STABILITY OF ITERATIVE PROCEDURES FOR HYBRID MAPS IN b-METRIC SPACE

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Abstract.

In this paper, we study the existence of coincidences and fixed points of hybrid contractions maps on more general setting than metric spaces.

The same utilize to discuss the problem of stability of iterative procedures in generalized Hausdorff metric space.

Our work generalize and improve some results due to Ostrowski 1967, Czerwik et al. 1997 ,Singh et al 2005 and Singh et al. 2008.

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1. INTRODUCTION

Let (X,d) be a metric space and an operator $T: X \to CL(X)$ where $CL(X)$ the collection of all nonempty closed subsets of X.Problem of existence of coincidence and fixed points of T lead to an iteration procedure $x_{n+1} \in f(T,x_n)$. However, in computation, an approximate sequence $\{y_n\}$ is applied instead of $\{x_n\}$. This leads to a concept of stability of iteration procedure with respect to T.

The first stability result in metric space was proved by Ostrowski (1967) . For any $x_0 \in X$, we put

$$x_{n+1} = f(T,x_n), n = 1,2,..... \quad (1)$$

Let $\{x_n\}$ be a sequence converges to a fixed point $u \in X$ of T . Let $\{y_n\} \subseteq X$ be an arbitrary sequence , and set

$$\varepsilon_n = d(y_{n+1}, Ty_n), \quad n = 0, 1, 2, ... \quad (2).$$

The iteration procedure (1) is said to be T -stable (cf Czerwik 1998) provided that $\lim_{n\to\infty} \varepsilon_n = 0$ implies that $\lim_{n\to\infty} y_n = u$.

This subject was developed by Harder and Hicks 1988a, b ,Rhoades 1993, Osilike1995 , Osilike1996 and berinde 2002 (see also Ciric 1972, Ciric 1974, Czerwik 1997 , Czerwik et al.2002, Singh & Chanda 1995, Singh & Prassad 2008 , Singh et al. 2005 and Singh & Hashim 2005 .

Czerwik 1997 and Czerwik et al. 2002 has been extended Ostrowski's classical theorem for the stability of iterative procedures for multi-valued maps in b-metric spaces. The purpose of this paper is to study the stability of iteration process for hybrid contraction mappings involving single-valued and multi-valued maps in b-metric spaces .

2. PRELIMINARIES

Consistent with Czerwik 1998 and Singh & Hashim 2005 , we use the following notations and definitions.

Definition 2.1: Czerwik 1998 .

Let X be a nonempty set and $s \geq 1$ be a given real number. A function $d : X \times X \to R_+$ is a b-metric provided that for all $x, y, z \in X$,

$$d(x, y) = 0 \text{ iff } x = y, \quad (3)$$

$$d(x, y) = d(y, x), \quad (4)$$

$$d(x, z) < s[d(x, y) + d(y, z)] \quad (5)$$

The pair (X, d) is called a b-metric spaces.

Notice that the class of b-metric spaces is effectively larger than that of metric spaces. The following example shows that a b-metric on X need not a metric on X (see also Singh & Prassad 2008).

Example 2.1: Let $X = \{x_1, x_2, x_3\}$ and $d : X \times X \to R_+$ such that

$d(x_1, x_2) = a \geq 2$, $d(x_1, x_3) = d(x_2, x_3) = 1$ and

$d(x_n, x_n) = 0$, $d(x_n, x_k) = d(x_k, x_n)$,

$$d(x_n, x_k) = \frac{a}{2}[d(x_n, x_i) + d(x_i, x_k)], \; n, k, i = 1, 2, 3$$

Then (X, d) is a b-metric space.

$CL(X) = \{A : A$ is nonempty closed subset of $X\}$

$$H(A,B)=max\begin{cases} \sup_{x\in A} D(x,B), \sup_{y\in B} D(y,A), & \text{if the maximum exists} \\ \infty, & \text{otherwise} \end{cases}$$

$A,B\in CL(X)$

H is called a generalized Hausdorff b-metric on $CL(X)$

$$D(x,A)=\inf_{\alpha\in A} d(x,a)$$

Lemma 2.1 Czerwik 1998:

For any $A,B,C\in CL(X)$,

$D(x,B)\le d(x,y)$ For any $y\in B$, (6)

$D(A,B)\le H(A,B)$, (7)

$H(A,C)\le s[H(A,B)+H(B,C)]$. (8)

Lemma 2.2 (Czerwik 1998):

Let A and B be distinct elements of $CL(X)$

Let $x\in A$. Then for a given number $\lambda>1$, there exists a point y in B such that $d(x,y)\le\lambda H(A,B)$.

Definition 2.2 (Singh & Chanda 1995):

Let X be a metric space and $T:X\to CL(X)$. Let $x_o\in X$, and

(*) $x_{n+1}\in f(T,x_n)$,

denote some iteration procedure. Let the sequence $\{x_n\}$ be convergent to a fixed point P of T. Let $\{y_n\}$ be an arbitrary sequence in X and set

$\varepsilon_n=H(y_{n+1},f(T,y_n)),\quad n=0,1,2,......,$

If limit $\varepsilon_n=0$ implies that $\lim_{n\to\infty} y_n=p$ then the iteration process defined in (*) is said to be T-stable or stable with respect to T.

Recall that this definition for a single-valued operator is due to Harder &Hicks 1988a, b.

The following definition of stability general iterative procedure (9) for the coincidence point equation $fx=Tx$ is due to Singh et al. 2008 when S and T are single –valued maps.

Definition 2.3 [Singh et al. 2008]:

Let (X,d) be a b-metric space and $Y\subseteq X$. Let $T:Y\to CL(X)$, $S:Y\to X$, $T(Y)\subseteq S(Y)$ and z a coincidence point of T and S, that is $Sz\in Tz=u$. For any $x_o\in Y$, let the sequence $\{Sx_n\}$ generated by the general iterative procedure.

$Sx_{n+1}\in f(T,x_n)\qquad n=1,2,.....$ (9)

Converge to an element $u\in X$. Let $\{Sy_n\}\subseteq X$ be an arbitrary sequence and let $\varepsilon_n=H(Sy_{n+1},f(T,y_n)),\quad n=0,1,2,......$.

Then the iterative procedure $f(T, x_n)$ is (S, T), stable or stable with respect to (S, T) if $\lim_{n\to\infty} \varepsilon_n = 0$ implies that $\lim_{n\to\infty} Sy_n = 0$.

We remark that the iterative procedure (9) reduce to $x_{n+1} \in f(T, x_n)$ $n = 1, 2, \ldots\ldots$

When $Y = X$ and S is the identity map on X . The general definition of stability of iterative procedures for general fixed equation due to Czerwik et al. 2002 and Singh et al. 2005.

Consider the following conditions for $T : Y \to CL(X)$ and $S : Y \to X$ for all $x, y \in Y$, where $0 < q < 1$ and X is a metric space;

$$H(Tx,Ty) \le q\, d(Sx,Sy); \quad (10)$$

$$H(Tx,Ty) \le q \max\{ d(Sx,Sy), D(Sx,Tx), D(Sy,Ty), \frac{1}{2}[D(Sx,Ty) + d(Sy,Tx)]\}; \quad (11)$$

$$H(Tx,Ty) \le q\, d(Sx,Sy) + LD(Sx,Tx). \quad 0 < q < 1, L \ge 0 \quad (12)$$

Notice that (10) is included in (11) while (11) itself is called a generalized hybrid contraction and when $S = id$ identity map on X (11) is called a generalized multi-valued contraction . These conditions with $X = Y$, $S = id$, the identity map on X and T is a single valued mapping are compared in Rhoades 1977.

Lemma 2.3: see also (Rhoades 1993 , Czerwik et al. 2002).

Let $\{ \varepsilon_n \}$ be a sequence of non negative real numbers.

Let $s_n = \sum_{i=0}^{n} k^{n-i} \varepsilon_i$ where $0 \le k < 1$.

Then $\lim_{n\to\infty} \varepsilon_n = 0$ iff $\lim_{n\to\infty} s_n = 0$. (13)

Now we can present the following fixed point theorem (see Singh et al. 2005)

Theorem 2.1: (Singh et al. 2005) :

Let X be a complete b-metric space and T a generalized multi-valued contraction on X . Then

(i) For every $x_o \in X$ there exists an orbit $\{ x_n \}$ of T at x_o and $u \in X$ such that $\lim_{n\to\infty} x_n = u$; .

(ii) The point u is fixed point of T , i.e. $u \in Tu$.

3. Main Results

Theorems 3.1:

let (X, d) be a b-metric and $T : Y \to CL(X)$, $S : Y \to X$ such that $T(Y) \subseteq S(Y)$ and (11) holds with $qs < 1$ and $\alpha s < 1$ where $\alpha = \max\{ k, \frac{ks}{2 - qs}\} < 1$ and one of

TY or SY is a complete subspace of X , then T and S have a coincidence point , i.e there exists a $v \in Y \ni Sv \in Tv$.

Proof:

Let $x_o \in Y$ since $T(Y) \subseteq S(Y)$, choose x_1, so that $y_1 = Sx_1 \in Tx_o$

In general choose x_{n+1} so that $y_{n+1} = Sx_{n+1} \in Tx_n$. let λ be a real number such that $0 < \lambda < 1$. By lemma (2.2) and condition (11) ,

$$\begin{aligned} d(y_n, y_{n+1}) &\le q^{-\lambda} H(Tx_{n-1}, Tx_n) \\ &\le q^{-\lambda}.q\, max\{ d(Sx_{n-1}, Sx_n), \\ &D(Sx_{n-1}, Tx_{n-1}), D(Sx_n, Tx_n), \\ &\frac{1}{2}[D(Sx_{n-1}, Tx_n) + D(Sx_n, Tx_{n-1})]\} \end{aligned}$$

$$\begin{aligned} d(y_n, y_{n+1}) \le k\, max\{ & d(y_{n-1}, y_n), d(y_{n-1}, y_n), \\ & d(y_n, y_{n+1}), \frac{1}{2}[d(y_{n-1}, y_{n+1}) + d(y_n, y_n)]\} \end{aligned}$$

where $k = q^{1-\lambda}$.

Since $k < 1$ the relation $d(y_{n-1}, y_{n+1}) \le kd(y_n, y_{n+1})$ implies $d(y_{n-1}, y_{n+1}) = 0$.

We may assume that $y_n, \ne y_{n+1}$ and

$$d(y_n, y_{n+1}) \le k\, max\{ d(y_{n-1}, y_n), \frac{1}{2} d(y_{n-1}, y_{n+1})\}$$

so $d(y_n, y_{n+1}) \le kd(y_{n-1}, y_n)$,or else

$$d(y_n, y_{n+1}) \le \frac{ks}{2}[d(y_{n-1}, y_n) + d(y_n, y_{n+1})],$$

so $d(y_n, y_{n+1}) \le \dfrac{ks}{2-ks} d(y_{n-1}, y_n)$

This yield

$$d(y_n, y_{n+1}) \le max\{ k, \frac{ks}{2-ks}\}[d(y_{n-1}, y_n)] \qquad n = 1,2,.....,$$

where $max\{ k, \dfrac{ks}{2-ks}\} < 1$.

Therefore, $\{ y_n \}$ is a Cauchy sequence and its sub sequence $\{ y_{2n} \}$ has a limit in SY call it u . let $v \in S^{-1}u$ .Then $Sv = u$.

Notice that the subsequence $\{ y_{2n=1} \}$ also converge to u . by (11)

$$\begin{aligned} D(Tv, y_{2n+2}) &\le H(Tv, Tx_{2n+1}) \\ &\le q\, max\{ d(Sv, Sx_{2n+1}), D(Sv, Tv), D(Sx_{2n+1}Tx_{2n+1}) \\ &\frac{1}{2}[D(Sv, Tx_{2n+1}) + D(Sx_{2n+1}, Tv)]\}, \\ &\le q\, max\{ d(Sv, y_{2n+1}), D(Sv, Tv), d(y_{2n+1}, y_{2n+2}), \\ &\frac{1}{2}[d(Sv, y_{2n}) + D(y_{2n+1}, Tv)]\}. \end{aligned}$$

Making $n \to \infty$ this obtains

$D(Tv, Sv) \le q\, D(Sv, Tv)$, and $Sv \in Tv$

We remark that theorem (3.1) with $S = Id$ (the identity map) is the main result of Singh et al. 2005 .

If T is a single-valued map in theorem 3.1 with $s = 1$ is also the main result of Ciric 1972 .

Theorem 3.2: let (X, d) be a b-metric and $Y \subseteq X$ let $T : Y \to CL(X)$, $S : Y \to X$ and SY or TY is a complete subspace of X .

Let z be a coincidence point of T and S ,i.e , $u = Sz \in Tz$. For any $x_o \in Y$, let the sequence $\{ Sx_n \}$, generated by $Sx_{n+1} \in Tx_n$ converge to u .Let $\{ Sy_n \} \subseteq X$ and define $\varepsilon_n = H(Sy_{n+1}, Ty_n), \quad n = 0, 1, 2, \ldots\ldots,$

If S and T satisfy (11) for all $x, y \in Y$ and $k = sq < 1$, then

$$d(u, Sy_{n+1}) \le sd(u, Sx_{n+1}) + s(s\alpha)^{n+1} d(Sx_o, Sy_o) + s^3\alpha \sum_{r=0}^{n} (s\alpha)^{n-r} d(Sx_n, Sx_{n+1}) + s^2 \sum_{r=0}^{n} (s\alpha)^{n-r} \varepsilon_r \qquad (14)$$

$$\alpha = s^2 q \,/\, 1 - s^2 q$$

$$\lim_{n\to\infty} Sy_n = u \iff \lim_{n\to\infty} \varepsilon_n = 0 \qquad (15)$$

Proof: From (11) for any $x, y \in Y$, one of the following holds;

$$H(Tx, Ty) \le qd(Sx, Sy),$$
$$H(Tx, Ty) \le qD(Sx, Tx) \le qH(Sx, Tx);$$
$$\begin{aligned} H(Tx, Ty) &\le qD(Sy, Ty) \le qH(Sy, Ty) \le qs[H(Sy, Sx) + H(Sx, Ty)] \\ &\le qsd(Sy, Sx) + qs[sH(Sx, Tx) + sH(Tx, Ty)] \\ &\le qsd(Sy, Sx) + qs^2 H(Sx, Tx) + qs^2 H(Tx, Ty) \end{aligned}$$

i.e, $H(Tx, Ty) \le \dfrac{qs}{1 - qs^2} d(Sy, Sx) + \dfrac{qs^2}{1 - qs^2} H(Sx, Tx)$

$$\le \frac{k}{1 - ks} d(Sy, Sx) + \frac{ks}{1 - ks} H(Sx, Tx)$$

$$\begin{aligned} H(Tx, Ty) &\le \frac{q}{2}\{ D(Sx, Ty) + D(Sy, Tx)\} \\ &\le \frac{q}{2}\{ H(Sx, Ty) + H(Sy, Tx) \\ &\le \frac{qs}{2}\{ H(Sx, Tx) + H(Tx, Ty) + H(Sy, Sx) + H(Sx, Tx)\} \\ &\le \frac{qs}{2}\{ 2H(Sx, Tx) + H(Sx, Sy) + H(Tx, Ty)\} \end{aligned}$$

i.e,

$$H(Tx, Ty) \le \frac{qs}{2 - qs}\{ 2H(Sx, Tx) + d(Sx, Sy)\}$$

$$\le \frac{k}{2 - k} d(Sx, Sy) + \frac{2k}{2 - k} H(Sx, Tx)$$

Therefore, in all the cases we get,

$$H(Tx, Ty) \le \alpha d(Sx, Sy) + s\alpha H(Sx, Tx)$$

Where $\alpha = \dfrac{s^2 q}{1 - s^2 q}$

Then by virtue of (11) we obtain , For any non-negative integer n .

$$\begin{aligned} d(Sx_{n+1}, Sy_{n+1}) &\le H(Tx_n, Sy_{n+1}) \le s[H(Tx_n, Ty_n) + H(Ty_n, Sy_{n+1})] \\ &\le s[\alpha d(Sx_n, Sy_n) + s\alpha H(Sx_n, Tx_n) + \varepsilon_n] \\ &\le s\alpha d(Sx_n, Sy_n) + s^2\alpha H(Sx_n, Tx_n) + s\varepsilon_n \\ &\le s^2\alpha[H(Tx_{n-1}, Ty_{n-1}) + H(Ty_{n-1} Sy_n)] + s^2\alpha H(Sx_n, Tx_n) + s\varepsilon_n \\ &\le s^2\alpha[\alpha d(Sx_{n-1}, Sy_{n-1}) + \alpha sd(Sx_{n-1}, Tx_{n-1})] + \\ &\qquad s^2\alpha\varepsilon_{n-1} + s^2\alpha H(Sx_n, Tx_n) + s\varepsilon_n \\ &\le s^2\alpha^2 d(Sx_{n-1}, Sy_{n-1}) + (s^2\alpha^2) sd(Sx_{n-1}, Tx_{n-1}) + \\ &\qquad s^2\alpha\varepsilon_{n-1} + s\varepsilon_n + s^2\alpha H(Sx_n, Tx_n) \\ &\le (s\alpha)^2 d(Sx_{n-1}, Sy_{n-1}) + (s\alpha)^2 sd(Sx_{n-1}, Tx_{n-1}) + \\ &\qquad s^2\alpha H(Sx_n, Tx_n) + s[\alpha s\varepsilon_{n-1} + \varepsilon_n] \end{aligned}$$

Repeating this process (n - 1) times we get

$$d(Sx_{n-1}, Sy_{n-1}) \le (s\alpha)^{n-1} d(Sx_o, Sy_o) + s^2\alpha \sum_{r=o}^{n} (s\alpha)^{n-r} d(Sx_r, Ty_r) + s\sum_{r=0}^{n} (s\alpha)^{n-r} \varepsilon_r$$

Consequently,

$$d(u, Sy_{n+1}) \le sd(u, Sx_{n+1}) + sd(Sx_{n+1}, Sy_{n+1})$$

This yields the inequality (14).

Assume that $\lim_{n\to\infty} Sy_n = u$

$$\begin{aligned} \varepsilon_n = H(Sy_{n+1}, Ty_n) &\le sH(Sy_{n+1}, u) + sH(u, Ty_n) \\ &\le sd(Sy_{n+1}, u) + s[\alpha d(u, Sy_n,) + s\alpha H(Su, Tu)] \end{aligned}$$

Letting $n \to \infty$ we obtain $\varepsilon_n \to 0$

Suppose $\lim_{n\to\infty} \varepsilon_n = 0$ since $0 \le s\alpha < 1$ and

$$\lim_{n\to\infty} Sx_n = u$$

First two terms on the right hand side (14) vanish in the limit. Appling lemma (2.3) to the last term of inequality (14) we see that

$$\lim_{n\to\infty} s^2 \sum_{r=0}^{\infty} (s\alpha)^{n-r} \varepsilon_r = 0$$

Finally we show that,

$$\lim_{n\to\infty} s^3 \alpha \sum_{r=0}^{n} (s\alpha)^{n-r} d(Sx_r, Tx_r) = 0$$

Let A denote the lower triangular matrix with entries $a_{nr} = (s\alpha)^{n-r}$, then $\lim_{n\to\infty} a_{nr} = 0$ for each r and $\lim_{n\to\infty} \sum_{r=0}^{n} a_{nr} = \dfrac{1}{1-s\alpha}$ therefore, A is multiplicative i.e for any converge sequence $\{s_n\}, \lim_{n\to\infty} A(s_n) = \dfrac{1}{1-s\alpha} \lim_{n\to\infty} s_n.$ thus,

$$\lim_{n\to\infty} s^3 \alpha \sum_{r=0}^{n} (s\alpha)^{n-r} d(Sx_r, Tx_r) = 0$$

(cf. Czerwik 1997 p. 692).

Remark 3.1:

Theorem (3.2) with $S = I$ (identity map) generalize the main result of Singh et al. 2005.

Corollary 3.1: Singh et al. 2005 .

Let (X, d) be a complete b-metric space and $T : X \to CL(X)$ satisfy condition (11) with $S = I$ (identity map), let $x_o \in X$ and $\{x_n\}$ be an orbit for T at x_n , i.e. $x_{n+1} \in Tx_n \quad n = 0, 1, 2, \ldots\ldots$ and $\{x_n\}$ converge to a fixed point u of T . Let $\{y_n\}$ be a sequence in X and set

$$\varepsilon_n = H(y_{n+1}, Ty_n), \quad n = 0, 1, 2, \ldots\ldots$$

then

$$d(u, Sy_{n+1}) \le s(sq)^{n+1} d(x_o, y_o) + s^2 mq \sum_{r=0}^{n} (sq)^{n-1} d(x_r, x_{r+1}) + s^2 \sum_{r=0}^{n} (sq)^{n-r} \varepsilon_n$$

Where $m = (1 - sq)^{-1}$

If Tu is singleton, then $\lim_{n\to\infty} y_n = u$ iff $\lim_{n\to\infty} \varepsilon_n = 0$

Corollary3.2: (Czerwik et al. 2002)

Let (X, d) be a complete b-metric space and let $T : X \to CL(X)$ satisfy

$$H(Tx, Ty) \le \alpha d(x, y), \; x, y \in \lambda$$

where $0 < \alpha < s^{-1}$ let $x_o \in X$ and $\{x_n\}$ be an orbit of T at x_o . i.e. $x_{n+1} \in Tx_n, \quad n = 0, 1, 2, \ldots.$ and $\{x_n\}$ converge to fixed point u of T , moreover let $\{y_n\}$ be a sequence in X and set

$$\varepsilon_n = H(y_{n+1}, Ty_n), \quad n = 0, 1, 2, \ldots\ldots$$

then

$$d(u, y_{n+1}) \le sd(u, x_{n+1}) + s(s\alpha)^{n+1} d(x_o, y_o) + s^2 \sum_{r=0}^{n} (s\alpha)^{n-r} \in r$$

if moreover, Tu is singleton, then $\lim_{n\to\infty} y_n = u$ iff $\lim_{n\to\infty} \varepsilon_n = 0$

Remark 3.2: 1- Theorem (3.2) generalize theorem 4.3 of Singh et al. 2008. when S and T are single valued maps .

2- If $s=1$, $S=Id$ (the identity map), S and T are single valued map in the theorem 3.2, we obtain the main result of Osilike 1995 .

Theorem 3.3: let (X,d) be a b-metric and $Y\subseteq X$ let $T:Y\to CL(X)$, $S:Y\to X$ such that $T(Y)\subseteq S(Y)$ and $S(Y)$ or $T(Y)$ is a complete subspace of X . Let z be a coincidence point of T and S , i.e. $u=Sz\in Tz$ generated by $Sx_{n+1}\in Tx_n$ converge to u . Let $\{Sy_n\}\subset X$ and define

$$\varepsilon_n=H(Sy_n,Ty_n),\ n=0,1,2,\ldots \quad .$$

If S and T satisfies (12) with $s^2q<1$, then

$$d(u,Sy_{n+1})\le sd(u,Sx_{n+1})+s(sq)^{n+1}d(Sx_o,Sy_o)+ s^2L\sum_{r=0}^{n}(sq)^{n-r}d(Sx_r,Tx_r)+s^2\sum_{r=0}^{n}(sq)^{n-r}\varepsilon_r. \quad 16)$$

$$\lim_{n\to\infty}Sy_n=u\Leftrightarrow\lim_{n\to\infty}\varepsilon_n=0. \quad (17)$$

Proof : In view of the fact that $Sx_{n+1}\in Tx_n$ for $n=0,1,2,\ldots..$ and the definition of H , we get $d(Sx_{n+1},Sy_{n+1})\le H(Tx_n,Sy_{n+1})$.

Now, since H is a b-metric, so for any $A,B,C\in CL(X)$ we have

$$H(A,C)\le s[H(A,B)+H(B,C)]$$

therefore by (12) we obtain

$$\begin{aligned}d(Sx_{n+1},Sy_{n+1})&\le H(Tx_n,Sy_{n+1})\le s[H(Tx_n,Ty_n)+H(Ty_n,Sy_{n+1})]\\&\le s[qd(Sx_n,Sy_n)+LD(Sx_n,Tx_n)]+s\varepsilon_n\\&\le(sq)^2d(Sx_{n-1},Sy_{n-1})+s^2qLd(Sx_{n-1},Tx_{n-1})+\\&\quad sLd(Sx_n,Tx_n)+s^2q\varepsilon_{n-1}+s\varepsilon_n\end{aligned}$$

Since

Repeating this process $n-1$ times, we get

$$d(u,Sy_{n+1})\le sd(u,Sx_{n+1})+s(sq)^{n+1}d(Sx_o,Sy_o)+s^2L\sum_{r=0}^{n}(sq)^{n-r}d(Sx_r,Tx_r)+ s^2\sum_{r=0}^{n}(sq)^{n-r}\varepsilon_r$$

The proof (17) is similar to (15) in Theorem 3.2.

Remark 3.3:

1**:** In Theorem 3.3 when we put S and T maps on arbitrary set Y with values in X we get Theorem 4.2 in Singh and Prassad 2008.and when s generalize the main result of Singh et al. 2005.

2: Theorem 3.3 generalize the main result of Rhoades 1993 in case of single value map and when $s=1$ and $S=Id$ (Identity map) under the following condition ;

$$d(Tx,Ty)\le q\max\{d(x,y),\frac{d(x,Tx)+d(y,Ty)}{2},d(x,Ty),d(y,Tx)\}$$

We remark that the above condition implies (12). For more detail see Rhoades 1977 .

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استقرار العمليات المكررة للدوال الهجينية في الفضاء المتري – ب

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الملخص

تضمن هذا البحث دراسة وجود النقاط الثابتة والمتطابقة للدوال الهجينية على فضاء اكثر تعميما من الفضاء المتري وكذلك تم استخدام نفس الطريقة لمناقشة استقرار العمليات المكررة في الفضاء الهاوزدورفي المتري المعمم.

في هذا البحث تم تعميم وتطوير نتائج العديد من الباحثين مثل, Ostrowski 1967, Czerwik et al. 1997 Singh et al 2005. و Singh et al. 2008.
