 2_2^+$	$10^{+2} \times 0.1118$	$10^{-15} \times 0.5513$
5_2^+	—	4.7910		$4_3^+ - 2_1^+$	$10^{-14} \times 0.8273$	0.2024
2_7^+	—	4.8322		$5_1^+ - 6_1^+$	0.6159	0.2195
4_6^+	—	4.8510		$5_1^+ - 4_1^+$	$10^{+2} \times 0.1034$	0.2195
6_3^+	—	5.0034		$6_2^+ - 6_1^+$	0.8290	$10^{-15} \times 0.2172$
6_4^+	—	5.2404		$4_4^+ - 2_3^+$	0.0000	$10^{-1} \times 0.8871$
7_1^+	—	5.2512		$6_2^+ - 4_1^+$	$10^{+1} \times 0.7387$	$10^{-15} \times 0.2172$
8_2^+	—	5.5344		$8_1^+ - 6_2^+$	0.0000	$10^{-15} \times 0.6719$
10_1^+	—	6.2070		$8_1^+ - 6_1^+$	$10^{+1} \times 0.2606$	0.1420



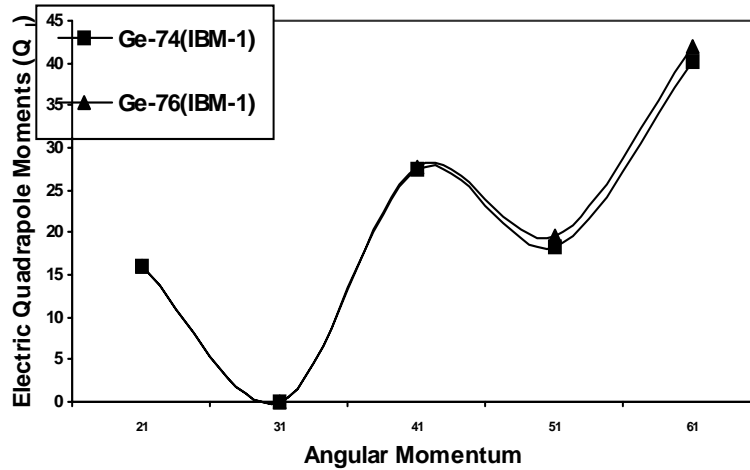


Table (5,6) listed gamma energy transitions by using IBS1. For program from IBM-1, reduced matrix elements $T^{(E2)}$, $T^{(M1)}$ (equation-3,6) using IBMT. For and M5. For which help us to calculate delta mixing ratios $\delta(E2/M1)$ and reduced mixing ratios $\square(E2/M1)$. For knowledge of gamma transition, is the transition either to be electric, magnetic, or mixed transition, the delta mixing ratios for the transitions belonging to the present isotopes have been calculated from the electric quadrupole transition probabilities and the magnetic dipole transition probabilities (equation-8,9). The delta maxing ratios $\delta(E2/M1)$ equality (E2) that means the transition is pure electric transition, while the delta equal to zero, we will found two probabilities, the first probability occurs nonbeing any transition that means inside change (E0), the second probability occurs the magnetic transition is pure. However, the appearance value of delta maxing ratio is existence the mixture between electric and magnetic components in the transition.

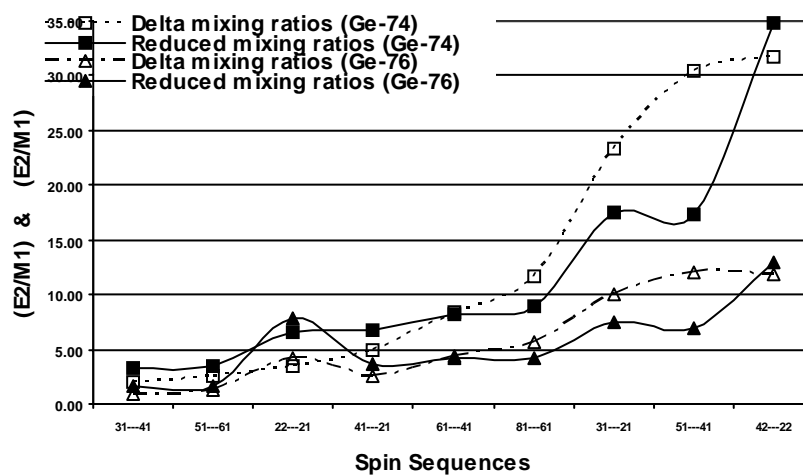
Figure (5) show the gamma energy transitions as a function of spin sequences, the transitions $(3_1 \rightarrow 2_1, 8_1 \rightarrow 6_1)$ has a very high reduced probability of gamma ray by using IBM-1 comparison with experimental data. Figure (6) explain the relationship between delta mixing ratios and reduced mixing ratios as a function of spin sequences for Ge-74,76.

Table (5): gamma transitions, reduced matrix elements for electric and magnetic transitions, delta mixing ratios and reduced mixing ratios compare with experimental data for Ge-74 isotope⁽⁷⁾.

$I_i \rightarrow I_f$	E_γ (MeV)		$\langle I_f \hat{T}^{(E2)} I_i \rangle$ (eb)	$\langle I_f \hat{T}^{(M1)} I_i \rangle$ (μ_N)	$\delta(E2/M1)$	$\Delta(E2/M1)$
	Exp.	IBM-1				
$2_1^+ \rightarrow 0_1^+$	0.5959	0.5290	-0.3106	0.0000	∞	∞
$2_2^+ \rightarrow 2_1^+$	0.6084	0.6450	3.7885	0.5804 Exp. ⁽¹¹⁾ =0.5	3.5155 Exp. ⁽¹¹⁾ = 3.3	6.5274 Exp. ⁽¹¹⁾ =6.4
$2_2^+ \rightarrow 0_1^+$	1.2043	1.1740	-11.2296	0.0000	∞	∞
$4_1^+ \rightarrow 2_1^+$	0.8678	0.8760	5.0828	0.7440	4.9971	6.8317
$0_2^+ \rightarrow 0_1^+$	1.4826	1.4875	0.0000	0.0000	—	—
$3_1^+ \rightarrow 4_1^+$	0.2335	0.7280	-2.4177	0.7440	-1.9754	-3.2497
$3_1^+ \rightarrow 2_2^+$	0.4929	0.9590	3.8228	-0.5176	-5.9142	-7.3857
$3_1^+ \rightarrow 2_1^+$	1.1013	1.6040	-12.9668	0.7440	-23.3427	-17.4285
$4_2^+ \rightarrow 4_1^+$	0.7021	0.8600	-3.5392	0.5804	-4.3789	-6.0979
$4_2^+ \rightarrow 2_2^+$	0.9615	1.0910	-3.7119	0.1066	-31.7213	-34.8209
$4_2^+ \rightarrow 2_1^+$	1.5699	1.7360	-12.5909	0.5804	-31.4460	-21.6935
$2_3^+ \rightarrow 2_2^+$	0.9937	0.8425	0.0000	0.1825×10^{-6}	0.0000	0.0000
$2_3^+ \rightarrow 4_1^+$	0.7343	0.6115	0.0000	-0.1667×10^{-6}	0.0000	0.0000
$6_1^+ \rightarrow 4_1^+$	—	1.2230	6.1640	0.7440	8.4606	8.2849
$2_4^+ \rightarrow 4_1^+$	—	1.2565	0.0000	-0.1345	0.0000	0.0000
$2_4^+ \rightarrow 4_2^+$	—	0.3965	0.0000	-0.5812	0.0000	0.0000
$5_1^+ \rightarrow 6_1^+$	—	0.8770	-2.5953	0.7440	-2.5545	-3.4883
$5_1^+ \rightarrow 4_1^+$	—	2.1000	-12.9196	0.7440	-30.4496	-17.3650
$6_2^+ \rightarrow 6_1^+$	—	1.0750	-3.2735	0.5804	-5.0627	-5.6401
$8_1^+ \rightarrow 6_1^+$	—	1.5700	6.6363	0.7440	11.6934	8.9198

Table (6): gamma transitions, reduced matrix elements for electric and magnetic transitions, delta mixing ratios and reduced mixing ratios comparison with experimental data for Ge-76 isotope⁽⁷⁾.

$I_i \rightarrow I_f$	E_γ (MeV)		$\langle I_f \hat{T}^{(E2)} I_i \rangle$ (eb)	$\langle I_f \hat{T}^{(M1)} I_i \rangle$ (μ_N)	$\delta(E2/M1)$	$\Delta(E2/M1)$
	Exp.	IBM-1				
$2_2^+ \rightarrow 2_1^+$	0.5462	0.6390	-4.1304	-0.5314 Exp. ⁽¹¹⁾ = 0.5	4.1472 Exp. ⁽¹¹⁾ = 3.5	7.7726 Exp. ⁽¹¹⁾ = 7.7
$2_2^+ \rightarrow 0_1^+$	1.1090	1.1712	10.5267	0.0000	∞	∞
$4_1^+ \rightarrow 2_1^+$	0.8472	0.8868	5.5415	-0.1554×10^{-1}	-2.6405	-3.5659
$3_1^+ \rightarrow 4_1^+$	0.1294	0.7104	-2.5628	-0.1554×10^{-1}	0.9783	1.6492
$3_1^+ \rightarrow 2_1^+$	0.9766	1.5972	-11.6377	-0.1554×10^{-1}	9.9876	7.4889
$0_2^+ \rightarrow 0_1^+$	1.9111	1.9170	0.0000	0.0000	—	—
$4_2^+ \rightarrow 4_1^+$	0.6099	0.8520	-3.7516	0.0000	∞	∞
$4_2^+ \rightarrow 2_2^+$	0.9109	1.0998	3.9347	-0.3052	-11.8393	-12.8922
$4_2^+ \rightarrow 2_1^+$	1.4571	1.7388	-11.3003	0.0000	∞	∞
$6_1^+ \rightarrow 4_1^+$	—	1.2414	6.5340	-0.1554×10^{-1}	-4.3584	-4.2046
$2_3^+ \rightarrow 4_1^+$	—	1.6692	4.3986	-0.1350×10^{-1}	-4.5413	-3.2583
$2_3^+ \rightarrow 2_2^+$	—	1.9170	-7.8685	0.0000	∞	∞
$2_4^+ \rightarrow 4_1^+$	—	1.6932	0.0000	0.3115	0.0000	0.0000
$4_3^+ \rightarrow 2_4^+$	—	1.0650	-3.2829	0.1332×10^{-1}	-2.1917	-2.4646
$4_3^+ \rightarrow 4_1^+$	—	1.9170	-3.8255	-0.1350×10^{-1}	4.5359	2.8337
$4_3^+ \rightarrow 2_1^+$	—	2.8038	00.000	-0.1350×10^{-1}	0.0000	0.0000
$5_1^+ \rightarrow 6_1^+$	—	0.8526	-2.6028	-0.1554×10^{-1}	1.1924	1.6749
$5_1^+ \rightarrow 4_1^+$	—	2.0940	-10.6661	-0.1554×10^{-1}	12.0010	6.8636
$6_2^+ \rightarrow 6_1^+$	—	1.0650	-3.2829	0.0000	∞	∞
$4_3^+ \rightarrow 3_1^+$	—	1.2066	0.0000	-0.1350×10^{-1}	0.0000	0.0000
$8_1^+ \rightarrow 6_1^+$	—	1.5960	6.6554	-0.1554×10^{-1}	-5.7075	-4.2828



References

- Abrahams. K. Allaart and A.E.L. Dieperink nuclear structure: Ed. Pub. PLENUM press. New York and London P.P. 67. 53. (1981).
- Arima A. and Iachello F.: Ann.phys. ,vol.21,No.75, NewYork (1981).
- Arima A. and Iachello F.: Interacting Boson Model. Ed. Iachello F., Pub. University of Cambridge, England. pp. 3-236 (1987).
- Bonatsos D.: Interacting Boson Models of nuclear structure Ed. Hodgson P.E., Pub. Oxford University press, NewYork. pp.1-264 (1988)..
- Casten R. F. and Warner D. D. : Rev. Mod. Phys. 60, 391 (1988).
- Ensdf, nuclear data sheet (2006).
- Lany J., Kumark and Hamilton J.H.: Rev. of Mod. Phys., Vol. 54, 119 (1982).
- Lederer G. M., Brown E., and Snihab: Table of Isotopes, 7th edition (1978).
- Mitsuo Saka I.: Atomic data and nuclear data tables Vol. 31. 399(1984).
- Pfeifer W.: (Introduction to interacting boson model of atomic (nuclei) V1, PP. 5, (1998).
- Pramana , j. phys.G: Vol.64, No.2, P.214 (2005).
- Raghavan et al., Atom. Nucl. Data Tables, Vol.42, No.189 (1989).