THEORETICAL ANALYSIS TO THE RADIAL MOTION OF A SPHERICAL GAS BUBBLE

Abbas Z. AL-Asady
Mechanical Engineering Department, Engineering College, Basrah University, Iraq

Abstract

A perturbation analysis is made of a nonlinear wave equation describing a solution to the problem of oscillation of gas-filled spherical cavity in an infinite compressible liquid. Numerical solution has been made of radial motion of the bubble in the liquid. The mathematical formulation takes into account the effects of the thermal conduction both inside and outside the bubble. The gas in the bubble is air and the liquid is water. The numerical results indicate that the thermal conduction strongly influence the dynamical behavior of the bubble in acoustic field. The calculated result fits with the experimental data of radius-time curve. It is clarified that the effect of thermal conduction stabilizes bubble oscillations. It is also clarified that the bubble transducers the energy of the acoustic wave into heat.

تحليل نظري للحركة ألقطريه لفقاعة غازيه كرويه

عباس زكي الأسدي

العراق–قسم الهندسة الميكانيكية - كلية الهندسة - جامعة البصرة

الخلاصة

تم تكوين تحليل إحصائي لمعادله الموجه اللاخطيه لوصف حل مسألة تذبذب فقاعه كرويه مملوءة بالغـاز فـي سائل انضغاطي غير محدود. تم تقديم حل عددي لمعادله الحركة ألقطريه للفقاعة في السائل. تعطي الصيغ ألرياضـيه تأثيرات التوصيل الحراري داخل وخارج الفقاعة. الغاز داخل الفقاعة هو الهواء والسائل المحيط بالفقاعة هـو المـاء. وضحت النتائج العددية أن التوصيل الحراري مؤثر بقوة على التصرف الحركي للفقاعة في وسط صوتي. تطابق النتائج المحسوبة مع البيانات التجريبية لمنحني القطر - الزمن. وضحت الحسابات أن تأثير التوصيل الحراري يجعل تذبـذب الفقاعة مستقراً. كذلك وضحت أن الفقاعة تحول الطاقة للموجة الصوتية إلى حرارة.

1- Introduction

Cavitation is the formation and activity of bubbles (or cavities) in a liquid. Here the world formation refers both to creation of a new cavity or to the expansion of a pre-existing one. In a non-following system, the ambient pressure can be varied by sending ultrasound waves through the liquid, as in a sonochemical reactor. If the amplitude of the pressure variation is great enough to bring the pressure locally down to the cavitation threshold pressure, in the negative parts of the sound cycle, any minute cavity will grow. Thus tiny bubbles grow and contract in the sound field [1].

A gas bubble in a liquid performs forced radial oscillations when a sound wave impinges upon it. Oscillations of large bubbles were originally analyzed by Rayleigh (in 1917 as is reviewed in [2]) derived an equation of the bubble collapse. He assumed that the surrounding liquid is incompressible and inviscid, that the bubble remains spherical, and the surface tension is negligible. Plesst [3], Noltingk and Neppiras [4], and Poritsky [5] modified equation of Rayleigh to include the effects of viscosity, surface tension, and an incident sound wave. Keller and Kolodner [6], who included the effects of acoustic radiation by treating the surrounding liquid as slightly compressible, made a different modification.

We shall combine these modifications to derive a new equation for the bubble radius. It includes the effects of acoustic radiation, viscosity, surface tension and an incident sound wave. In the present paper, the perturbation method which used by Benjamin [7], Jahsman [8], Tomita and Shima [9], and Fujikawa and Akamatsu [10] will be used to control the behavior of the functions appearing in the series.

In this paper, the pressure and temperature inside the bubble shall be calculated by solving numerically (using finite difference method) the energy equation of a gas (air) inside the bubble. Temperature distribution in liquid (water) will be calculated from the energy equation in the liquid (which is solved numerically).

2. The Bubble Model

2.1 Statement of the problem

There is a spherical bubble of initial radius R_o containing non-condensable gas in a viscous compressible liquid. At time zero, the ambient pressure is increased instantaneously to $P_{L\infty}$, and then the bubble beings to oscillate accompanied with heat conduction through the bubble wall. The problem is to investigate these physical effects on the bubble oscillations. Schematic diagram depicting a model is illustrated in Fig. A.

In writing the basic equations, the following assumptions are made:
(a) The bubble is spherically symmetric. (b) The effects of gravity and diffusion are negligible. (c) The evaporation and condensation at bubble wall are neglected. (d) The pressure inside the bubble is uniform throughout. (e) The non-condensable gas inside the bubble obeys the perfect-gas law. (f) The temperature of the gas is not uniform. (g) The thermal boundary layers developing both inside and outside the bubble are thin enough compared with the bubble radius. (h) The physical properties (viscosity, surface tension, thermal conductivity and density) of liquid and the gas are variable. The analysis based on the present assumptions will be the first step in an attempt to understand the behavior of the oscillation of cavitation bubble in the liquid.

Under the above assumptions, the governing equations in the external region occupied by the liquid [10]:

Eq. of Continuity:

$$\frac{\partial \rho_L}{\partial t} + \frac{\partial}{\partial r}(\rho_L u_L) + \frac{2\rho_L u_L}{r} = 0 \quad \ldots(1)$$

Eq. of Momentum:

$$\frac{\partial u_L}{\partial t} + u_L \frac{\partial u_L}{\partial r} = -\frac{1}{\rho_L}\frac{\partial P_L}{\partial r} \quad \ldots(2)$$

Eq. of State:

$$\frac{P_L + B}{P_{L\infty} + B} = \left(\frac{\rho_L}{\rho_\infty}\right)^n \quad \ldots(3)$$

In eq. (3) (Tait equation), B is 3047 bar and n=7.15 for water [11]

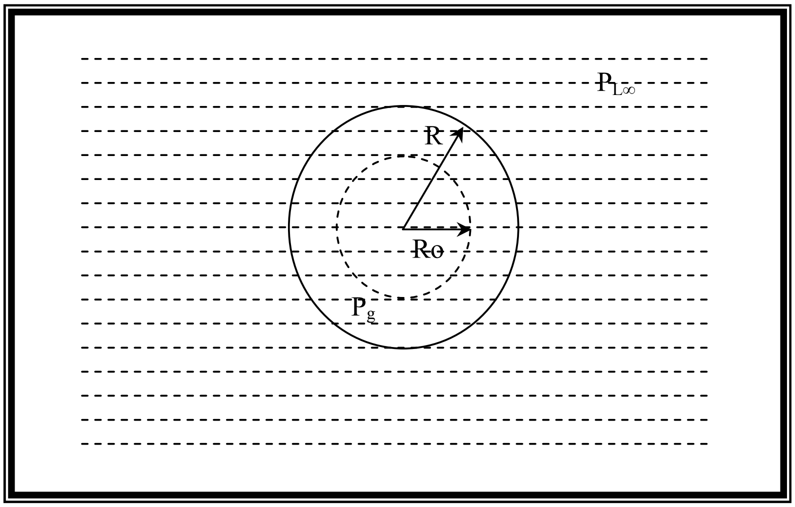

Fig. A. The bubble model

2.2 The Equation of Radial Motion of the bubble

In this section, we construct a mathematical formulation that enables us to study the radial motion of the bubble and the effects of heat conduction, shear viscosity, compressibility, and surface tension on the dynamical behavior.

Let $\phi(r,t)$ be the velocity potential for the liquid. The sound speed in the liquid [10].

$$\tilde{c} = \left\{ c^2 - (n-1)\left[\frac{\partial \phi}{\partial t} + \frac{1}{2}\left(\frac{\partial \phi}{\partial t}\right)^2\right]\right\} \quad \ldots(4)$$

From the equations (1), (2), (3) and (4) Fujikawa and Akamatsu [10] obtained a partial differential equation concerning the velocity potential ϕ of the liquid,

$$\nabla^2 \phi - \frac{1}{c^2}\frac{\partial^2 \phi}{\partial t^2} = \frac{1}{c^2}\left[\begin{array}{l} 2\dfrac{\partial \phi}{\partial r}\dfrac{\partial^2 \phi}{\partial r\,\partial t} + \\ \dfrac{2(n-1)}{r}\dfrac{\partial \phi}{\partial r}\dfrac{\partial \phi}{\partial t} + \\ (n-1)\dfrac{\partial^2 \phi}{\partial r^2}\dfrac{\partial \phi}{\partial t} + \\ \dfrac{n+1}{2}\left(\dfrac{\partial \phi}{\partial r}\right)^2\dfrac{\partial^2 \phi}{\partial r^2} + \\ \dfrac{n-1}{r}\left(\dfrac{\partial \phi}{\partial r}\right)^3 \end{array}\right] \quad \ldots(5)$$

The boundary conditions are:

(i) Continuity at the phase interface

$$\left(\frac{\partial \phi}{\partial r}\right)_{r=R} = \dot{R} \quad \ldots(6)$$

(ii) The pressure equation in the liquid at the interface

$$\left[\frac{\partial \phi}{\partial t}+\frac{1}{2}\left(\frac{\partial \phi}{\partial r}\right)^2\right]_{r=R}= \qquad \ldots (7)$$
$$\frac{c^2}{n-1}\left[1-\left(\frac{P_{LR}+B}{P_{L\infty}+B}\right)^{\frac{n-1}{n}}\right]$$

(iii) At infinity

$$\phi = 0 \text{ as } r \to \infty \qquad \ldots (8)$$

Now, consider the problem along the outgoing characteristic η(r,t) = constant such that [10],

$$dt = (u_L + c)^{-1}\ dt \qquad \ldots(9)$$

According to Fujikawa and Akamatsu method [10],

$$\left.\begin{aligned}
\phi(r,\eta) &= \vartheta_0(r,\eta)+\frac{1}{c}\vartheta_1(r,\eta)+ \\
&\qquad \frac{1}{c^2}\vartheta_2(r,\eta)+ ..., \\
r &= r, \\
t &= \eta+\frac{1}{c}\,t_1(r,\eta)+\frac{1}{c^2}\,t_2(r,\eta)+...,
\end{aligned}\right\} \qquad ...(10)$$

where η is the initial time on outgoing characteristic and satisfies the condition $\eta = t$ on $r = R$.

(a) First Perturbation Procedure. The first-order approximation $\phi_1 = \vartheta_0 + \frac{1}{c}\vartheta_1$ is determined by

$$\frac{\partial^2 \phi_1}{\partial r^2}+\frac{2}{r}\frac{\partial \phi_1}{\partial r}-\frac{1}{c^2}\frac{\partial^2 \phi_1}{\partial t^2}=0 \qquad \ldots(11)$$

with the boundary conditions eqs. (6), (7) and (8). The appropriate solution can be written as follows:

$$\phi_1(r,\eta) = -\frac{f(\eta)}{r} \qquad \ldots(12)$$

On the other hand, from eq. (9), Fujikawa and Akamatsu [10] obtained,

$$t=\eta+\frac{r-R(\eta)}{c} \qquad \ldots(13)$$

An unknown function $f(\eta)$ in eq. (12) can be determined from the boundary condition eq. (6) by using eq. (13):

$$f(\eta)=R^2\dot{R}-\frac{R^2}{c^2}(2\dot{R}^2+R\ddot{R}) \qquad \ldots(14)$$

Therefore, the velocity potential (ϕ_1) can be written as follows,

$$\phi_1(r,\eta)=-\frac{1}{r}\left[R^2\dot{R}-\frac{R^2}{c^2}(2\dot{R}^2+R\ddot{R})\right] \qquad \ldots(15)$$

From equations (6), (7) and (15), we obtain the equation of motion of the bubble with the first-order correction of the liquid compressibility:

$$R\ddot{R}\left(1-\frac{2\dot{R}}{c}\right)+\frac{3}{2}\dot{R}^{2}\left(1-\frac{4\dot{R}}{3c}\right)+ \frac{P_{L\infty}-P_{LR}}{\rho_\infty}-\frac{R}{\rho_\infty c}\frac{dP_{LR}}{dt}=0 \qquad \ldots(16)$$

where

$$P_{LR}=P_g-\frac{2\sigma}{R}-\frac{4\mu}{R}\dot{R} \qquad \ldots(17)$$

Raleigh equation (in 1917) is deduced from eq. (16) for special case when $\frac{\dot{R}}{c}\to 0$.

(b) Second Perturbation Method. Now, the right-hand side of eq. (5) is obtained from the first-order solution ϕ_1, then the second-order correction φ_2 is determine by

$$\frac{\partial^2 \vartheta_2}{\partial r^2}+\frac{2}{r}\frac{\partial \vartheta_2}{\partial r}=-\frac{2f^{3}}{r^7} \qquad \ldots(18)$$

Under the boundary condition $\varphi_2 = 0$ at و$r\to\infty$ we have

$$\vartheta_2=\frac{F(\eta)}{r}-\frac{R^6\dot{R}^3}{10\,r^5} \qquad \ldots(19)$$

On the other hand, from eq. (9), we obtain

$$t=\eta+\frac{r-R(\eta)}{c}+\frac{f(\eta)}{c^2}\left[\frac{1}{r}-\frac{1}{R(\eta)}\right] \qquad \ldots(20)$$

An unknown function $F(\eta)$ can be determined from the boundary condition eq. (6) by using eq. (20),

$$F(\eta)=-\frac{3}{2}R^2\dot{R}^3-2R^2\dot{R}(\dot{R}^2+R\ddot{R})-R^3\ddot{R}(5\dot{R}-R^4\ddot{R}) \quad ...(21)$$

Thus, we can obtain the solution of ϕ for the second-order approximation as follows:

$$\vartheta_2(r,\eta)=-\frac{1}{r}\left[R^2\dot{R}-\frac{R^2}{c^2}(2\dot{R}^2+R\ddot{R})+\frac{1}{c^2}\left\{\frac{3}{2}R^2\dot{R}^3+2R^2\dot{R}(\dot{R}^2+R\ddot{R})+R^3\ddot{R}(5\dot{R}-R^4\ddot{R})+\frac{R^6\dot{R}^3}{10r^4}\right\}\right] \qquad \ldots(22)$$

Finally, we obtain the equation of motion of the bubble with the second-order correction of the liquid compressibility:

$$R\ddot{R}\left[1-\frac{2\dot{R}}{c}+\frac{23}{10}\,\frac{\dot{R}^2}{c^2}\right]+\frac{3}{2}\dot{R}\left[\dot{R}-\frac{4\,\dot{R}^2}{3\,c}+\frac{7\,\dot{R}^3}{5c^2}\right]$$

$$+\frac{1}{\rho_\infty}\left[\begin{array}{l}(P_{L\infty}-P_{L2})-\dfrac{R}{c}\dfrac{dP_{LR}}{dt}+\dfrac{1}{c^2}\\ \left\{\begin{array}{l}2\,R\dot{R}\dfrac{dP_{LR}}{dt}+(P_{L\infty}-P_{LR})\\ \left[\dfrac{1}{2}\dot{R}^2+\dfrac{3}{2}\dfrac{(P_{L\infty}-P_{LR})}{\rho_\infty}\right]\end{array}\right\}\end{array}\right]=0 \quad ...(23)$$

where $\mathrm{P_{L2}}=\mathrm{P}_{\mathrm{LR}}-\dfrac{4\mu}{3c^2\,\rho_\infty}\,\dfrac{\mathrm{dP_{LR}}}{\mathrm{dt}}$

2.3 The Temperature Profile both Inside and Outside the Bubble

The dynamics of a gas bubble in a liquid is strongly dependent on the pressure of the gas contained in it. In principle, this quantity must be determined from the solution of the energy equations inside and outside the bubble joined together by suitable boundary conditions at the bubble interface.

The mathematical model for the bubble interior is described in detail by Kamath and Prosperetti [12]. The model accounts for the compressibility of the gas and heat transport by conduction inside the bubble. The main assumptions, discussed below, are those of perfect gas behavior and of spatial uniformity of the gas pressure.

The internal pressure (P_g) is found by integrating [13].

$$\frac{\mathrm{dP_g}}{\mathrm{dt}}=\frac{3}{\mathrm{R}}\left[(\gamma-1)\;\mathrm{k_g}\frac{\partial \mathrm{T_g}}{\partial \mathrm{r}}\bigg|_{\mathrm{R}}-\gamma\;\mathrm{P_g}\;\dot{\mathrm{R}}\right] \quad ...(24)$$

where γ is the ratio of the specific heats at the gas and k_g is the gas thermal conductivity. The gas temperature field T_g (r, t) is obtained from the energy equation [14].

$$\frac{\gamma}{\gamma-1}\,\frac{P_g}{T_g}\left(\frac{\partial T_g}{\partial t}+u_g\frac{\partial T_g}{\partial r}\right)-\frac{dP_g}{dt}=\frac{1}{r^2}\frac{\partial}{\partial r}\left(k_g\,r^2\,\frac{\partial T_g}{\partial r}\right) \quad \ldots(25)$$

with $\mathrm{u_g}=\dfrac{1}{\gamma\,\mathrm{P_g}}\left[(\gamma-1)\mathrm{k_g}\,\dfrac{\partial \mathrm{T_g}}{\partial \mathrm{r}}-\dfrac{\mathrm{r}}{3}\dfrac{\mathrm{dP_g}}{\mathrm{dt}}\right]$ ... (26)

The complete mathematical formulation of the problem of spherical-bubble growth and collapse is needed a partial differential equation for the energy (or temperature) in the liquid [15],

$$\rho_\infty\,cp_L=\left[\frac{\partial T_L}{\partial t}+\frac{R^2}{r^2}\dot{R}\frac{\partial T_L}{\partial r}\right]=\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\,k_L\,\frac{\partial T_L}{\partial r}\right) \quad \ldots(27)$$

Eq. (25) is used in the domain $0 \le r \le R$, while eq. (27) is used in the domain $R \le r \le \infty$.

The two-temperature field obtained from eqs. (25) and (27) must be match at the bubble interface, where the continuity of heat flux [16].

$$k_g \frac{\partial T_g}{\partial r}\bigg|_{r=R} = k_L \frac{\partial T_L}{\partial r}\bigg|_{r=R} \qquad \ldots(28)$$

and of temperatures

$$T_g(R, t) = T_L(R,t) \qquad \ldots (29)$$

where $T_L(R,t) = T_{LB}$

The bubble is irradiated by an acoustic wave. The liquid pressure at infinity is [17].

$$P_{L\infty} = P_0 - P_m \sin \omega t \qquad \ldots(30)$$

where P_m is the pressure amplitude of the acoustic wave and ω is its angular frequency $(\omega = 2\pi f)$.

To solve the bubble model, eqs. (25) and (27) are solved using finite difference method, and coupled with numerical solution of the bubble radius equation (eq. (23)) (see Appendix). In this model, the liquid is water and the gas in the bubble is air.

3. Results and Discussion

Calculations are performed under the following conditions. The initial bubble radius is 4.5 μm. The frequency and the amplitude of acoustic wave are 26.5 kHz and 1 bar, respectively. The ambient liquid temperature (T_o) and the ambient liquid pressure (P_o) are chosen to be 20 ^oC and 1 bar, respectively. The specific heat of liquid water and the ratio of the specific heats of the gas (air) inside the bubble are chosen to be 4.2 kJ/kg.K and 1.4, respectively. Calculations start from the time t=0 μs with the initial conditions that:

$$R = R_o, \dot{R} = 0, T_g = T_{LB} = T_o, P_{L\infty} = P_o$$

Results of the calculation under the above conditions are shown in Figs. (1~6) for three acoustic cycles. The time axes (the horizontal axes) in the figures are the same. All the physical quantities of a bubble change with time periodically with the frequency of the acoustic field applied on the bubble. In Fig. 1, the bubble radius (R) is shown as a function of time. It is seen that the bubble expands when the acoustic pressure applied on the bubble is negative and that it collapses strongly when the acoustic pressure changes to positive. After the strongest collapse, it oscillates softly a few times with its own frequency. In Fig. 2, the pressure inside the bubble (P_g) is shown as a function of time with logarithmic vertical axis. It is seen that P_g increases up to 100 bar at the strongest collapse and that soft oscillations follow due to the soft oscillations of the bubble radius.

In Fig. 3, the temperature inside a bubble (T_g) is shown as a function of time with a linear vertical axis. It is seen that the expansion of the bubble is the isothermal process. The temperature becomes very high at collapses of the bubble. In Fig. 4, the liquid temperature at bubble wall (T_{LB}) is shown as a function of time. It is seen that T_{LB} is identical to the ambient liquid temperature (T_o) except for the strongest collapses. At the strongest collapse, the liquid temperature increases. The heated liquid layer is very thin. In Fig. 5, the bubble wall velocity ($\dot{R}$) is shown as a function of time.

In Fig. 6, the comparison is given between the calculated result and the experimental data [18] of radius-time curve for one acoustic cycle. The calculated result fits well with the experimental data.

4. Conclusions

A new equation of a bubble radius is derived. A new model of bubble dynamics is constructed in which effects of thermal conduction both inside and outside the bubble are included. The energy equations of the gas (air) inside the bubble and the liquid near the bubble are solved numerically without assuming a profile of the gas or liquid temperatures. At the slow expansion of the bubble T_g and T_{LB} are almost identical to the ambient liquid temperature during bubble oscillations except at strong collapses. At strong collapses, T_g and T_{LB} are increased. T_{LB} increases mainly due to the thermal conduction from the heated interior of the bubble. The assumption of the spatial uniformity of the temperature (T_g) in a bubble is no more realistic one at the strong collapse of the bubble. It is concluded that the effects of thermal conduction both inside and outside the bubble are considerable on bubble dynamics in acoustic field. The calculated results fit with the experimental data of radius time curve. It is concluded that the effect of thermal conduction stabilizes bubble oscillations.

5. Nomenclature

g : Refers to the gas in the bubble (air)
L: Refers to the liquid (water)
LB: Refers to the liquid at bubble wall
o: Refers to the equilibrium value

∞: Refers to the condition at a great distance from the bubble

Symbol	Definition	Unit
B	Constant is used in eq. (3)	bar
c	Sound speed in liquid at infinity	m /s
cp	Heat capacity at constant pressure	J /kg.K
f	Acoustic field frequency	Hz
f	Function is defined in eq. (12)	
F	Function is defined in eq. (19)	
k	Thermal conductivity	W /m.K
n	Constant is used in eq. (3)	
P	Pressure	Pa
P_o	Surrounding liquid pressure	Pa
P_m	Acoustic pressure amplitude	Pa
P_{LR}	Liquid pressure at bubble wall	Pa
$P_{L\infty}$	Liquid pressure at infinity	Pa
r	Radial distance from bubble center	m
R	Bubble radius	m
$\dot{R}$	Bubble wall velocity	m /s
$\ddot{R}$	Second derivative of bubble radius	m /s^2
R_o	Initial bubble radius	m
t	Time	s
T	Temperature	K
T_o	Ambient liquid temperature	K
u	Velocity	m/s
γ	Ratio of specific heat for gas	
μ	Liquid viscosity	N.s /m^2
ρ	Liquid density	kg /m^3
ρ_∞	Ambient liquid density	kg /m^3
σ	Liquid surface tension	N /m
ϕ	Velocity potential in liquid	m^2 /s
ω	Angular frequency	rad /s

Appendix
Numerical Method

The system to be solved consists of the radius equation, the pressure equation, and the temperature equations of the gas and liquid. The first two are ordinary differential equations, while the last two are a partial differential equation.

(1) The Temperature Equation of Air

We begin by carring out a spatial discretization on this equation by introducing NN+1, equispaced points $\zeta_{kk} = (kk-1)\,\Delta\zeta$, kk = 1, 2, …, NN+1, with $\zeta_1=0$, $\zeta_{NN+1}=1$, and $\Delta\zeta = \frac{1}{NN}$. This equation is:

$$\frac{\gamma P_g}{(\gamma-1)T_g}\left[\begin{array}{l}\dfrac{\partial T_g}{\partial t}+\\ \dfrac{1}{\gamma P_g}\left\{\begin{array}{l}(\gamma-1)k_g\left.\dfrac{\partial T_g}{\partial r}\right|_{r=R}\\ -\dfrac{r\dot{P}_g}{3}\end{array}\right\}\dfrac{\partial T_g}{\partial r}\end{array}\right]$$

$$-\dot{P}_g=\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2k_g\frac{\partial T_g}{\partial r}\right)...(A-1)$$

Let

$$\tau=\int_{T_{LB}}^{T_g}k_g(T')\,dT' \quad , \quad \zeta=\frac{r}{R} \quad \text{and}$$

$$D_g=\frac{(\gamma-1)\,k_g(T_g)\,T_g}{\gamma P_g}$$

Then the energy equation (A-1) takes the form:

$$\frac{\partial\tau}{\partial t}+\frac{\gamma-1}{\gamma R^2 P_g}\left[\frac{\partial\tau}{\partial\zeta}-\left.\frac{\partial\tau}{\partial\zeta}\right|_{\zeta=1}\zeta\right]\frac{\partial\tau}{\partial\zeta}-$$

$$D_g\,\dot{P}_g=\frac{D_g}{R^2}\nabla^2\tau \quad ...(A-2)$$

The boundary conditions are, in terms of □

$\tau(\zeta=1,t)=0$ …(A-3a)

$\dfrac{\partial\tau(\zeta=0,t)}{\partial t}=0$ …(A-3b)

For Air,

$k_g(T)=A_gT+B_g$

where A_g and B_g are in ref.[19].

$$\tau=\int_{T_{LB}}^{T_g}\left(A_gT'+B_g\right)dT' \quad …(A-4a)$$

$$T_g=\left\{\left[k_g^2(T_{LB})+2A_g\tau\right]^{1/2}-B_g\right\}\Big/A_g \quad …(A-4b)$$

We use the following approximation for the spatial differential operators:

$$\left(\frac{\partial\tau}{\partial\zeta}\right)_i=\frac{\tau_{i+1}-\tau_{i-1}}{2\,\Delta\zeta} \quad …(A-5)$$

$$\left(\nabla^2\tau\right)_i=\frac{1}{\Delta\zeta^2}\left[\begin{array}{l}\left(1+\dfrac{\Delta\zeta}{\zeta_i}\right)\tau_{i+1}-\\ 2\,\tau_i+\left(1-\dfrac{\Delta\zeta}{\zeta_i}\right)\tau_{i-1}\end{array}\right] …(A-6)$$

Where the index i indicates evaluation at the node ζ_i. These two expressions are used for all internal nodes.

For the first node, the appropriate expressions are:

$$\left(\frac{\partial \tau}{\partial \zeta}\right)_1 = 0 \qquad \ldots\text{(A-7)}$$

$$\left(\nabla^2 \tau\right)_1 = \frac{6\left(\tau_2 - \tau_1\right)}{\Delta\zeta^2} \qquad \ldots\text{(A-8)}$$

while at the bubble wall

$$\left(\frac{\partial \tau}{\partial \zeta}\right)_{NN+1} = \frac{3\left(\tau_{NN+1} - 4\,\tau_{NN} + \tau_{NN-1}\right)}{2\,\Delta\zeta}$$

$$= \frac{\tau_{NN-1} - 4\,\tau_{NN}}{2\,\Delta\zeta} \qquad \ldots\text{(A-9)}$$

since $\tau_{NN+1} = 0$ according to the equation of τ. The generic equation of this system has the form:

$$A_i\,\tau_{i-1} + B_i\,\tau_i + C_i\,\tau_{i+1} = F_i \qquad \ldots\text{(A-10)}$$

where

$$A_i = -\frac{(\gamma-1)\,\Delta t}{2\,\gamma\,P_g\,R^2\,\Delta\zeta}\left[\left(\frac{\partial\tau}{\partial\zeta}\right)_i - \zeta_i\left(\frac{\partial\tau}{\partial\zeta}\right)_{NN+1}\right] - \frac{D_g\,\Delta t}{R^2\,\Delta\zeta^2}\left(1 - \frac{\Delta\zeta}{\zeta_i}\right)$$

$$B_i = 1 + \frac{2\,D_g\,\Delta t}{R^2\,\Delta\zeta^2}$$

$$C_i = \frac{(\gamma-1)\,\Delta t}{2\,\gamma\,P_g\,R^2\,\Delta\zeta}\left[\left(\frac{\partial\tau}{\partial\zeta}\right)_i - \zeta_i\left(\frac{\partial\tau}{\partial\zeta}\right)_{NN+1}\right] - \frac{D_g\,\Delta t}{R^2\,\Delta^2\zeta}\left(1 + \frac{\Delta\delta}{\zeta_i}\right)$$

$$F_i = \tau_i^{(t-\Delta t)} + \frac{1}{2}\,\Delta t\,D_g\,\dot{P}_g$$

These expressions apply for $2 \le i \le NN$ except that, for i = NN the last term in the left-hand side of eq.(A-10) vanishes since □$_{NN+1}$=0. At the first spatial node (i =1) the coefficients take a somewhat different form due to (A-7) and (A-8). Specifically, one has

$A_1=0$

$$B_1 = 1 + \frac{6\,D_g\,\Delta t}{R^2\,\Delta^2\zeta}$$

$$C_1 = -\frac{6\ D_g\ \Delta t}{R^2\ \Delta^2\zeta}$$

$$D_1 = \tau_1^{(t-1)} + D_g\ \dot{P}_g\ \Delta t$$

Once the system eq.(A-10) has been solved. For stability, we have used values of NN between 100 and 200.

(2) The Temperature Equation of Water

This equation is

$$\rho_L C_{PL}\left[\frac{\partial T_L}{\partial t} + \frac{R^2\,\dot{R}}{r^2}\frac{\partial T_L}{\partial r}\right] = \frac{1}{r^2}\frac{\partial}{\partial r}\left(k_L r^2\,\frac{\partial T_L}{\partial r}\right) \quad \ldots\text{(A-11)}$$

Let

$$\theta = \int_{T_o}^{T_L} k_L\,(T')\ dT' \quad , \quad \eta = \frac{r}{R} \qquad \text{and}$$

$$D_L = \frac{k_L(T_L)}{\rho_L C_{PL}}$$

The energy equation (A-11) takes the form

$$\frac{\partial\theta}{\partial t} + \frac{\dot{R}}{R\,\eta^2}\frac{\partial\theta}{\partial\eta} = \frac{D_L}{R^2}\ \nabla^2\theta \quad \ldots\text{(A-12)}$$

The boundary conditions are, in terms of

$$\theta(\eta = 1, t) = \int_{T_o}^{T_{LB}} k_L\,(T')\ dT' \quad \ldots\text{(A-13a)}$$

$$\theta(\eta \to \infty, t) = 0 \quad \ldots\text{(A-13b)}$$

For Water

$$k_L(T) = A_L T + B_L$$

where A_L and B_L are in ref.[16].

$$\theta = \int_{T_o}^{T_L} (A_L T' + B_L)\ dT' \quad \ldots\text{(A-14a)}$$

$$T_L = \left\{\left[k_L^2(T_o) + 2A_L\theta\right]^{1/2} - B_L\right\}\Big/A_L \quad \ldots\text{(A-14b)}$$

We use the following approximations for the spatial differential operators:

$$\left(\frac{\partial\theta}{\partial\eta}\right)_j = \frac{\theta_{j+1} - \theta_{j-1}}{2\Delta\eta} \quad \ldots\text{(A-15)}$$

$$\left(\nabla^2\theta\right)_j = \frac{1}{\Delta\eta^2}\begin{bmatrix}\left(1+\frac{\Delta\eta}{\Delta\eta_j}\right)\theta_{j+1} - \\ 2\ \theta_j + \left(1-\frac{\Delta\eta}{\eta_j}\right)\theta_{j-1}\end{bmatrix}$$
...(A-16)

Where the index j indicates evaluation at the node η_j and $\Delta\eta = \frac{1}{MM}$; where MM is number of nodes in liquid side. These two expressions are used for all internal nodes.

For the first node, at the bubble wall, the expressions are:

$$k_L \frac{\partial T_L}{\partial r}\bigg|_{r=R} = k_g \frac{\partial T_g}{\partial r}\bigg|_{r=R} \qquad \ldots\text{(A-17)}$$

$$\frac{\partial\theta}{\partial\eta}\bigg|_{\eta=1} = R\ k_L \frac{\partial T_L}{\partial r}\bigg|_{r=R} \qquad \ldots\text{(A-18)}$$

while at $r \to \infty$

$$\theta(r=\infty, t) = 0 \qquad \ldots\text{(A-19)}$$

The generic equation of this system has the form:

$$A_j\,\theta_{j-1} + B_j\,\theta_j + C_j\,\theta_{j+1} = F_j \quad \ldots\text{(A-20)}$$

where

$$A_j = -\left[\frac{\dot{R}\,\Delta t}{4R\,\eta_j^2\,\Delta\eta} + \frac{D_L\,\Delta t}{2\,R^2\,\Delta^2\eta}\left(1+\frac{\Delta\eta}{\eta_j}\right)\right]$$

$$B_j = 1 + \frac{D_L\,\Delta t}{R^2\,\Delta\eta^2}$$

$$C_j = \frac{\dot{R}\,\Delta t}{4R\,\eta_j^2\,\Delta\eta} - \frac{D_L}{2R^2\Delta\eta^2}\,(1+\frac{\Delta\eta}{\eta_j})$$

$$F_j = \theta_j^{(t-\Delta t)}$$

These expressions apply for 1 < j < MM. Once the system (A-20) has been solved. We have used values of MM between 100 and 200.

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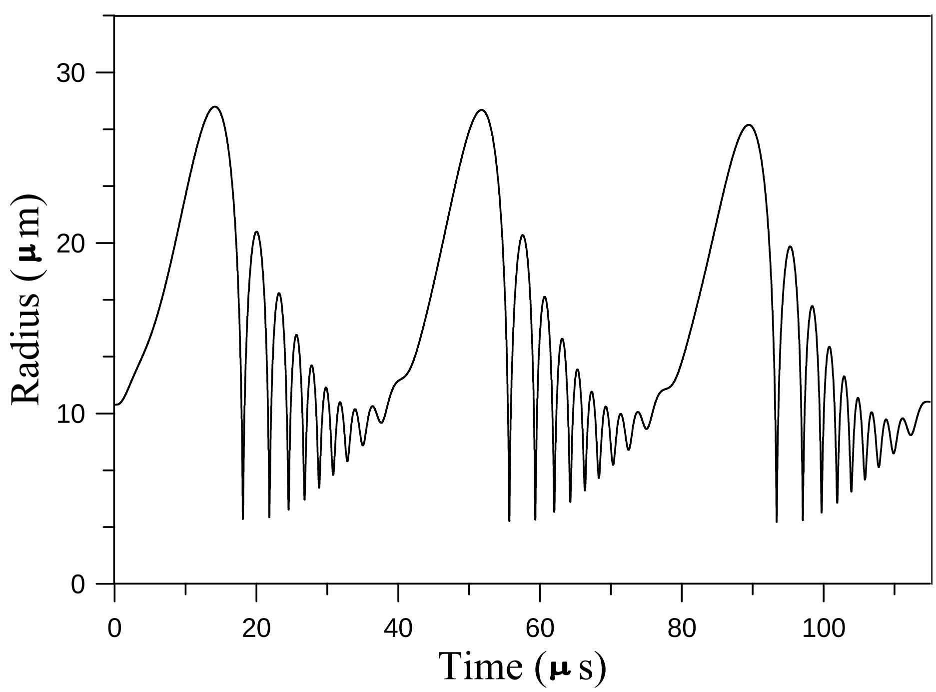

Fig. 1. The bubble radius (R) as a function of time.

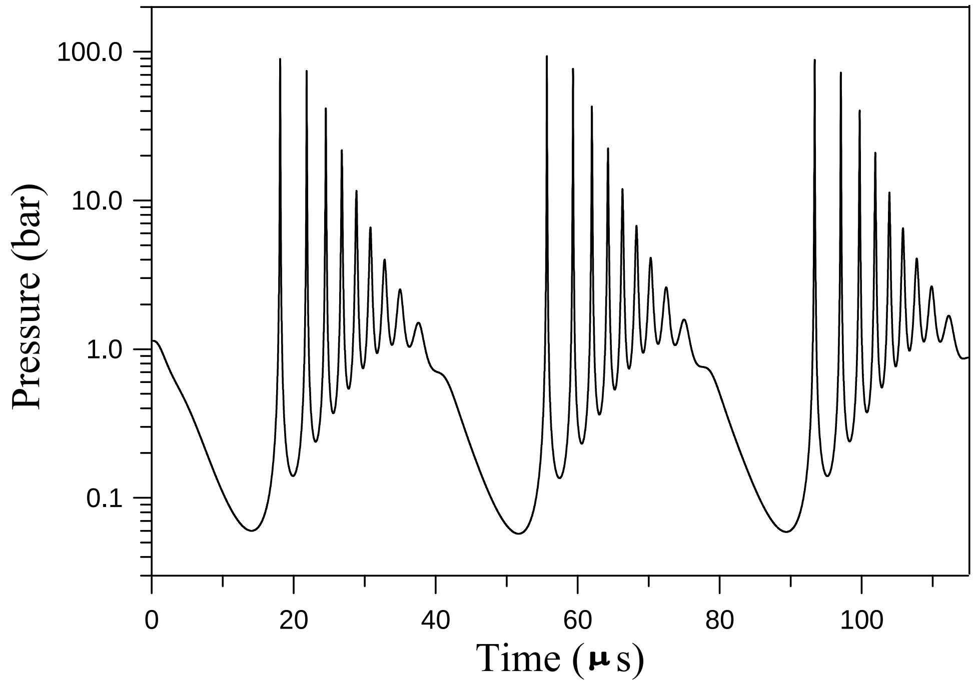

Fig. 2. The pressure inside the bubble (P_g) as a function of time with logarithmic vertical axis.

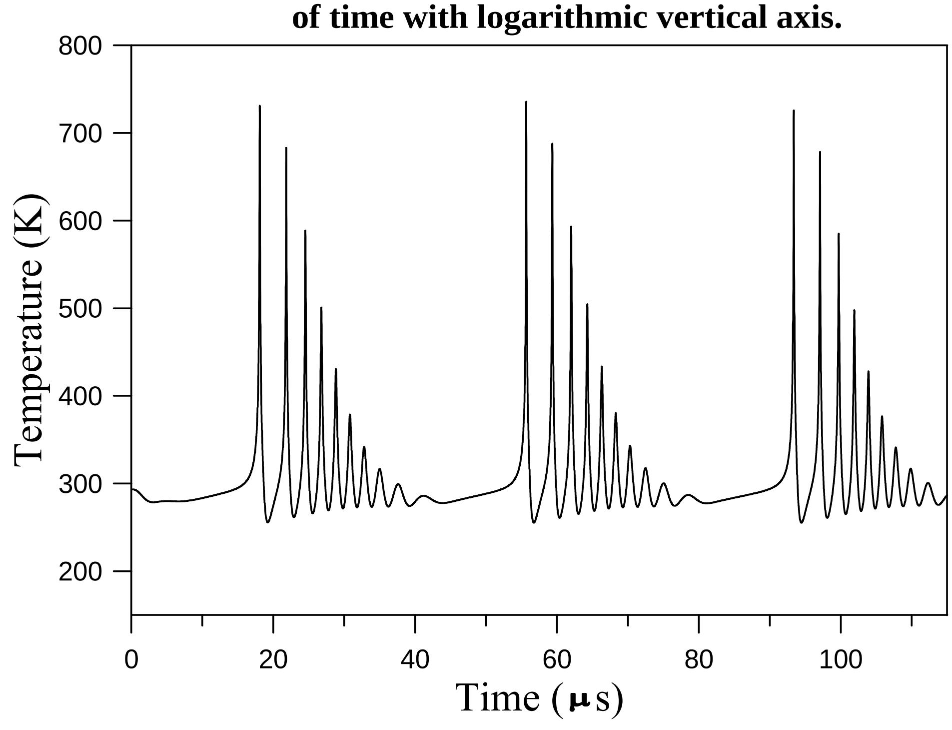

Fig. 3. The temperature at the bubble center as a function of time.

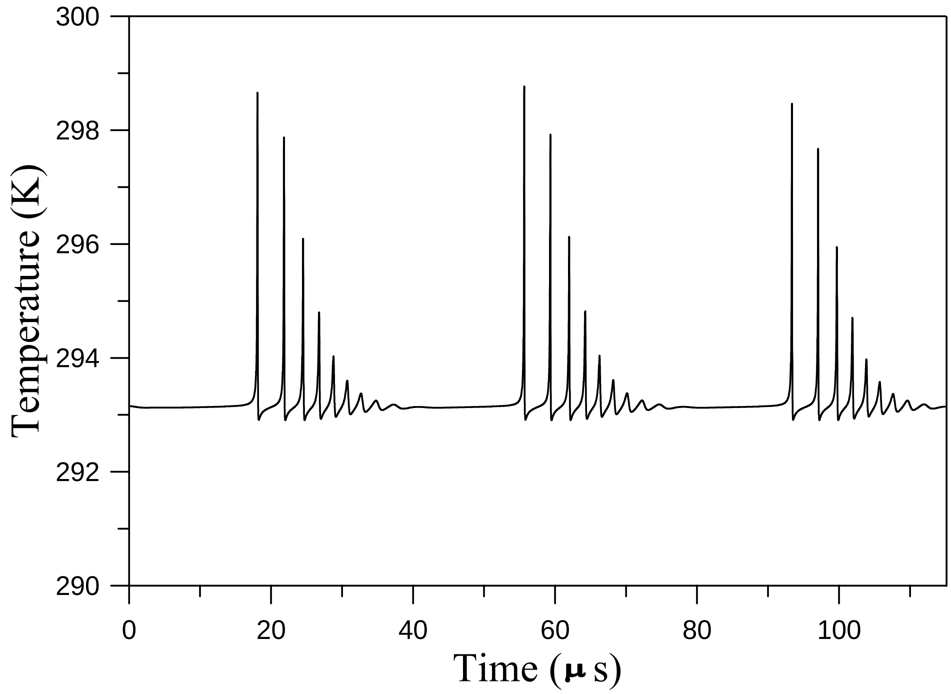

Fig. 4. The liquid temperature at the bubble wall(T_{LB}) as a function of time.

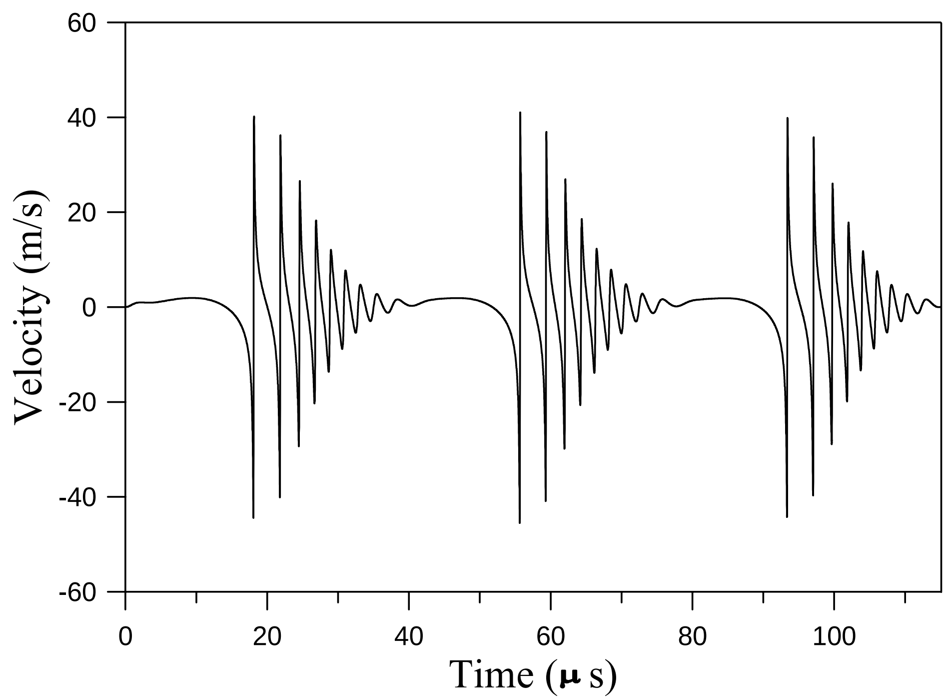

Fig. 5. The bubble wall velocity ($\dot{R}$) as a function of time.

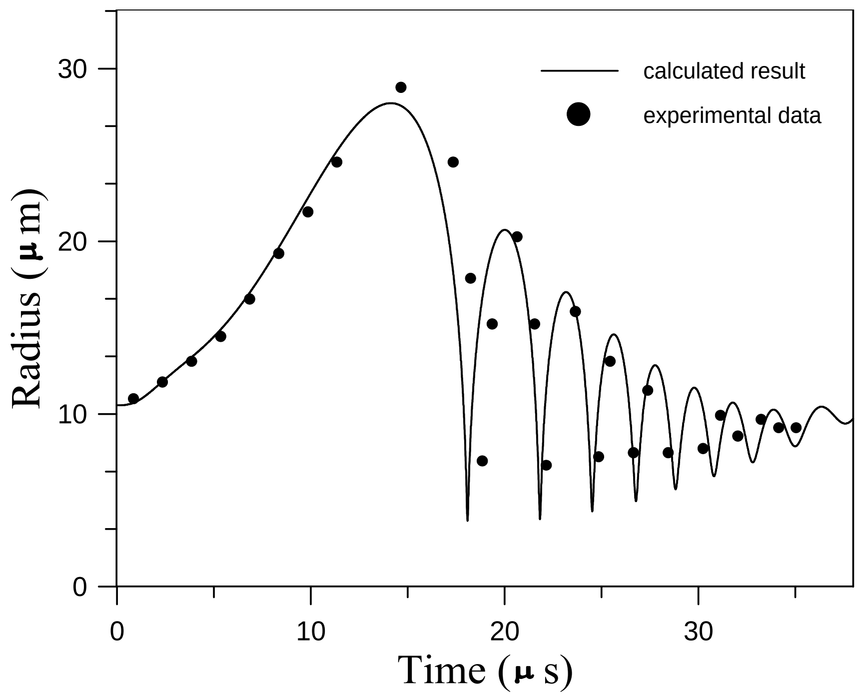

Fig. 6. Comparison between the calculated result and the experimental data[18] of radius-time curve for acoustic cycle.