Theoretical analysis of the radiation fields of short backfire antenna fed by a coaxial waveguide

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Abstract
This paper present a study of characteristics of the radiation from a coaxial waveguide as an antenna and a short backfire antenna (SBFA) fed by this coaxial waveguide. It is shown that the performance of such antenna is preferable in comparison with that fed by a circular waveguide, and a good agreement has been obtained between the predicated results and those reported by other research workers.

Keywords : Antenna ; Short backfire antenna ; Coaxial waveguide

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Introduction

The coaxial line is one of the oldest and most extensively used transmission line. The transverse cross section of a coaxial line is a doubly connected region. In dominant mode the propagation of electromagnetic wave along a line with doubly connected transverse cross section and in particular, along a coaxial waveguide, is independent of the relation-ship between the transverse dimensions of the line and the

wavelength
(Abduall 1977).

The interest in coaxial waveguide has increased considerably. This has been due to various engineering applications of coaxial lines or finite structures based on these lines in antenna-waveguide technology (Gerhard 1973) and microwave electronics [Paul and Richard (1974), Joseph and et al. (1974)].

The radiation characteristics of a TE$_{11}$-mode excited semi-infinite coaxial waveguide are examined, Kirchoff-Huygen analysis illustrates a pattern sensitivity to the transverse coaxial dimension ratio which can render the TE$_{11}$-mode coaxial radiator unattractive for many antenna applications (Lester and Charles 1977).

The SBFA consists of a leaky cavity resonator formed from two plane reflector of different diameter, spaced a half-wavelength apart, with a source (feed) placed between them, and fed by either a dipole placed normal to the axis of the reflectors, or an open-ended waveguide coaxial with the reflector axis. Multiple reflections of electromagnetic wave and maximum radiation occurs along the outward edge from the larger reflector are two characteristics of such a system (Ehernspeck 1965).

The SBFA is a highly-efficient radiator of simple and compact construction, and highly directional radiator. So it has been the subject of extensive experimental studies, which enable us to optimize the most important parameters (Subber 1997). However, a rigorous theoretical solution for SBFA excited by a coaxial waveguide has not yet been found for understanding the basic operational principles of this antenna.

Analysis of the radiation fields

The coaxial waveguide is aligned along the z-axis with its aperture positioned symmetrically about the origin in the xy-plane as shown in Fig.(1). The electromagnetic fields in the waveguide can be obtained by solving Maxwell's equations subject to the boundary conditions on the waveguide wall. When the TEM-mode in coaxial waveguide can not be us as a feeder to the SBFA because the radiation pattern of this waveguide in this mode have two main lobes and than the value of this field is equal zero at angle $(\theta_1 = 0°)$. The TE$_{11}$-mode, which has the lowest cut-off frequency, is the one most commonly used in coaxial-guide antenna feeds. Hence the discussion will be confined to this mode. The field components of the TE$_{11}$-mode are derived in this work as:-
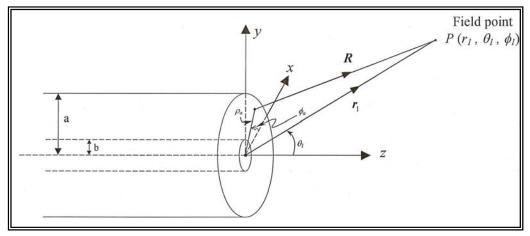

Fig.(1): Definition of orthogonal coordinate system employed in the

analysis.

$$E_\rho(\rho_a,\varphi_a) = jA_1\frac{\omega\mu}{k_c^2\rho_a}[J_1(k_c\rho_a) - E_1N_1(k_c\rho_a)\sin\varphi_a] \qquad \dots\dots\dots (1\text{-}a)$$

$$E_\varphi(\rho_a,\varphi_a) = jA_1\frac{\omega\mu}{k_c}[J_1'(k_c\rho_a) - E_1N_1'(k_c\rho_a)\cos\varphi_a] \qquad \dots\dots\dots (1\text{-}b)$$

$$H_\rho(\rho_a,\varphi_a) = -E_\varphi(\rho_a,\varphi_a)/Z_g \qquad\qquad \dots\dots\dots (1\text{-}c)$$
$$H_\varphi(\rho_a,\varphi_a) = \quad E_\rho(\rho_a,\varphi_a)/Z_g \qquad\qquad \dots\dots\dots (1\text{-}d)$$

where

$$E_1 = \frac{J_1'(k_cb)}{N_1'(k_cb)} = \frac{J_1'(k_ca)}{N_1'(k_ca)} \qquad\qquad \dots\dots\dots (2)$$

$J_1(k_c\rho_a)$ and $N_1(k_c\rho_a)$ are the Bessel functions of order one of the first and second kind respectively, while $J_1'(k_c\rho_a)$ and $N_1'(k_c\rho_a)$ denotes the derivative of $J_1(k_c\rho_a)$ and $N_1(k_c\rho_a)$ with respect to ρ_a respectively, A_1 is constant, k_c -wave number at cut-off frequency, $Z_g = \omega\mu/\beta$ is the waveguide impedance for TE-waves, β is the wave number corresponding to the free space wavelength, a and b are outer and inner radii of coaxial waveguide and ω is angular frequency.

The first root for TE-waves is x_{11} and the cut-off fr1equency f_{c11} below which transmission of a wave will not occur with belong to TE-waves:-

$$f_{c11} = \frac{cr}{2\pi a}x_{11} \qquad\qquad \dots\dots\dots (3)$$

where $r = a/b$.

The scalar radiation pattern field components of the TE11-mode excited coaxial waveguide are:-

$$E_{1\theta}(\theta_1,\varphi_1) = \beta A_1\frac{\omega\mu}{k_c}\frac{e^{-j\beta r_1}}{4r_1}\left(1+\frac{\beta_{11}}{\beta}\cos\theta_1\right)[I_2\sin(2\varphi_1)\cos(\varphi_1) - \sin\varphi_1 \quad \dots(4\text{-}a)$$
$$\{I_0 + I_2\cos(2\varphi_1)\}]$$

$$E_{1\varphi}(\theta_1,\varphi_1) = -\beta A_1\frac{\omega\mu}{k_c}\frac{e^{-j\beta r_1}}{4r_1}\left(\cos\theta_1+\frac{\beta_{11}}{\beta}\right)[I_2\sin(2\varphi_1)\sin(\varphi_1) - \cos\varphi_1 \quad \dots(4\text{-}b)$$
$$\{I_0 + I_2\cos(2\varphi_1)\}]$$

Where

$$\beta_{11} = \sqrt{\beta^2 - k_c^2}$$

$$I_0 = \int_b^a [J_0(k_c\rho_a) - E_1N_0(k_c\rho_a)]J_0(\beta\rho_a\sin\theta_1)\rho_a d\rho_a$$

$$I_2 = \int_b^a [J_2(k_c\rho_a) - E_1N_2(k_c\rho_a)]J_2(\beta\rho_a\sin\theta_1)\rho_a d\rho_a$$

In SBFA the radiating field in the free space can be considered to consists of two parts, as shown in Fig.(2). The principle part is the radiating field reflected by the sub-reflector and in turn reflected by the main-reflected. The second part consists all diffracted fields through the edge of the sub-reflector, we shall restrict ourselves in the paper   to   the   contribution
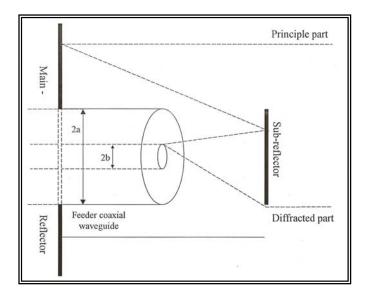
Fig.(2): Sketch of SBFA or radiation sources of SBFA.

of the first part, (i.e. the second part very small). This is because it was demonstrated that the sub-reflector diffraction can significantly increase both side-lobes and cross-polarization (Volker 1988), and hence it was assumed to have a negligible effect on the main-lobe pattern.

Fig.(3) shows a SBFA with its circular aperture situated on the xy-plane with its center at the origin **o** of the rectangular coordinate system (x_3,y_3,z_3). The axis of the SBFA is along the z-direction. The primary feed is placed at midway between and parallel to the reflectors which is taken to be the origin of another rectangular coordinate system (x_1,y_1,z_1) of the feed. Whereas the secondary feed (sub-reflector) is placed at distance z_2 from main-reflector which the origin of rectangular coordinate system (x_2,y_2,z_2) of the sub-reflector.

The tangential field at the aperture of the exciter is :-

$$\overline{E}_a = jA_1 \frac{\omega\mu}{2k_c}[\hat{x}\sin(2\varphi_a)\{J_2(k_c\rho_a) - E_1 N_2(k_c\rho_a)\}+ \qquad \dots (5)$$
$$\hat{y}\{J_0(k_c\rho_a) - E_1 N_0(k_c\rho_a) - \cos(2\varphi_a)J_2(k_c\rho_a) - E_1 N_2(k_c\rho_a)\}]$$

The tangential magnetic field at an observation point $P_1(r_1,\theta_1,\varphi_1)$ may be calculated by a well-known technique (Stutzman and Thiele 1981) based on the equivalent electric and magnetic surface current densities, as :-

$$\overline{H}_{ts} = -j\frac{\beta^2 A_1}{k_c}\frac{e^{-j\beta r_1}}{2r_1}\left[\hat{x}(I_0+I_2)\left\{\left(1+\frac{\beta_{11}}{\beta}\cos\theta_1\right)-\frac{1}{2}\sin^2\theta_1(1+\cos(2\varphi_1))\right\}+ \right.$$
$$\left. \hat{y}\sin(2\varphi_1)\left\{I_2\left(1-\frac{\beta_{11}}{\beta}\cos\theta_1\right)-\frac{1}{2}\sin^2\theta_1(I_0+I_2)\right\}\right]$$

$$\ldots\ldots\ldots (6)$$

where

$$\theta_1 = \tan^{-1}(\rho_s / z_1) \qquad\qquad \ldots\ldots\ldots (7)$$

and the tangential magnetic field at an observation point $P_2(r_2, \theta_2, \varphi_2)$ due to the electric surface current density after use the boundary condition :-

$$\overline{H}_{tm} = -j\beta \frac{e^{-j\beta r_2}}{2\pi r_2} \overline{Q}_s \qquad\qquad \ldots\ldots\ldots (8)$$

where

$$\overline{Q}_s = \frac{\beta^2 A_1 \pi}{k_c} \frac{e^{-j\beta\beta_1}}{2r_1}\left[\hat{x}\left\{\int_0^{r_s}(I_0+I_2)J_0(\beta\rho_s\sin\theta_2)\left(1+\frac{\beta_{11}}{\beta}\cos\theta_1 - \frac{1}{2}\sin^2\theta_1\right) + \cos(2\varphi_2) \right.\right.$$

$$\int_0^{r_s}(I_0+I_2)J_0(\beta\rho_s\sin\theta_2)\bigg\} + \hat{y}\sin(2\varphi_2)\left\{\int_0^{r_s}I_2 J_2(\beta\rho_s\sin\theta_2)\left(1-\frac{\beta_{11}}{\beta}\cos\theta_1\right) - \right.$$

$$\frac{1}{2}\sin^2\theta_1(I_0+I_2)J_2(\beta\rho_s\sin\theta_2)\bigg\}\bigg]\rho_s d\rho_s \qquad \ldots\ldots (9\text{-a})$$

$$\theta_2 = \tan^{-1}(\rho_m / z_2) \qquad\qquad \ldots\ldots\ldots (9\text{-b})$$
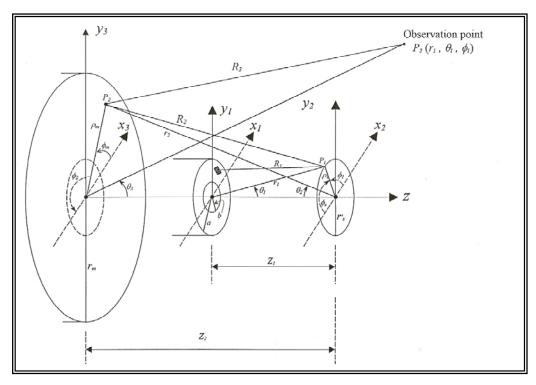

Fig.(3): Coordinate system of SBFA excited by a coaxial waveguide.

The scalar radiation electric field set up at an observation point $P_3(r_3, \theta_3, \varphi_3)$ in the far-field due to the electric surface current density, which is associated with $\overline{H}_{tm}$ on the main-reflector, is :-

$$E_{3\theta} = -j\omega \frac{e^{-j\beta r_3}}{4\pi r_3} \cos\theta_3 \left(Q_{xm} \sin\varphi_3 - Q_{ym} \cos\varphi_3 \right) \qquad \text{......... (10-a)}$$

$$E_{3\varphi} = -j\omega \frac{e^{-j\beta r_3}}{4\pi r_3} \left(Q_{xm} \cos\varphi_3 + Q_{ym} \sin\varphi_3 \right) \qquad \text{......... (10-b)}$$

where

$$Q_{xm} = -j \frac{\beta^3 A_1 \pi}{k_c} \frac{e^{-j\beta(r_1+r_2)}}{2r_1 r_2} \left[\int_a^{r_m} \cos\theta_2 J_0(\beta\rho_m \sin\theta_3) \int_0^{r_s} (I_0 + I_2) J_0(\beta\rho_s \sin\theta_2) \right.$$

$$\left(1 + \frac{\beta_{11}}{\beta}\cos\theta_1 - \frac{1}{2}\sin^2\theta_2 \right)\rho_s d\rho_s + \cos(2\varphi_3)\int_a^{r_m} J_2(\beta\rho_m \sin\theta_3)$$

$$\left. \int_0^{r_s} (I_0 + I_2) J_2(\beta\rho_s \sin\theta_2) \rho_s d\rho_s \right] \rho_m d\rho_m$$

$$Q_{ym} = -j \frac{\beta^3 A_1 \pi}{k_c} \frac{e^{-j\beta(r_1+r_2)}}{2r_1 r_2} \left[\sin(2\varphi_3)\int_a^{r_m} \cos\theta_2 J_2(\beta\rho_m \sin\theta_3) \int_0^{r_s} I_2 J_2(\beta\rho_s \sin\theta_2) \right.$$

$$\left. \left(1 - \frac{\beta_{11}}{\beta}\cos\theta_1 \right) - \frac{1}{2}\sin^2\theta_1 (I_0 + I_2) J_2(\beta\rho_s \sin\theta_2)\rho_s d\rho_s \right] d\rho_m d\rho_m$$

Computed results and discussion

The E-plane and H-plane normalized pattern functions for the coaxial TE$_{11}$-mode, found from equations (4), are plotted in Fig.(4) for several values of (a/b). The H-plane pattern beamwidth and side lobe levels are relatively insensitive to difference of (a/b) ratio. The E-plane pattern is sensitive to difference of (a/b) ratio, particularly where the ratio is less than 10. E-plane beamwidth is determined by this ratio with side lobe amplitude.
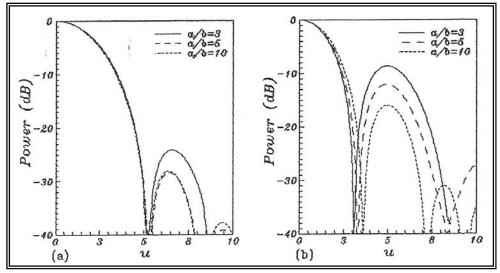

Fig.(4): Normalized (a) E-plane and (b) H-plane patterns TE$_{11}$-mode, compared with (Lester and Charles 1977). Where $(u = \beta \sin\theta)$

These patterns are compared in Fig.(4) with the corresponding theoretical results of Lester (Lester and Charles 1977), a very good agreement between these results, because these results are identical quite.

Examination of the radiation pattern functions of the TE_{11}-mode coaxial waveguide reveals an E-plane beamwidth and side lobe sensitivity to the transverse dimension ratio.

The radiation field patterns of a SBFA, excited by a coaxial waveguide propagating the TE_{11}-mode, in the principle E- and H-planes are obtained from equations (10) by numerically evaluating the appropriate integrals. These patterns are compared in Fig.(5) with the corresponding theoretical results of (Kishk and Shafai 1986). A good agreement between these results is achieved in the figure for the main lobe, and nearly identical -3 dB and -10 dB beamwidths are obtained in both principle planes. Some deviation is noted in the side lobe may be due to the rim or to the diffracted field through the edge of the sub-reflector, as mentioned in analysis of the radiation field.
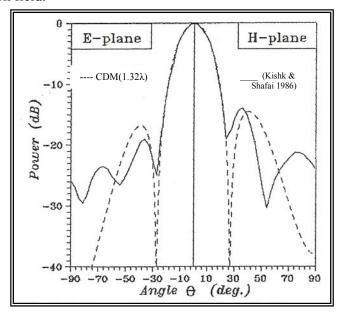

Fig.(5): Radiation pattern of SBFA excited by a coaxial waveguide, propagating the TE_{11}-mode.
CDM: current distribution method used in this analysis.

The calculated parameters and theoretical results of (Kishk and Shafai 1986)related to the radiation field patterns are summarized in table (1).

The excitation of a SBFAs by the circular and coaxial waveguide provides more symmetric excitations. In particular, an excitation by a coaxial waveguide was found to yield higher gain than circular waveguide as predicated in table (2).

Table (1): Comparison between theoretical results of the radiation parameters of SBFA excited by a coaxial waveguide, with dimensions:-

$$r_m = 1.32\lambda \ , \ r_s = 0.4\lambda \ , \ z_1 = 0.35\lambda \ , \ z_2 = 0.5\lambda \ , \ a = 0.4\lambda \ , \ r = a/b = 10 \ , \ f = 10 \text{ GHz}$$

	H – plane				E – plane				Directivity	Aperture efficiency
	-3 dB B.W. (deg.)	-10 dB B.W. (deg.)	S.L.L (dB)	Post. of SLL (deg.)	-3 dB B.W. (deg.)	-10 dB B.W. (deg.)	S.L.L (dB)	Post. of SLL (deg.)	(dB)	(%)
Theore*	22.5	34.6	-14.0	36.0	20.0	33.0	-19.5	39.0	18.035	92.45
CDM	22.6	38.5	-14.5	40.5	22.0	37.8	-16.8	39.0	17.739	95.098

* Theoretical results reproduced from (Kishk, and Shafai 1986)

Table (2): Comparison between theoretical results of SBFA excited by a coaxial and circular waveguides, with dimensions:-

$$r_m = 1.0\lambda \ , \ r_s = 0.3\lambda \ , \ z_1 = 0.465\lambda \ , \ z_2 = 0.72\lambda \ , \ a = 0.33\lambda \ , \ r = a/b = 10 \ , \ f = 9 \text{ GHz}$$

Type of feeder	H – plane				Co – polar				E – plane				Max. cross-polar		Directivity
	-3 dB B.W. (deg.)	-10 dB B.W. (deg.)	S.L.L (dB)	Post. of SLL (deg.)	-3 dB B.W. (deg.)	-10 dB B.W. (deg.)	S.L.L (dB)	Post. of SLL (deg.)	-3 dB B.W. (deg.)	-10 dB B.W. (deg.)	S.L.L (dB)	Post. of SLL (deg.)	(dB)	Position of cross-polar (deg.)	(dB)
Coaxial waveguide	28.9	49.5	-14.2	56.0	28.0	48.7	-16.3	53.0	27.5	47.9	-18.6	50.0	-22.1	90.0	15.52
Circular waveguide*	29.3	50.2	-13.4	57.5	28.7	49.5	-15.6	54.0	28.1	48.8	-18.1	51.0	-21.8	90.0	15.30

* Theoretical results reproduced from (Subber 1997)

Conclusions

It concluded that the radiation patters of a SBFA fed by a coaxial waveguide are preferable from the SBFA fed by a circular waveguide. Examination of the radiation pattern functions of the TE_{11} coaxial waveguide mode reveals an E-plane beamwidth and side lobe sensitivity to the transverse dimension ratio.

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