Expected Mean Squares For 3-way Crossed Model With Unbalanced Correlated Data

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ABSTRACT

In this study we calculate the expected mean squares for 3-way crossed unbalanced model with correlated data and notice the effect of the correlated data on F statistic.

Key word : The crossed model , analysis of variance

1. INTRODUCTION

The use of limited way analysis of variance on the assumption that the terms of random error for the experiment are independent of each other. So it seems that the imposition of independence between observations is to impose logical to examine observation using the designs of experiment. Under this, the basis of analysis of variance was built on the fact that observations are independent of each other , but the imposition of independence rarely achieved.

Pavur and Davenport (1985) study the effect of correlated data on the analysis of variance results and on the type I error for 2-way balanced model. Pavur (1988) studied simple linear model with correlated error expression and notice the effect of correlated on multiple comparison procedures for this model, Al-Shahiry (1997) studied the effect of correlation on F statistic and the correction factor for one way model. Al-Kaabawi (2000) found the expected mean squares for balanced crossed 2-way model with correlated data , Abdullah and Al-Kaabawi (2007) found the expected mean squares for balanced crossed3-way model with correlated data , Al-Kaabawi (2007) studied the effect of dependent data on type I error rates for multiple comparison procedures for 3-way crossed balanced model , Al-Kaabawi (2010) found the expected mean squares for balanced crossed 4-way model with correlated data.

This research aims to account for expected mean squares in the absence of independence between the limits of errors, and then lack of independence between observations, although the independence between the errors is one of the usual assumptions in the analysis of variance, which is rarely achieved. It also deals with this research study the effect of lack of independence between observations when the formation of the distribution of F to test the effects of these factors by using the correction factor and by which are recognized over the potential impact of type I error because of the existence of correlations between observations and that the model adopted in this study is to model a 3-way cross unbalanced.

2.THE MODEL

Consider the 3-way crossed model

$$Y_{ijkl} = \theta + \alpha_i + \beta_j + \gamma_k + (\alpha\beta)_{ij} + (\alpha\gamma)_{ik} + (\beta\gamma)_{jk} + (\alpha\beta\gamma)_{ijk} + e_{ijkl} \quad \ldots(1)$$

With i= 1, ….., a, j= 1, …., b, k= 1, ……, c, l= 1, …., d_{ijk} and θ unknown parameter

and

$$\sum_{i=1}^{a}\alpha_i = \sum_{j=1}^{b}\beta_j = \sum_{k=1}^{c}\gamma_k = 0 \ , \quad \sum_{i=1}^{a}(\alpha\beta)_{ij} = 0 = \sum_{j=1}^{b}(\alpha\beta)_{ij},\ \sum_{i=1}^{a}(\alpha\gamma)_{ik} = 0 = \sum_{k=1}^{c}(\alpha\gamma)_{ik},\ \sum_{j=1}^{b}(\beta\gamma)_{jk} = 0 =$$

$$\sum_{k=1}^{c}(\beta\gamma)_{jk}\ ,\sum_{i=1}^{a}(\alpha\beta\gamma)_{ijk} = 0 = \sum_{j=1}^{b}(\alpha\beta\gamma)_{ijk} = \sum_{k=1}^{c}(\alpha\beta\gamma)_{ijk} = 0$$

and e_{ijkl} are independent random variable having normal with zero mean and

$$COV(Y_{ijkl}, Y_{i'j'k'l'}) = COV(e_{ijkl}, e_{i'j'k'l'}) \quad \ldots(2)$$

where

$$COV(Y_{ijkl}, Y_{i'j'k'l'}) = \begin{cases} \sigma^2 & ; i=i', j=j', k=k', l=l' \\ \sigma^2\rho_1 & ; i=i', j=j', k=k', l\neq l' \\ \sigma^2\rho_2 & ; i=i', j=j', k\neq k' \\ \sigma^2\rho_3 & ; i=i', j\neq j', k=k' \\ \sigma^2\rho_4 & ; i\neq i', j=j', k=k' \\ \sigma^2\rho_5 & ; i=i', j\neq j', k\neq k' \\ \sigma^2\rho_6 & ; i\neq i', j=j', k\neq k' \\ \sigma^2\rho_7 & ; i\neq i', j\neq j', k=k' \\ \sigma^2\rho_8 & ; i\neq i', j\neq j', k\neq k' \end{cases} \quad \ldots(3)$$

let

$$Y_{ijkl} = \mu_{ijkl} + e_{ijkl} \quad \ldots(4)$$

Where

$$\mu_{ijkl} = \theta + \alpha_i + \beta_j + \gamma_k + (\alpha\beta)_{ij} + (\alpha\gamma)_{ik} + (\beta\gamma)_{jk} + (\alpha\beta\gamma)_{ijk} \quad \ldots(5)$$

3. ANALYSIS OF VARIANCE

$Y_{ijkl} \approx N(\mu_{ijkl}, \sigma^2)$

From cockran's theorem

SSTO= SSA + SSB + SSC + SSAB + SSAC + SSBC + SSABC + SSE …(6)

Where

$$SSTO = \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c}\sum_{l=1}^{d_{ijk}} (Y_{ijkl} - \overline{Y}_{....})^2 \qquad \ldots(7)$$

$$SSA = \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c} d_{ijk} (\overline{Y}_{i...} - \overline{Y}_{....})^2 \qquad \cdots(8)$$

$$SSB = \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c} d_{ijk} (\overline{Y}_{.j..} - \overline{Y}_{....})^2 \qquad \ldots(9)$$

$$SSC = \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c} d_{ijk} (\overline{Y}_{..k.} - \overline{Y}_{....})^2 \qquad \cdots(10)$$

$$SSAB = \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c} d_{ijk} (\overline{Y}_{ij..} - \overline{Y}_{i...} - \overline{Y}_{.j..} + \overline{Y}_{....})^2 \qquad \cdots(11)$$

$$SSAC = \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c} d_{ijk} (\overline{Y}_{i.k.} - \overline{Y}_{i...} - \overline{Y}_{..k.} + \overline{Y}_{....})^2 \qquad \cdots(12)$$

$$SSBC = \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c} d_{ijk} (\overline{Y}_{.jk.} - \overline{Y}_{.j..} - \overline{Y}_{..k.} + \overline{Y}_{....})^2 \qquad \ldots(13)$$

$$SSABC = \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c} d_{ijk} (\overline{Y}_{ijk.} - \overline{Y}_{ij..} - \overline{Y}_{i.k.} - \overline{Y}_{.jk.} + \overline{Y}_{i...} + \overline{Y}_{.j..} + \overline{Y}_{..k.} - \overline{Y}_{....})^2 \qquad \cdots(14)$$

$$SSE = \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c}\sum_{l=1}^{d_{ijk}} (\overline{Y}_{ijkl} - \overline{Y}_{ijk.})^2 \qquad \ldots(15)$$

SSTO are (a-1), (b-1), (c-1), (a-1)(b-1), (a-1)(c-1), (b-1)(c-1), (a-1)(b-1)(c-1),

and the degrees of freedom for SSA, SSB, SSC, SSAB, SSAC, SSBC, SSABC, SSE and

$$\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c} d_{ijk} - abc \text{ and } \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c} d_{ijk} - 1 \text{ respectively}$$

MSB, MSC, MSAB, MSAC, MSBC and MSABC.

4.EXPECTED MEAN SQUARES E(MS)

Now we calculate the expected mean squares with correlated data for MSE, MSA,

$$MSE = \frac{SSE}{\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c} d_{ijk} - abc}$$

$$= \left(\frac{1}{\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c} d_{ijk-abc}} \right) \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c} (Y_{ijkl} - \overline{Y}_{ijk.})^2$$

Since $Y_{ijkl} - \overline{Y}_{ijk.} = e_{ijkl} - \overline{e}_{ijk.}$

There fore

$$MSE = \left(\frac{1}{\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c} d_{ijk} - abc} \right) \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c}\sum_{l=1}^{d_{ijk}} (e_{ijkl} - \overline{e}_{ijk.})^2$$

$$= \left(\frac{1}{\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c} d_{ijk} - abc} \right) \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c}\sum_{l=1}^{d_{ijk}} (e_{ijkl}^2 - 2e_{ijkl}\overline{e}_{ijk.} + \overline{e}_{ijk.}^2)$$

$$= \left(\frac{1}{\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c} d_{ijk} - abc} \right) \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c} \left[\sum_{l=1}^{d_{ijk}} e_{ijkl}^2 - d_{ijk}\overline{e}_{ijk.}^2 \right]$$

There fore

$$E(MSE)=\left(\frac{1}{\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c}d_{ijk}-abc}\right)\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c}\left[\sum_{l=1}^{d_{ijk}}E(e_{ijkl}^{2})-d_{ijkl}E(\bar{e}_{ijk.}^{2})\right]$$

$$=\left(\frac{1}{\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c}d_{ijk}-abc}\right)\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c}\left[\sum_{l=1}^{d_{ijk}}\sigma^{2}-\sigma^{2}(1-\rho_{1}+d_{ijk}\rho_{1})\right]$$

$$=\left(\frac{1}{\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c}d_{ijk}-abc}\right)\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c}\left[d_{ijk}\sigma^{2}-\sigma^{2}(1-\rho_{1}+d_{ijk}\rho_{1})\right]$$

$$=\left(\frac{\sigma^{2}}{\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c}d_{ijk}-abc}\right)\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c}(d_{ijk}-1)(1-\rho_{1})$$

$$=\left(\frac{\sigma^{2}(1-\rho_{1})}{\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c}d_{ijk}-abc}\right)\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c}d_{ijk}-abc=\sigma^{2}(1-\rho_{1}) \qquad \ldots(16)$$

Also $MSE=\frac{SSA}{a-1}=\frac{1}{a-1}\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c}d_{ijk}(\bar{Y}_{i...}-\bar{Y}_{....})^{2}$

Since $\bar{Y}_{i...}-\bar{Y}_{....}=\alpha_{i}+\bar{e}_{i...}-\bar{e}_{....}$

There fore

$$MSA=\frac{SSA}{a-1}=\frac{1}{a-1}\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c}d_{ijk}(\alpha_{i}+\bar{e}_{i...}-\bar{e}_{....})^{2}$$

$$=(\frac{1}{a-1})\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c}d_{ijk}(\alpha_{i}^{2}+2\alpha_{i}(\bar{e}_{i...}-\bar{e}_{....})+\bar{e}_{i...}^{2}-2\bar{e}_{i...}\bar{e}_{....}+\bar{e}_{....}^{2})$$

Then

$$E(MSA)=(\frac{1}{a-1})\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c}d_{ijk}(\alpha_{i}^{2}+2\alpha_{i}E(\bar{e}_{i...}-\bar{e}_{....})+E(\bar{e}_{i...}^{2})-2E(\bar{e}_{i...}\bar{e}_{....})+E(\bar{e}_{....}^{2})) \qquad \ldots(17)$$

Since

$$E(\bar{e}_{i...}-\bar{e}_{....})=0 \qquad \ldots(18)$$

$$E(\bar{e}_{i...}^{2})=\frac{\sigma^{2}}{bcd_{ijk}}\left[1-\rho_{1}+d_{ijk}(\rho_{1}-\rho_{2}-\rho_{3}+\rho_{5})+cd_{ijk}(\rho_{2}-\rho_{5})+bd_{ijk}(\rho_{3}-\rho_{5})+bcd_{ijk}\rho_{5}\right] \qquad \ldots(19)$$

$$E(\bar{e}_{i...}\bar{e}_{....})=\frac{1}{ab^2c^2d_{ijk}^2}E\left[\sum_{j=1}^{b}\sum_{k=1}^{c}\sum_{l=1}^{d_{ijk}}e_{ijkl}\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c}\sum_{l=1}^{d_{ijk}}e_{ijkl}\right]$$

$$=\frac{\sigma^2}{abcd_{ijk}}\begin{bmatrix}1-\rho_1+d_{ijk}(\rho_1-\rho_2-\rho_3-\rho_4+\rho_5+\rho_6+\rho_7-\rho_8)+ad_{ijk}(\rho_4-\rho_6-\rho_7+\rho_8)\\+bd_{ijk}(\rho_3-\rho_5-\rho_7+\rho_8)+cd_{ijk}(\rho_2-\rho_5-\rho_6+\rho_8)+abd_{ijk}(\rho_7-\rho_8)\\+acd_{ijk}(\rho_6-\rho_8)+bcd_{ijk}(\rho_5-\rho_8)+abcd_{ijk}\rho_8\end{bmatrix} \quad \ldots(20)$$

$$E(\bar{e}_{....}^2)=\frac{1}{a^2b^2c^2d_{ijk}^2}E\left[\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c}\sum_{l=1}^{d_{ijk}}e_{ijkl}\right]^2$$

$$=\frac{\sigma^2}{abcd_{ijk}}\begin{bmatrix}1-\rho_1+d_{ijk}(\rho_1-\rho_2-\rho_3-\rho_4+\rho_5+\rho_6+\rho_7-\rho_8)+cd_{ijk}(\rho_2-\rho_5-\rho_6+\rho_8)\\+bd_{ijk}(\rho_3-\rho_5-\rho_7+\rho_8)+ad_{ijk}(\rho_4-\rho_6-\rho_7+\rho_8)+bcd_{ijk}(\rho_5-\rho_8)\\+abd_{ijk}(\rho_7-\rho_8)+acd_{ijk}(\rho_6-\rho_8)+abcd_{ijk}\rho_8\end{bmatrix} \quad \ldots(21)$$

By substituting (18), (19), (20) and (21) in (17) we get :

$$E(MSA)=\frac{1}{(a-1)}\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c}d_{ijk}\alpha_i^2+\sigma^2\phi_1$$

Similarly we obtain that

$$E(MSB)=\frac{1}{(b-1)}\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c}d_{ijk}\beta_j^2+\sigma^2\phi_2 \quad \cdots(22)$$

$$E(MSC)=\frac{1}{(c-1)}\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c}d_{ijk}\gamma_k^2+\sigma^2\phi_3 \quad \cdots(23)$$

$$E(MSAB)=\frac{1}{(a-1)(b-1)}\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c}d_{ijk}(\alpha\beta)_{ij}^2+\sigma^2\phi_4 \quad \ldots(24)$$

$$E(MSAC)=\frac{1}{(a-1)(c-1)}\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c}d_{ijk}(\alpha\gamma)_{ik}^2+\sigma^2\phi_5 \quad \cdots(25)$$

$$E(MSBC)=\frac{1}{(b-1)(c-1)}\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c}d_{ijk}(\beta\gamma)_{jk}^2+\sigma^2\phi_6 \quad \ldots(26)$$

$$E(MSABC)=\frac{1}{(a-1)(b-1)(c-1)}\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c}d_{ijk}(\alpha\beta\gamma)_{ijk}^2+\sigma^2\phi_7 \quad \ldots(27)$$

Where

$$\phi_8=1-\rho_1 \quad \cdots(28)$$

$$\phi_7=\phi_8+\left[\frac{\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c}d_{ijk}^2}{N}(\rho_1-\rho_2-\rho_3-\rho_4+\rho_5+\rho_6+\rho_7-\rho_8)\right] \quad ...(29)$$

$$\phi_6 = \phi_8 + \left[\frac{\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c} d_{ijk}^2}{N}(\rho_1 - \rho_2 - \rho_3 + \rho_5) + \frac{N^2 - bc\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c} d_{ijk}^2}{bcN}(\rho_4 - \rho_6 - \rho_7 + \rho_8) \right] \quad ...(30)$$

$$\phi_5 = \phi_8 + \left[\frac{\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c} d_{ijk}^2}{N}(\rho_1 - \rho_2 - \rho_4 + \rho_6) + \frac{N^2 - ac\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c} d_{ijk}^2}{acN}(\rho_3 - \rho_5 - \rho_7 + \rho_8) \right] \quad ....(31)$$

$$\phi_4 = \phi_8 + \left[\frac{\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c} d_{ijk}^2}{N}(\rho_1 - \rho_3 - \rho_4 + \rho_7) + \frac{N^2 - ab\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c} d_{ijk}^2}{abN}(\rho_2 - \rho_5 - \rho_6 + \rho_8) \right] \quad ...(32)$$

$$\phi_3 = \phi_8 + \left[\begin{array}{l} \frac{N^2 - ab\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c} d_{ijk}^2}{ab(c-1)N}(\rho_1 - \rho_2) + \frac{(b-1)(N^2 - ab\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c} d_{ijk}^2)}{ab(c-1)N}(\rho_3 - \rho_5) \\ + \frac{(a-1)(N^2 - ab\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c} d_{ijk}^2)}{ab(c-1)N}(\rho_4 - \rho_6) + \frac{(a-1)(b-1)(N^2 - ab\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c} d_{ijk}^2)}{ab(c-1)N}(\rho_7 - \rho_8) \end{array} \right] ...(33)$$

$$\phi_2 = \phi_8 + \left[\begin{array}{l} \frac{N^2 - ac\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c} d_{ijk}^2}{ac(b-1)N}(\rho_1 - \rho_3) + \frac{(c-1)(N^2 - ac\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c} d_{ijk}^2)}{ac(b-1)N}(\rho_2 - \rho_5) \\ + \frac{(a-1)(N^2 - ac\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c} d_{ijk}^2)}{ac(b-1)N}(\rho_4 - \rho_7) + \frac{(a-1)(c-1)(N^2 - ac\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c} d_{ijk}^2)}{ac(b-1)N}(\rho_6 - \rho_8) \end{array} \right] ...(34)$$

$$\phi_1 = \phi_8 + \left[\begin{array}{l} \frac{N^2 - bc\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c} d_{ijk}^2}{(a-1)bcN}(\rho_1 - \rho_4) + \frac{(c-1)(N^2 - bc\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c} d_{ijk}^2)}{(a-1)bcN}(\rho_2 - \rho_6) \\ + \frac{(b-1)(N^2 - bc\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c} d_{ijk}^2)}{(a-1)bcN}(\rho_3 - \rho_7) + \frac{(b-1)(c-1)(N^2 - bc\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c} d_{ijk}^2)}{(a-1)bcN}(\rho_5 - \rho_8) \end{array} \right] ...(35)$$

the analysis of variance table (ANOVA) can be written as table (1)

hypotheses to know that if the factor levels mean are equal and thses hypotheses are.

5.F-TEST

After finding (ANOVA) for study models we can discuss seven cases for null

$$H_0 : \alpha_i = 0 \qquad \forall i = 1,2,....,a$$
$$H_1 : \alpha_i \neq 0 \qquad for \;\; some \;\; i \qquad ...(36)$$

$$H_0 : \beta_j = 0 \qquad \forall j = 1,2,....,b$$
$$H_1 : \beta_j \neq 0 \qquad for \;\; some \;\; j \qquad ...(37)$$

$$H_0 : \gamma_k = 0 \qquad \forall k = 1,2,....,c$$
$$H_1 : \gamma_k \neq 0 \qquad for \;\; some \;\; k \qquad ....(38)$$

$$H_0 : (\alpha\beta)_{ij} = 0 \qquad \forall i = 1,2,....,a, \qquad j = 1...,b$$
$$H_1 : (\alpha\beta)_{ij} \neq 0 \qquad for \;\; some \;\; i \;\; or \;\; j \qquad ....(39)$$

$$H_0 : (\alpha\gamma)_{ik} = 0 \qquad \forall i = 1,2,....,a, \qquad k = 1,...,c$$
$$H_1 : (\alpha\gamma)_{ik} \neq 0 \qquad for \;\; some \;\; i \;\; or \;\; k \qquad ...(40)$$

$$H_0 : (\beta\gamma)_{jk} = 0 \qquad \forall j = 1,2,....,b, \qquad k = 1,...,c$$
$$H_1 : (\beta\gamma)_{jk} \neq 0 \qquad for \;\; some \;\; j \;\; or \;\; k \qquad ...(41)$$

$$H_0 : (\alpha\beta\gamma)_{ijk} = 0 \qquad \forall i = 1,....,a, \;\; j = 1,...,b, \;\; k = 1,...,c$$
$$H_1 : (\alpha\beta\gamma)_{ijk} \neq 0 \qquad for \;\; some \;\; i \;\; or \;\; j \;\; or \;\; k \qquad ...(42)$$

From cockran's theorem. Under null hypotheses we obtain

$$W_A = \frac{SSA}{\sigma^2} \sim \phi_1 x^2 (a-1) \qquad ...(43)$$

$$W_B = \frac{SSB}{\sigma^2} \sim \phi_2 x^2 (b-1) \qquad ...(44)$$

$$W_C = \frac{SSC}{\sigma^2} \sim \phi_3 x^2 (c-1) \qquad ...(45)$$

$$W_{AB} = \frac{SSAB}{\sigma^2} \sim \phi_4 x^2 ((a-1)(b-1)) \qquad ...(46)$$

$$W_{AC} = \frac{SSAC}{\sigma^2} \sim \phi_5 x^2 ((a-1)(c-1)) \qquad ...(47)$$

$$W_{BC} = \frac{SSBC}{\sigma^2} \sim \phi_6 x^2 ((b-1)(c-1)) \qquad ...(48)$$

$$W_{ABC} = \frac{SSABC}{\sigma^2} \sim \phi_7 x^2 ((a-1)(b-1)(c-1)) \qquad ...(49)$$

$$W_E = \frac{SSE}{\sigma^2} \sim \phi_8 x^2\left(\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c} d_{ijk} - abc\right) \qquad ...(50)$$

Where W_A, W_B, W_C, W_{AB}, W_{AC}, W_{BC}, W_{ABC}, and W_E are independent therefore we can write the F* distribution for test to equal factor levels mean as:

$$F_1^* = \frac{W_A \phi_8\left(\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c} d_{ijk} - abc\right)}{W_E \phi_1 (a-1)} = C_1 F_1 \sim F((a-1), \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c} d_{ijk} - abc) \qquad \cdots(51)$$

Where $C_1 = \frac{\phi_8}{\phi_1}$ *and* $F_1 = \frac{MSA}{MSE}$

$$F_2^* = \frac{W_B \phi_8\left(\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c} d_{ijk} - abc\right)}{W_E \phi_2 (b-1)} = C_2 F_2 \sim F((b-1), \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c} d_{ijk} - abc) \qquad ...(52)$$

Where $C_2 = \frac{\phi_8}{\phi_2}$ *and* $F_2 = \frac{MSB}{MSE}$

$$F_3^* = \frac{W_C \phi_8\left(\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c} d_{ijk} - abc\right)}{W_E \phi_3 (c-1)} = C_3 F_3 \sim F((c-1), \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c} d_{ijk} - abc) \qquad ...(53)$$

Where $C_3 = \frac{\phi_8}{\phi_3}$ *and* $F_3 = \frac{MSC}{MSE}$

$$F_4^* = \frac{W_{AB} \phi_8\left(\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c} d_{ijk} - abc\right)}{W_E \phi_4 (a-1)(b-1)} = C_4 F_4 \sim F((a-1)(b-1), \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c} d_{ijk} - abc) \qquad ...(54)$$

Where $C_4 = \frac{\phi_8}{\phi_4}$ *and* $F_4 = \frac{MSAB}{MSE}$

$$F_5^* = \frac{W_{AC} \phi_8\left(\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c} d_{ijk} - abc\right)}{W_E \phi_5 (a-1)(c-1)} = C_5 F_5 \sim F((a-1)(c-1), \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c} d_{ijk} - abc) \qquad ...(55)$$

Where $C_5 = \frac{\phi_8}{\phi_5}$ *and* $F_5 = \frac{MSAC}{MSE}$

$$F_6^* = \frac{W_{BC} \phi_8\left(\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c} d_{ijk} - abc\right)}{W_E \phi_6 (b-1)(c-1)} = C_6 F_6 \sim F((b-1)(c-1), \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c} d_{ijk} - abc) \qquad ...(56)$$

Where $C_6 = \frac{\phi_8}{\phi_6}$ *and* $F_6 = \frac{MSBC}{MSE}$

$$F_7^* = \frac{W_{ABC}\phi_8\left(\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c} d_{ijk} - abc\right)}{W_E\phi_7(a-1)(b-1)(c-1)} = C_7F_7 \sim F((a-1)(b-1)(c-1), \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c} d_{ijk} - abc) \quad ...(57)$$

Where $C_7 = \frac{\phi_8}{\phi_7}$ *and* $F_7 = \frac{MSABC}{MSE}$

After finding F^* statistic we find table values from F- distribution table with significance level α and degree of freedom and a compare between these values.

6.CORRECTING FOR CORRELATION

The correction constants $C_h : h = 1,2,3,4,5,6,7$ may be $= (or > or <)$ one. If the correction constant equal to 1 , then no correction is needed to the F test , where as if the correction constant $> (or <) 1$, then we need correction. (see Al-Shahiry (1997)).

Tables (2),(3),(4),(5),(6),(7),(8) shows the values of true α for a variety of values for significance level of α was calculated for some hypothetical values for $C_1, C_2, \cdots, C_7$. To calculate the true α in tables from 1 to 7 we used the formula $X = X_1 + (Y - Y_1)\left[\frac{(X_2 - X_1)}{(Y_2 - Y_1)}\right]$ where X_1, X_2 represents significance level which take from the tables in the statistical distribution F and be known , Y_1, Y_2, Y represents the values corresponding to spreadsheet X_1, X_2 and $X = True\ alpha$ respectively and be known also where Y represents the value of a statistical spreadsheet F at the level α multiplied by correction factor.

7.CONCLUSIONS

1- This model is an extension of some models studied earlier by many authors. But under certain condition we may get the same models discussed before by Pavur and Davenport (1985), Al-Shahiry (1997) , Al-Kaabawi (2000) and Abdullah , Al-Kaabawi (2007).

2- Tabels 2,…,8 , show that the true alpha level inflate (deflate) when the correction constant $<(or >)1$, and this lead to have a smaller (bigger) rejection region for the complete null hypothesis on testing factors.

3- This study may be extended to n-way model.

4- Can be done statistical tables values of real alpha for different values of the coefficient is greater or less than one.

5- We Can use the existing laws in this research to find a table of analysis of variance in the case of the correlated data form a unbalanced cross with three factors.

6- The practical side is the important part so we can discuss the latest work supports the findings of this research.

S.O.V	d.f	SS	MS	E(MS)	
				For indep. Data	For dep. data
A	(a-1)	SSA	$\frac{SSA}{(a-1)}$	$\frac{\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c} d_{ijk}\alpha_i^2}{(a-1)}+\sigma^2$	$\frac{\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c} d_{ijk}\alpha_i^2}{(a-1)}+\sigma^2\phi_1$
B	(b-1)	SSB	$\frac{SSB}{(b-1)}$	$\frac{\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c} d_{ijk}\beta_j^2}{(b-1)}+\sigma^2$	$\frac{\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c} d_{ijk}\beta_j^2}{(b-1)}+\sigma^2\phi_2$
C	(c-1)	SSC	$\frac{SSC}{(c-1)}$	$\frac{\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c} d_{ijk}\gamma_k^2}{(c-1)}+\sigma^2$	$\frac{\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c} d_{ijk}\gamma_k^2}{(c-1)}+\sigma^2\phi_3$
AB	(a-1) (b-1)	SSAB	$\frac{SSAB}{(a-1)(b-1)}$	$\frac{\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c} d_{ijk}(\alpha\beta)_{ij}^2}{(a-1)(b-1)}+\sigma^2$	$\frac{\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c} d_{ijk}(\alpha\beta)_{ij}^2}{(a-1)(b-1)}+\sigma^2\phi_4$
AC	(a-1) (c-1)	SSAC	$\frac{SSAC}{(a-1)(c-1)}$	$\frac{\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c} d_{ijk}(\alpha\gamma)_{ik}^2}{(a-1)(c-1)}+\sigma^2$	$\frac{\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c} d_{ijk}(\alpha\gamma)_{ik}^2}{(a-1)(c-1)}+\sigma^2\phi_5$
BC	(b-1) (c-1)	SSBC	$\frac{SSBC}{(b-1)(c-1)}$	$\frac{\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c} d_{ijk}(\beta\gamma)_{jk}^2}{(b-1)(c-1)}+\sigma^2$	$\frac{\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c} d_{ijk}(\beta\gamma)_{jk}^2}{(b-1)(c-1)}+\sigma^2\phi_6$
ABC	(a-1) (b-1) (c-1)	SSABC	$\frac{SSABC}{(a-1)(b-1)(c-1)}$	$\frac{\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c} d_{ijk}(\alpha\beta\gamma)_{ijk}^2}{(a-1)(b-1)(c-1)}+\sigma^2$	$\frac{\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c} d_{ijk}(\alpha\beta\gamma)_{ijk}^2}{(a-1)(b-1)(c-1)}+\sigma^2\phi_7$
E	$\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c} d_{ijk}-abc$	SSE	$\frac{SSE}{\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c} d_{ijk}-abc}$	σ^2	$\sigma^2(1-\rho_1)$
Total	$\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c} d_{ijk}-1$	SSTO	$\frac{SSTO}{\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c} d_{ijk}-1}$		

Table (2) true α for different values of C_1

C_1 \ α	0.05	0.01
0.8	0.081	0.021
0.9	0.066	0.015
1	0.05	0.01
1.1	0.043	0.008
1.2	0.036	0.005

a=2 , b=2 , c=2 , N=24

Table (3) true α for different values of C_2

C_2 \ α	0.05	0.01
0.8	0.089	0.023
0.9	0.069	0.016
1	0.05	0.01
1.1	0.041	0.007
1.15	0.036	0.006

a=2 , b=3 , c=2 , N=34

Table (4) true α for different values of C_3

C_3 \ α	0.05	0.01
0.6	0.191	0.076
0.7	0.156	0.059
1	0.05	0.01
1.05	0.045	0.009
1.1	0.041	0.007

a=2 , b=2 , c=3 , N=36

Table (5) true α for different values of C_4

C_4 \ α	0.05	0.01
0.75	0.148	0.038
0.9	0.089	0.021
1	0.05	0.01
1.05	0.045	0.009
1.1	0.039	0.007

a=2 , b=4 , c=2 , N=28

Table (6) true α for different values of C_5

C_5 \ α	0.05	0.01
0.65	0.154	0.041
0.95	0.067	0.015
1	0.05	0.01
1.16	0.038	0.006
1.2	0.035	0.005

a=2 , b=4 , c=2 , N=44

Table (7) true α for different values of C_6

C_6 \ α	0.05	0.01
0.85	0.089	0.023
0.95	0.063	0.0143
1	0.05	0.01
1.1	0.041	0.0063
1.12	0.039	0.0055

a=2 , b=3 , c=3 , N=78

Table (8) true α for different values of C_7

C_7 \ α	0.05	0.01
0.7	0.226	0.089
0.85	0.138	0.049
1	0.05	0.01
1.05	0.045	0.007
1.09	0.041	0.005

a=4 , b=2 , c=4 , N=152

Note : $N = \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c} d_{ijk}$

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حساب توقع معدلات المربعات لنموذج ذو ثلاث اتجاهات متقاطع

غير متوازن لبيانات مترابطة

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الخلاصة

في هذا البحث تم حساب توقع معدلات المربعات لنموذج ذو ثلاث اتجاهات متقاطع غير متوازن لبيانات مترابطة ولاحظنا التأثير للترابط بين البيانات على الإحصائية F .