NEW COINCIDENCE AND FIXED POINT THEOREMS FOR GREGUŠ TYPE HYBRID MAPS

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Abstract

In this paper, we obtain coincidence and fixed point theorems for single-valued and multivalued maps satisfying Greguš type contractive conditions. We extend and generalize some well known results obtained by many authors.

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1. INTRODUCTION

The following result is usually called the Greguš fixed point theorem.

Theorem G. (Greguš ,1980) Let C be a closed convex subset of a Banach space X. If T is a map from C into itself satisfying the inequality

(1.1) $||Tx-Ty|| \le a\,||x-y|| + b\,||x-Tx|| + c\,||y-Ty||$

for all $x, y \in C$, where $0 < a < 1, b > 0, c > 0$ and $a + b + c = 1$. Then T has a unique fixed point.

A map satisfying the inequality (1.1) with $a = 1$, $b = c = 0$ (respectively $a = 0$, $b = c = ½$) is called non-expansive (respectively Kannan map) and it was considered by Kirk (1965) (respectively Wong ,1975). Further, Fisher and Sessa, 1986 established a generalization of Theorem G as follows:

Theorem FS (Fisher and Sessa, 1986) Let C be a nonempty closed convex subset of a Banach space X and T, f two weakly commuting maps from C into itself satisfying the following condition:

$$|| Tx - Ty|| \le a || fx - fy|| + (1-a)\max\{\|Tx - fx\|, \|Tx - fy\|\}$$

for all $x, y \in C$, where $0 < a < 1$. If f is linear and nonexpansive in C such that fC contains TC, then T and f have a unique common fixed point in C.

In recent years, fixed point theorems for Greguš type maps have been obtained by Ćirić, 1972, Ćirić,1991, Davies and Sessa,1992, Diviccaro et al.1987, Jungck ,1990, Khan and Imdad 1983, Mukherjee and Verma, 1988, Murthy et al. 1995, and Pathak et al. 1998. Some results closely related to Theorem G have also been extended to multivalued maps (see, for instance, Li-Shan 1992, Li-Shan 1993,Rashwan and Ahmed, 2002 and Singh et al. 1989).

2. PRELIMINARIES

We generally follow the definitions and notations used in Nadler 1969, Naimpally et. Al 1986, Singh & Hashim 2005, Singh & Hashim 2004 and Singh & Mishra 2001. Given a metric space *(X, d)*, let *(CL(X), H)* and *(C(X), H)* denote respectively the hyperspaces of nonempty closed and nonempty compact subsets of X, where H is the Hausdorff metric inducted by d. Throughout, $d(A, B)$ will denote the ordinary distance between subsets A and B of X and $d(x, B)$ will stand for $d(A, B)$ when A is singleton $\{x\}$. Further, let Y be an arbitrary nonempty set and $C(f, S) = \{u : fu \in Su\}$, the collection of coincidence points of maps $f : Y \to X$ and $S : Y \to CL(X)$.

The following result is well-known (see, for instance, [Ćirić, 1972, Nadler 1969, Singh et al. 1989].

Lemma 2.1. Let $A, B \in CL(X)$ and $a \in A$. Then there exists ,an element $b \in B$ such that $d(a, b) \le \lambda\, H(A, B)$ for some $\lambda > 1$.

We remark that if A, B are compact then this lemma is true with $\lambda = 1$.

The following definition is due to Ito and Takahashi 1977 (see also Singh & Hashim 2005, Singh & Mishra 2001).

Definition 2.2. Let $f : Y \to Y$ and $S : Y \to 2^Y$, the collection of nonempty subsets of Y. Then hybrid pair of maps of (f, S) is IT-commuting at a point $u \in Y$ if $fSu \subset Sfu$. They are IT-commuting on Y if $fSu \subset Sfu$ for each $u \in Y$.

Maps S and f are commuting at a point $u \in Y$ if $Sfu=fSu$. Clearly a commuting hybrid pair of maps is IT-commuting and the reverse implication is not true. For details, one may refer to Singh & Mishra 2001.

Example 2.3. Let $Y = [0, \infty)$, $fx = 3 + 5x$ and $Sx = [2 + 3x, \infty)$, $x \in Y$. Then $Sfx = [11 + 15x, \infty) \neq [13 + 15x, \infty) = fSx$. So f and S are not commuting, but they are IT-commuting as $fSx \subset Sfx$.

Definition 2.4 [Singh & Hashim 2005]. Let $S : Y \to CL(X)$ and $f : Y \to X$. Then S and f satisfy the (EA)-property if there exists a sequence $\{x_n\}$ in Y such that $\lim_{n\to\infty} Sx_n = M \in CL(X)$ and $\lim_{n\to\infty} fx_n = t \in M$.

For the definition of (EA)-property for a pair of self-maps of a metric space, one may refer to Aamri and El Moutawakil 2002.

Let Ø denote the class of maps ϕ from $[0, \infty)$ into itself such that ϕ iis non-decreasing upper semi-continuous and $\phi(t) < t$ for all $t > 0$. Also let ψ denote the class of maps ϕ from $[0, \infty)$ into itself such that ϕ is non-decreasing upper semi-continuous and $\phi(t) \leq qt$ for some $q \in (0, 1)$. The following lemma is due to Matkowski 1977 .

Lemma 2.5. If $\phi \in$ Ø then $\lim_{n\to\infty} \phi^n(t) = 0$, where ϕ^n is the n times composition of ϕ.

3. MAIN RESULTS

Theorem 3.1 Let Y be an arbitrary nonempty set and (X, d) be a metric space, $S, T : Y \to C(X)$ and $f, g : Y \to X$ such that

(3.1.1) $SY \subset gY, \quad TY \subset fY$;

(3.1.2) $H^p(Sx, Ty) \leq \phi\,(ad^p(fx, gy) + (1-a)\max\{d^p(fx, Sx), d^p(gy, Ty)\})$

for all $x, y \in Y$, $p > 0$, where $a \in (0,1)$ and $\phi \in$ Ø. If one of SY, TY, fY or gY is a complete subspace of X then $C(f, S)$ and $C(g, T)$ are nonempty. Further, if $Y = X$ then:

(i) Maps S and f have a common fixed point provided that S and f are (IT)-commuting at some $u \in C(f, S)$ and $ffu = fu$;

(ii) Maps T and g have a common fixed point provided that T and g

are (IT)–commuting at some $u \in C(g,T)$ and $ggu = gu$;

(iii) Maps S,T,f and g have a common fixed point provided that (I) and (II) are true.

Proof. Pick $x_0 \in Y$. Construct two sequences $\{x_n\} \subset Y$ and $\{y_n\} \subset X$ in the following manner. Since $x_0 \in Y$ and $SY \subset gY$, there exists a point $x_1 \in Y$ such that

$y_1 = g x_1 \in Sx_o$. Since $TY \subset fY$, in view of the remark following Lemma 2.1, we can choose $x_2 \in Y$ such that $y_2 = fx_2 \in Tx_1$ and

$$d(y_1, y_2) = d(gx_1, fx_2) \le H(Sx_0, Tx_1)$$

Inductively, having already chosen $y_{2n} = fx_{2n} \in Tx_{2n-1}$, we choose $y_{2n+1} = gx_{2n+1} \in Sx_{2n}$ and $y_{2n+2} = fx_{2n+2} \in Tx_{2n+1}$ such that

$$d(y_{2n}, y_{2n+1}) \le H(Tx_{2n-1}, Sx_{2n}) \text{ and } d(y_{2n+1}, y_{2n+2}) \le H(Sx_{2n}, Tx_{2n+1}).$$

For the sake of convenience, assume that $d_n = d(y_n, y_{n+1})$. We claim that $d_{n+1} \le d_n$ for each n. Suppose that $d_{2n} > d_{2n-1}$ for some n. Then by (3.1.2),

$$
\begin{aligned}
d_{2n} &\le H(Sx_{2n}, Tx_{2n-1})\\
&\le [\phi(ad^p(fx_{2n}, gx_{2n-1}) + (1-a)max\{d^p(fx_{2n}, Sx_{2n}), d^p(gx_{2n-1}, Tx_{2n-1})\})]^{\frac{1}{p}}\\
&\le [\phi(ad^p{}_{2n} + (1-a)max\{d^p{}_{2n}, d^p{}_{2n-1}\})]^{\frac{1}{p}}\\
&< [\phi(ad^p{}_{2n} + (1-a)max\{d^p{}_{2n}, d^p{}_{2n}\})]^{\frac{1}{p}}\\
&= [\phi(ad^p{}_{2n} + (1-a)ad^p{}_{2n})]^{\frac{1}{p}}\\
&= [\phi(d^p{}_{2n})]^{\frac{1}{p}} < [d^p{}_{2n}]^{\frac{1}{p}} = d^p{}_{2n},
\end{aligned}
$$

a contradiction, and $d_{2n} \le d_{2n-1}$. Similarly we obtain $d_{2n+1} \le d_{2n}$. This establishes our claim that $d_{n+1} \le d_n$ for each n.

By Lemma 2.5, it follows that

$$\lim_{n\to\infty} d^p(y_n, y_{n+1}) = 0. \quad 3.1.3$$

In order to prove that $\{y_n\}$ is a Cauchy sequence, it is sufficient to show that $\{y_{2n}\}$ is a Cauchy sequence. Suppose that $\{y_{2n}\}$ is not a Cauchy sequence. Then there is an $\varepsilon > 0$ such that for each positive integer *2k* there exist integers *2n(k)* and *2m(k)* such that

$$2k < 2n(k) < 2m(k) \qquad 3.1.4$$

$$\text{and } d\left(y_{2n(k)}, y_{2m(k)}\right) \ge \varepsilon . \qquad 3.1.5$$

Let $d_{i,j} = d\left(y_i, y_j\right)$ and $d_i = d\left(y_i, y_{i+1}\right)$. Then for each integer $2k.$,

$$\varepsilon \le d_{2n(k),2m(k)} \le d_{2n(k),2m(k)-2} + d_{2m(k)-2} + d_{2m(k)-1} \qquad 3.1.6$$

Let *2m(k)* denote the smallest integer satisfying (3.1.4) and (3.1.5), so that

$d_{2n(k),2m(k)-2} < \varepsilon$, and it follows from (3.1.6) that

$$\lim_k d_{2n(k),2m(k)} = \varepsilon . \qquad 3.1.7$$

By the triangle inequality,

$$|\, d_{2n(k),2m(k)-1} - d_{2n(k),2m(k)} \,| < d_{2m(k)-1},$$
$$|\, d_{2n(k)+1,2m(k)-1} - d_{2n(k),2m(k)} \,| < d_{2n(k)} + d_{2m(k)-1}$$

These relations in view of (3.1.3) and (3.1.7) yield

$$\lim_k \; d_{2n(k),2m(k)-1} = \lim_k \; d_{2n(k)+1,2m(k)-1} = \varepsilon . \qquad 3.1.8$$

Using (3.13) and (ii), we have

$$d_{2n(k),2m(k)} \le d_{2n(k)} + d_{2n(k)+1,2m(k)}$$

$$\le d_{2n(k)} + H\left(Sx_{2n(k)}, Tx_{2m(k)-1}\right)$$

$$\le d_{2n(k)} + \left(\phi\left[ad^{p}\left(fx_{2n(k)}, gx_{2m(k)-1}\right)\right]\right)$$

$$+\left(1-a\right)max\{\,d^{\,p}(\,fx_{2n(k)},Sx_{2n(k)}\,),d^{\,p}(\,gx_{2m(k)-1},Tx_{2m(k)-1}\,)\}\})^{1/p}$$

$$\le d_{2n(k)}+\left(\phi\left[\,ad^{\,p}\left(\,fx_{2n(k)},gx_{2m(k)-1}\right)+\left(1-a\right)max\{\,d^{\,p}(\,fx_{2n(k)},gx_{2n(k)+1}\,),\right.\right.$$

$$d^{\,p}(\,gx_{2m(k)-1},\,fx_{2m(k)}\,)\}\})^{1/p}$$

$$\le d_{2n(k)}+(\,\phi[\,ad^{\,p}_{2n(k),2m(k)-1}+\left(1-a\right)max\{\,d^{\,p}_{2m(k)},d^{\,p}_{2m(k)-1}\,\}])^{1/p}$$

Taking $k\to\infty$ and using (3.1.3) and (3.1.8) with upper semi-continuity of ϕ it follows that

$$\varepsilon\le\left[\phi\left(\varepsilon^{p}\right)\right]^{1/p}<\varepsilon,$$

a contradiction. So $\{\,y_{2n}\,\}$ is a Cauchy sequence in *X*. Consequently $\{\,y_n\,\}$ is Cauchy sequence in X .

Now suppose that fY is complete. Then the subsequence $\{\,y_{2n+1}\,\}\subset fY$ has a limit in fY . Call it z. Let $u\in f^{-1}z.$ So $z=fu$.

Notice that the subsequence $\{\,y_{2n}\,\}$ also converges to z , and we claim that

$fu\in Su.$ Suppose otherwise. Then $d\left(Su,fu\right)$ > 0, and

$$d^{\,p}\left(Su,y_{2n+2}\right)\le H^{\,p}\left(Su,Tx_{2n+1}\right)$$
$$\le\phi(\,ad^{\,p}(\,fu,y_{2n+1}\,)+(1-a\,)max\{\,d^{\,p}\left(\,fu,Su\right),d^{\,p}\left(y_{2n+1},y_{2n+2}\right)\}).$$

Making $n\to\infty$, $d^{\,p}\left(Su,fu\right)<d^{\,p}\left(\,fu,Su\right)$, a contradiction.

Consequently $C(\,f\,,S\,)$ is nonempty. Since $SY\subset gY$, there is a point $v\in Y$ such that $gv=fu\in Su.$. So by (ii),

$$d^{\,p}\left(gv,Tv\right)=d^{\,p}\left(\,fu,Tv\right)\le H^{\,p}\left(Su,Tv\right)$$
$$\le\phi(\,ad^{\,p}(\,fu,gv\,)+(1-a\,)max\{\,d^{\,p}\left(\,fu,Su\right),d^{\,p}\left(gv,Tv\right)\})$$

$$=\phi(1-a\,)d^{\,p}(\,gv,Tv\,).$$

This yields that $C(g,T)$ is nonempty.

Analogous arguments establish that $C(f,S)$ and $C(g,T)$ are nonempty when gY or SY or TY is a complete subspace of X .

Notice that $fu \in Su$, $fu = ffu \in fSu \subset Sfu$. So fu is a common fixed point of S as well. This proves (i) and (ii) comes in a similar way. Now, (III) is immediate.

Multivalued maps in Theorem 3.1 are compact-valued. We may dispense with this requirement by changing a slightly the condition $"\phi(t) < t"$ of Theorem 3.1. We, indeed inspired by Rhoades et al.(1984) and Singh et al. (1989) do so below.

Theorem 3.2. Let Y be an arbitrary nonempty set and (X, d) a metric space. Let $S,T : Y \to C(X)$ and $f,g : Y \to X$ be such that conditions (3.1.1) and (3.1.2) of Theorem 3.1 are satisfied, where $a \in (0,1)$ and $\phi \in \psi$. If one of SY, TY, fY or gY is a complete subspace of X then $C(f,S)$ and $C(g,T)$ are nonempty. Further, if $Y = X$ then all other conclusions of Theorem 3.1 are also true.

Proof. We give only a brief sketch of it. Pick $x_0 \in$ Y. Choose a point $x_1 \in Y$ such that $y_1 = gx_1 = Sx_0$. Now, in view of Lemma 2.1, choose a point $x_2 \in Y$ such that $y_2 = fx_2 \in Tx_1$, and

$$d(y_1, y_2) = d(gx_1, fx_2) \le q^{-\frac{1}{2}} H(Sx_0, Tx_1) .$$

In general, we construct sequences $\{x_n\} \subset Y$ and $\{y_n\} \subset X$ in such a way that

$y_{2n+1} = gx_{2n+1} \in Sx_{2n}$ and $y_{2n+2} = fx_{2n+2} \in Tx_{2n+1}$ satisfying

$$d(y_{2n}, y_{2n+1}) \le q^{-\frac{1}{2}} H(Tx_{2n-1}, Sx_{2n}) \quad \text{and} \quad d(y_{2n+1}, y_{2n+2}) \le q^{-\frac{1}{2}} H(Sx_{2n}, Tx_{2n+1}).$$

Now, in view of the proof of Theorem 3.1, it is enough to show that $\{y_n\}$ is a Cauchy sequence. Using (ii) and simplifying, we get

$$d_{2n} \le q^{-\frac{1}{2}} H\left(Sx_{2n}, Tx_{2n-1}\right) \le q^{-\frac{1}{2}} [\phi(ad^{p}_{2n-1} + \left(1-a\right) max\{ d^{p}_{2n}, d^{p}_{2n-1} \})]^{\frac{1}{p}} .$$

This gives $d_{2n} \leq \sqrt{q}\, d_{2n-1}$. Similarly we obtain $d_{2n+1} \leq \sqrt{q}\, d_{2n}$. So $d_{n+1} \leq \sqrt{q}\, d_n$ for each n. Hence the sequence $\{y_n\}$ is Cauchy, and this complete the proof.

As regards the coincidence parts of maps f, S and g, T in Theorems 3.1 & 3.2, we remark that the sets $C(f,S)$ and $C(g,T)$ may be different, that is, the maps f, S and g, T may have different coincidence points. This may easily be verified by using [Singh & Mishra 2001, Example 4]. Further, this observation remains true even if $S = T$ (see Example 3.5 below). However, if $f = g$, then maps f, S and T have a common coincidence point. So, we have a slightly improved version of the above theorems.

Theorem 3.3. Let Y be an arbitrary nonempty set and (X, d) a metric space. Let $S, T : Y \to C(X)$ (respectively $S, T : Y \to CL(X)$ and $f : Y \to X$ such that

(3.3.1) $SX \subset fX$ and $TX \subset fX;$

(iv) $H^p(Sx, Ty) \leq \phi(ad^p(fx, fy) + (1-a)\max\{d^p(fx, Sx), d^p(fy, Ty)\}),$

for all x, y in X, $p > 0$ where $a \in (0,1)$, and $\phi \in$ Ø (respectively $\phi \in \psi$).

If one of SY, TY or fY is complete subspace of X then the maps f, S and T have a common coincidence point. Further, if Y = X and f is (IT)-commuting with each of S and T at a common coincidence point z then the maps f, S and T, have a common fixed point provided that fz is a fixed point of f.

Proof. It comes from the Theorems 3.1 and 3.2 with $f = g$.

We remark that Theorem 3.3 extends and generalizes several fixed point theorems for multivalued maps (see, for instance, Li-shan 1992, Negoescu 1989, Rashwan and Ahmed 2002, Singh et al 1989 and references thereof). Several results for single-valued maps satisfying Greguš type conditions may also be derived from the above theorems (see, for instance Davies &sessa 1992, Greguš 1980,Jungck 1990, Murthy et al 1995, Pathak et al 1998 and Singh et al 2001).

Now we prove a common fixed point theorem of Greguš type maps using a strict condition.

Theorem 3.4. Let (X,d) be a metric space, $S,T:Y\to CL(X)$ and $f,g:Y\to X$ satisfying (3.1.1) and

(v) $H(Sx,Ty)<M(axy)$, x,y in Y when $M(axy)>0$,

for some $p>0$ and $0<a<1$, where

$$M(axy)=ad\left(fx,gy\right)+\left(1-a\right)max\{d\left(fx,Sx\right),d\left(gy,Ty\right)\}$$

Suppose that (f,S) or (g,T) satisfies the (E.A)-property and one of SY,TY,fY or gY is a complete subspace of X, then $C(f,S)$ and $C(g,T)$ are nonempty.

Further, if $Y=X$, then the conclusions (i), (ii) and (iii) of Theorem 3.1 are true.

Proof: Suppose that (g,T) satisfies the (E.A)-property then there exists a sequence $\{x_n\}$ in Y such

that $\lim_{n\to\infty}(gx_n)=t\in M$, $\lim_{n\to\infty}Tx_n=M\in CL(X)$

Since $TY\subset fY$, there exists in Y a sequence $\{y_n\}$,such that $fy_n\in Tx_n$ and $\lim_{n\to\infty}fy_n=t\in M=\lim_{n\to\infty}Tx_n$. We show that $\lim_{n\to\infty}Sy=M$. If not then there exists a subsequence $\{y_k\}$ of $\{Sy_n\}$, a positive integer N, and real number $\varepsilon>0$ such that for same $k\ge N$ we have $H(Sy_k,M)\ge\varepsilon$. From (v)

$H(Sy_k,M)\le H(Sy_k,Tx_k)+H(Tx_k,M)$.

$$\le d(fy_k,gx_k)+(1-a)\max\{d(fy_k,Sy_k),d(gx_k,Tx_k)\}+H(Tx_k,M)$$

Taking the limit $k\to\infty$.

$\varepsilon\le(1-a)\,max\{\varepsilon\}<\varepsilon$. which is a contradiction.

Hence $\varepsilon=0$ i.e $\lim_{n\to\infty}Ay_n=M$.

Suppose TY or fY is a complete subspace of X , then there exists a point $u\in Y$ such that $t=fu$. To show that $fu\in Su$, we suppose otherwise and use the condition (v) to have

$$d(Su,Tx_n) \le H(Su,Tx_n) < ad(fu,gx_n) + (1-a)\max\{d(fu,Su), d(gx_n,Tx_n)\}$$

Taking the limit $n \to \infty$

$H(Su,M) < (1-a)d(fu,Su) < d(fu,Su) \le H(Su,M)$

Which is a contradiction. Then $C(f,S)$ is nonempty. Since $SY \subset gY$,there exists the point $v \in Y$ such that $fu = gv \in Su$, so by (v),

$H(gv,Tv) = d(fu,Tv) \le H(Su,Tv) < ad(fu,gv) + (1-a)\max\{d(fu,Su), d(gv,Tv)\}$

$d(gv,Tv) < (1-a)d(gv,Tv) < d(gv,Tv)$,

Which is a contradiction . Then $C(g,T)$ is nonempty.

Further, $ffu = fu$ and (IT)-commutativity of S and f at $u \in C(f,S)$ imply that $fu \in fSu \subset Sfu$. so fu is common fixed point of S and f.

The proof of (ii) is similar. Now (iii) is immediate .analogous argument establishes the theorem when SY or gY is a complete subspace of X .

We remark that if $f = g$ in theorem 3.4, then the maps S, T and f have a common coincidence point. The following example shows that Theorems 3.1 & 3.2 with $S = T$ and $f \ne g$ need not guarantee the existence of a point z such that $fz = gz \in Tz$.

Example 3.5. Let $X = [0,2]$ be endowed with the usual metric. Let $S = T = \{0, 2\}, fx = x$ and $gx = 2-x$. Then for any $a \in (0,1)$,

$$H(Tx, Ty) = 0 \le ad(fx, gy).$$

So, all the hypotheses of Theorems 3.1 & 3.2 are satisfied. Notice that $fz \in Tz$ and $gz \in Tz$ but $fz \ne gz$ when $z = 0$ or 2 .

We remark that (theorem 3.2 and 3.3 is false under the following condition:

(3.1.3) $H^p(Sx,Ty) \le \phi(ad^p(fx,gy) + (1-a)\max\{d^p(fx,Sx), d^p(gy,Ty),$

$1\backslash 2[d^p(fx,Ty) + d^p(gy,Sx)]\})$

Which is more general than the condition(3.1.2) for details, one may refer to Rhoades 1977.

Example 3.6. Let $X = \{a,b,c,d\}, d(x,x) = 0, d(a,b) = d(a,d) = d(b,c) = d(c,d) = 1,$

$d(a,c) = d(b,d) = 2$ *and* $Y = X$. Further, let $fx = gx = x$ for each $x \in Y$ and let $Sa = Sd = \{c\}, Sb = Sc = \{a\}, Ta = Tb = \{d\}$ and $Tc = Td = \{b\}$ Then evidently fY is a complete metric space and (3.1.1) is satisfied. If $a = 1\backslash 2,\ p = 4$ and $\phi(t) = t\backslash 2,$ then $\phi \in$ Ø. and (3.1.3) is satisfied, too. However, evidently $C(f,S)$ and $C(g,T)$ are empty.

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مبرهنات جديدة نظرية النقطة الصامدة والمتطابقة للدوال الهجينية من النوع Greguš

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الملخص

في هذا البحث حصلنا على النقطة الصامدة للدوال الاحادية والمتعددة القيم والتي حققت الشرط الذي وضعه Greguš. لقد تم توسيع وتعميم بعض النتائج التي حصل عليها العديد من الباحثين.