He's variational iteration method with orthogonal polynomials

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Abstract

The He's varational iteration method (VIM) depend on solving integration at each iterations, some times the integration is very difficult to overcome this difficulty, the orthogonal polynomials will be used. Test problems are discussed for comparison between VIM and Adomian Decomposition method (ADM) for different orthogonal polynomials M. M. Hosseini (2006).

Key words: ADM, VIM, orthogonal polynomials

1. Introduction

The VIM, which was proposed originally by J.H. He (1999), the VIM has been proved by many authors to be a powerful mathematical tool for various kinds of linear and nonlinear problems. Unlike the traditional numerical methods, VIM needs no discretization, linearization, transformation or perturbation. The method, has been widely applied to solve nonlinear problems, more and more merits have been discovered and some modifications are suggested to overcome the demerits arising in the solution procedure Syed Tauseef Mohyud-Din (2009), Yongxiang Zhao (2010) and L. Ahmed (2010). In the early eighties of the last century, researchers interested in new procedures and then used widely to solve the nonlinear equations, which called Adomian decomposition method (ADM) G. Adomian (1994). Many of modification on these methods have been introduced since the years past and even now.

Consider the following nonlinear differential equation

$$Lu + Nu = g(x) \qquad (1)$$

where L is a linear operator, N a nonlinear operator, and the continues function $g(x)$ is the remaining term. Some equations have a complicated functions $g(x)$, ADM and VIM need the integrations at each iteration, and some times the integration is very difficult so that the approximate $g(x)$ will be used. Wazwaz A. M. Wazwaz (1999) introduced the modified ADM to solve these problems by approximat $g(x)$ by Taylor series, M. M. Hosseini (2006) propose a new modification of ADM based on Chebyshev polynomials to approximate $g(x)$. The results obtained are better than the results obtained from solving equations with Taylor expansion for $g(x)$ A. M. Wazwaz (1999), as well as Yucheng Liu Yucheng Liu (2009) use the Legender polynomials to approximate $g(x)$, and show that Chebyshev polynomial is more accurate in approximation than Legender polynomial, but Legender polynomials are easier in applications because Legender polynomials have a unit weight function. Many researchers compare the VIM and ADM to show which of these methods are better in the applications. In this paper we introduce a new modification of He's VIM based on orthogonal polynomials that used to approximate $g(x)$. Two test problems are discussed, the results are compared with M. M. Hosseini (2006) and Yucheng Liu (2009). We use Maple 13 software for this purpose.

2. Variational Iteration method

To introduce the basic concepts of VIM, consider the ordinary differential equation (1). According to the VIM J.H. He (1999), we can construct the correction functional for equation (1) as,

$$u_{n+1} = u_n + \int_0^x \lambda(s)\left(Lu_n + N\tilde{u}_n - g(s)\right) ds$$

where $\lambda(s)$ is a general Lagrangian multiplier, which can be identified optimally via the variational theory, the subscript n denotes the nth order approximation, $\tilde{u}_n$ is considered as a restricted variation ($\delta\tilde{u}_n = 0$), in the VIM the zero term u_0 must satisfy the initial conditions of equation (1). The procedure continue until we have a better approximate solution.

3. Test problems

In order to assess the modified VIM by orthogonal polynomials to approximate functions, we will consider the following test problems:

Problem 3.1. Consider the following ordinary differential equation with variables coefficients

$$u'' + xu' + x^2u^3 = \left(2+6x^2\right)e^{x^2} + x^2e^{3x^2} \qquad 0 \le x \le 1$$

with boundary conditions $u(0) = 1$ and $u'(0) = 0$.

According to the VIM its correction functional can be written down as follows

$$u_{n+1} = u_n + \int_0^x \lambda(s)\left(u_n'' + x\tilde{u}_n' + x^2\tilde{u}_n^3 - g(s)\right) ds$$

Making the above correction functional stationary, and noticing that $\delta u(0)=0$, then the Lagrange multiplier, therefore, can be easily identified as follows $\lambda = s-x$, yields the following iteration formula

$$u_{n+1} = u_n + \int_0^x (s-x)\left(u_n'' + xu_n' + x^2u_n^3 - g(s)\right) ds$$

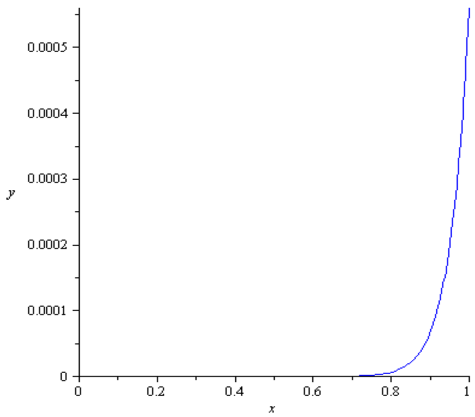

Figure 1

Now, we replace the function $g(x)$ with its Chebysheve approximation with $N=7$, and obtain the Maximum error as shown in figure 2.

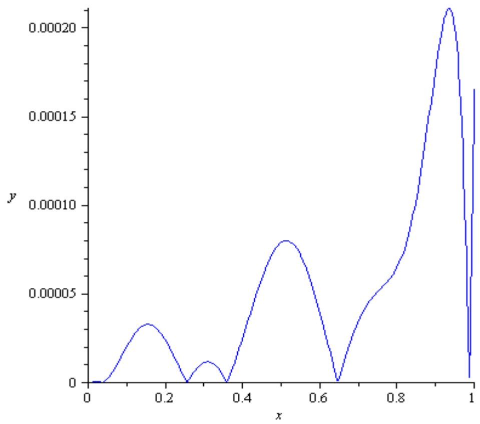

Figure 2

In similar manner, if we to replace the function $g(x)$ with its Legender approximation with $N=7$, this yield the Maximum error in figure 3.

Now, we replace the function $g(x)$ with its Taylor expansion we have figure 1 that show the Maximum error.

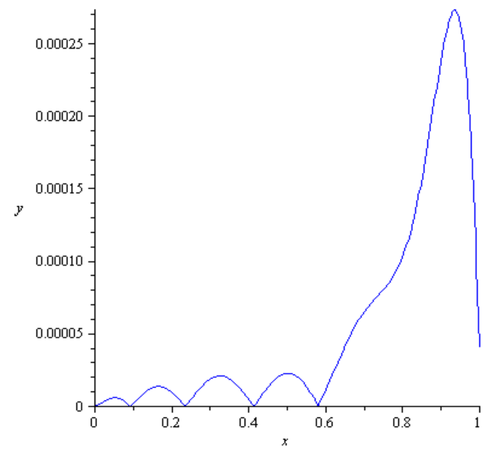

Figure 3

Problem 3.2. In the following ordinary differential equation for $0 \le x \le 1$

$$u'' + uu' = x\sin(2x^2) - 4x^2\sin(x^2) + 2\cos(x^2)$$

with boundary conditions $u(0)=u'(0)=0$.

The exact solution is $u(x)=\sin(x^2)$.

Now, if we expand $g(x)$ by Taylor series, and using VIM, we have the Maximum error shown in figures 4.

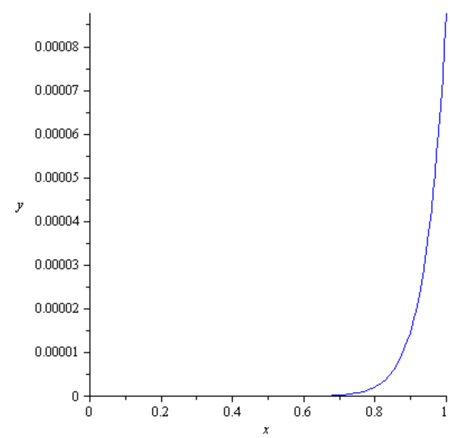

Figure 4

Also, by Chebyshev approximation for $g(x)$ with $N = 10$, we obtain the Maximum error in figure 5,

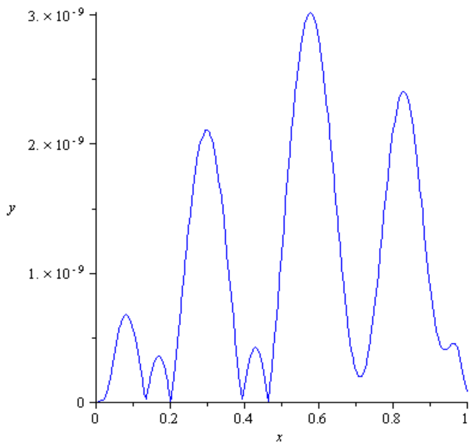

Figure 5

Finally, Legender Approximation for $g(x)$ with $N = 7$, produce the Maximum error in figure 6,

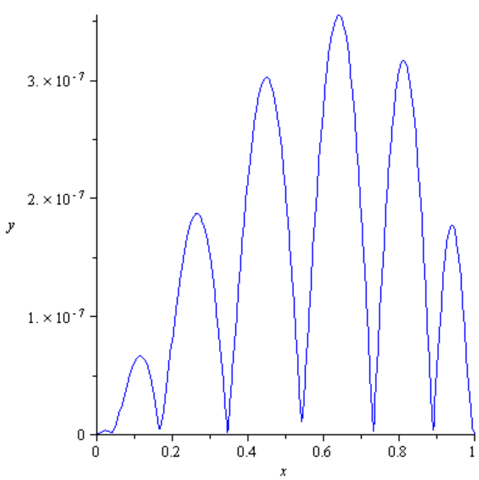

Figure 6

4. Conclusions

The conclusions can be summarized in the following three points:

1. By ADM M. M. Hosseini (2006), when the function $g(x)$ is approximated with $N = 10$ Problem 3.1 does not give an accurate results, while VIM given an accurate results even when $N = 10$.
2. In the VIM, the solution u_0 can be chosen such that it is satisfy the initially conditions to get a faster convergence of the solution.
3. ADM is complicated in applications because it use the Adomian polynomial that's complicated in computations, also when the order of ordinary differential equations raising the ADM will be complicated in applications, because of multiple integrations, while the VIM requires just ones integration in iterations for any order of equations.

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طريقة VIM مع متعددات الحدود التعامدية

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المستخلص

تعتمد طريقة VIM على حل تكامل في كل تكرار، في بعض الأحيان يكون حساب التكامل صعباً، لتجاوز هذه المشكلة استخدمنا متعددات الحدود التعامدية. للمقارنة بين طريقتي VIM و ADM طرقنا مسألتين مع عدة متعددات حدود تعامدية M. M. Hosseini (2006) .