Isovector M2 transitions in ^{30}Si

Ammar A. Al-Sa´ad, Ali A. Abbas and Muzahim M. Abdullah

Department of Physics, College of Science, University of Basrah, Basrah, Iraq.

Abstract

Transverse M2 form factors for isovector transitions in ^{30}**Si** peaked at low q values are very well predicted in the framework of simple particle-hole approach. Thus, the structure of the 2^- states at E_X=9.96, 10.27, 10.93, 11.18, 11.84, 12.02, 12.4, 12.7, 12.83, 13.14, 13.4, 14.0 and 14.19 MeV, respectively, are assumed to be the linear combination of the $(1f_{7/2})(1d^{-1}_{5/2})$ and $(1f_{5/2})(1d^{-1}_{5/2})$ configurations only. The amplitudes of the assumed two configurations are determined phenomenologically by reproducing the reduced probabilities of magnetic quadrupole transitions at the photon point.

Keywords:- M2 form factors, Reduced transition probability.

1. Introduction

^{30}**Si** is one of the 2s-1d shells nuclei which has closed sub-shells for protons (Z=14; fills $1d_{5/2}$ shell), and neutrons (N=16; fills $2s_{1/2}$ shell). The transitions from its ground state at J^π, T=0^+**,** 1 to the low-lying positive parity (isoscalar or isovector) states can be studied by using USD (Wildenthal) interaction. According to Wildenthal model, eigenvalues (energy levels) and eigenvectors (amplitudes) of ^{30}**Si** are obtained from the configuration mixing of 14 nucleons distributed over ($1d_{5/2}$ - $2s_{1/2}$ - $1d_{3/2}$**)** harmonic oscillator shells. Thus, the magnetic dipole (M1) and the electric quadrupole (E2) transitions in the nuclei of A=17-39 are studied by this model [Brown and Wildenthal, 1988]. The magnetic dipole transition strength B(M1) are measured in ^{30}**Si** (and other sd-shell nuclei) by Nuclear Resonance Fluorescence (NRF) technique [Berg , *et. al.*, 1984], and by inelastic electron scattering [Petraities, *et. al.*, 1994]. In both studies the researchers are compared their measurements with calculations accomplished in the framework of sd shell model. On the other hand, the odd parity transitions, such as magnetic quadrupole one, require the inclusion of $1\hbar\omega$ excitations to either 2p-1f or 1p shells. Rangacharyulu. presented a simple phenomenological two-state model for the description of the 2^{-} states at 11.62 and 12.1 MeV in ^{20}**Ne**. The best description of their measured isovector M2 form factors is obtained by trying various pairs of configurations and finally they found that the unique set for those two levels are mostly due to $\left|1d^{3}_{5/2}1f^{1}_{7/2}\right\rangle$ and $\left|1d^{5}_{5/2}1p^{-1}_{1/2}\right\rangle$ [Rangacharyulu *et al.*, 1985]. A simpler structure consist of the configuration $(1f_{7/2})(1d^{-1}_{5/2})$ only is used by [Lüttge *et al.*, 1996] to describe the form factor for the M2 transition to the E_X=14.36 MeV state in ^{28}**Si**. [Alford *et al.*,1986] used the transition amplitudes resulting from a full sd shell model calculations to study the structure of ^{30}**Si** (as well as ^{26}**Mg**) through the reaction ^{28}**Si(t,p)** ^{30}**Si.** For excitation energies higher than 6 MeV they require the $(1f_{7/2})^2$ configuration to identify 2^+ and 4^+ states in ^{30}**Si**. In order to provide a simple basis for comparing the strength of transitions to the negative parity states, they carried out their calculations assuming $(1f_{7/2}\ 1d_{5/2})$ configuration.

However, the electron scattering, as an excellent tool to reveal the nuclear structure, is used by [Petraitis *et al.*, 1994] to explore the form factors of 2^-(as well as 1^+) states in ^{30}**Si** (as well as ^{32}**S** and ^{34}**S**) at low momentum transfer values. In the present work, we adopt the J^{π} assignments of this reference to compare with our calculations of the M2 form factors of ^{30}**Si** assuming the $(1f_{7/2})(1d^{-1}_{5/2})$ and $(1f_{5/2})(1d^{-1}_{5/2})$ configurations only.

2. Theory

The magnetic electron scattering form factors involving momentum transfer, *q*, angular momentum, *J*, and isospin, *T*, is given by [Donnelly and Sick, 1984]:-

$$\left|F_J^m(q)\right|^2 = \frac{4\pi}{Z^2}\frac{1}{2J_i+1}\left|\sum_T(-1)^{T_f-T_z}\begin{pmatrix} T_f & T & T_i \\ -T_z & 0 & T_z\end{pmatrix}\left\langle J_fT_f\left\|\hat{T}^m_{jT}(q)\right\|J_iT_i\right\rangle\right|^2\left|\exp\left[-\frac{1}{4}\left(0.43-\frac{b^2}{A^2}\right)q^2\right]\right|^2 \qquad (1),$$

where the exponential factor corrects for the finite size of the nucleon and the center of mass motion [Tassie and Barker, 1958]. The atomic number, the mass number and the size parameter are represented respectively, by *Z*, *A* and *b*. On the other hand, the reduced transition probability, $B(MJ\uparrow,q)$, is correlated with the magnetic form factor through the relation [Brown , *et. al.*, 1985]

$$B(MJ\uparrow,q) = \frac{\left[(2J+1)!!\right]^2 Z^2 J}{4\pi(\mathrm{J}+1)\mathrm{q}^{2\mathrm{J}}}\left|F_J^m(q)\right|^2$$

(2).

As a simple approach, the wave functions of the excited states are assumed to be the mixing of two configurations of particle-hole excitations

$$\left|J_f T_f\right\rangle = \alpha\left|p_1 h_1^{-1}; J_f T_f\right\rangle + \beta\left|p_2 h_2^{-1}; J_f T_f\right\rangle \quad (3),$$

Where the parameters α and β obey the normalization condition $\alpha^2 + \beta^2 = 1$.

In the work of [Donnelly, 1970], the reduced many-particle matrix elements of the transverse magnetic electron scattering operator, $\hat{T}_{JT}^{m}(q)$, between the initial, $\left|J_i T_i\right\rangle$, and final, $\left|J_f T_f\right\rangle$, nuclear states are given by

$$\left\langle J_f T_f \left\| \hat{T}_{jT}^{m}(q) \right\| J_i T_i \right\rangle = \alpha\left\langle h_1 \left\| \hat{T}_{jT}^{m}(q) \right\| p_1 \right\rangle + \beta\left\langle h_2 \left\| \hat{T}_{jT}^{m}(q) \right\| p_2 \right\rangle \quad (4),$$

where the reduced single-particle matrix elements on the right-hand side of eq.(4) are calculated according to [Brown *et al.*,1985], while α and β are taken as adjusted parameters that reproduce the experimental reduced magnetic transition probabilities at the photon point.

3. Results and discussion

The electro-excitation of ^{30}**Si** from its ground state at $J_i^{\pi_i}, T_i = 0^+, 1$ to the final states at $J_f^{\pi_f}, T_f = 2^-, 1$ are transverse magnetic M2 transitions. The change of isospin $\Delta T = 1$ during these transitions is possible (isovector transitions). These transitions can be represented by the removing of one particle from the filled (1d) shell and promoting it to the unoccupied (1f) shells leaving a hole in the (1d) shell. Thus, our configuration mixing wave function for each $2^-,1$ state in ^{30}**Si** is given by;

$$\left|2^-,1\right\rangle = \alpha\left|1f_{7/2}1d_{5/2}^{-1}\right\rangle_{2^-,1} + \beta\left|1f_{5/2}1d_{5/2}^{-1}\right\rangle_{2^-,1} \quad (5).$$

In our simple approach, α and β are taken as adjusted parameters that reproduce the reduced transition probability at the photon point $q = k = \frac{E_X}{\hbar c}$ for each transition separately. The parameter α is varied freely, while β is calculated from $\sqrt{1-\alpha^2}$. Table (1) gives the resulting values of α and β which are used in turn for calculation of the magnetic form factors $\left|F_J^m(q)\right|^2$ and the reduced transition probability $B(M2\uparrow, q)$ which are displayed in figs.(1) to (13). We employ harmonic oscillator single-particle wave functions with a size parameter b=1.834 *fm* [Brown, *et. al.*, 1980] obtained from a fit to the nuclear charge radius of ^{30}**Si**.

The available data are extracted from Petraities, *et. al*. and displayed as open circles in the figures (1-13). The M2 form factors data are maximized at low momentum transfer values ($q \le 0.6\, fm^{-1}$)with expected positions for the diffraction minima at $q < 1\, fm^{-1}$. Our theoretical calculation for the M2 form factors is consistent with most of the available data and predicts the minima diffraction values at $0.7\, fm^{-1} < q < 1\, fm^{-1}$, as shown in figs.(1a-13a). Underestimation by a factor of magnitude of our calculation in comparison with the experimental data of $B(M2\uparrow, q)$ and $|F_J^m(q)|^2$ is observed for the M2 transitions to the states at E_X=10.27, 13.14, 13.4 and 14.20 MeV which are shown in figs.(2), (10), (11) and (13), respectively. However, the presence of considerable longitudinal strength for the transitions to the states at 10.27 and 13.4 MeV [Petraities, *et. al*., 1994] is reflected on the behavior of the data as a function of *q* which tend to be maximized at higher value of momentum transfer. On the other hand, the most probable assignment for the transition to the states at 10.27 and 14.20 MeV is M2 [Petraities, *et. al*., 1994] which admixed with small M1 component during the measurement. And this may account for the observed underestimation of figs.(2), (11) and (13).

4. Conclusions

An overall very well description for the data of M2 transition due to electron scattering from ^{30}**Si** is obtained in the framework of this particle-hole approach. The calculations of the present work reflect the simplicity of the structure of the M2 transitions in ^{30}**Si**. However, the available data have large error bars and thus experimental measurements, with high precision and extended to higher *q* values, are necessary to check our prediction.

References

Alford W.P., Cameron J. A, Habib E. and Wildenthal B. H., Nucl. Phys. A 454

(1986) 189.

Brown B. A. and Wildenthal B. H., Annu. Rev. Nucl. Part. Sci. 38 (1988) 29.

Brown B. A., Wildenthal B. H., Williamson C. F., Rad F. N., Kowalski S.,

Crannell H. and O´Brien J. T., Phys. Rev. C 32 (1985) 1127.

Brown B. A., Chung W. and Wildenthal B. H., Phys. Rev. C 21 (1980) 2600.

Berg U. E. P., Ackermann K., Bangert K., Blasing C., Naatz W., Stock R., Weinhard K., Brussel M. K., Chapuran T. E. and Wildenthal B. H., Phys. Lett. B 140 (1984) 191.

Donnelly T. W. and Sick I., Rev. Mod. Phys. 56 (1984) 461.

Donnelly T. W., Phys. Rev. C1 (1970) 833.

Lüttge C., von Neumann P.-Cosel, Neumeyer F. and Richter A., Nucl. Phys. A 606 (1996) 183.

Petraitis M., Connelly J. P., Crannell H., Fagg, L. W. O´Brien J. T., Sober D. I., Deininger J.R., Williamson S. E., Lindgren R. and Raman S., Phys. Rev C 49 (1994) 3000.

Rangacharyulu C., Ansaldo E. J., Stockhausen D., Bender D., Müller S., Richter A., Lo Iudice N. and Palumbo F., Phys. Rev. C 31 (1985) 1656.

Tassie L. J. and Barker F. C., Phys. Rev 111 (1958) 940.

Table (1):- The parameters α and β for some isovector M2 transitions in ^{30}Si.

E_X (MeV)[(a)]	α	$\beta = \sqrt{1-\alpha^2}$
9.96	0.45724	0.88934
10.27	0.43895	0.89851
10.93	0.45740	0.88926
11.18	0.42650	0.90449
11.84	0.46423	0.88571
12.02	0.49617	0.86823
12.4	0.47070	0.88229
12.7	0.48828	0.87269
12.83	0.46115	0.88732
13.14	0.45785	0.88903
13.4	0.48845	0.87259
14.0	0.50435	0.86350
14.20	0.45107	0.89249

ref. $_{(a)}$ [Petraities, *et. al*., 1994].

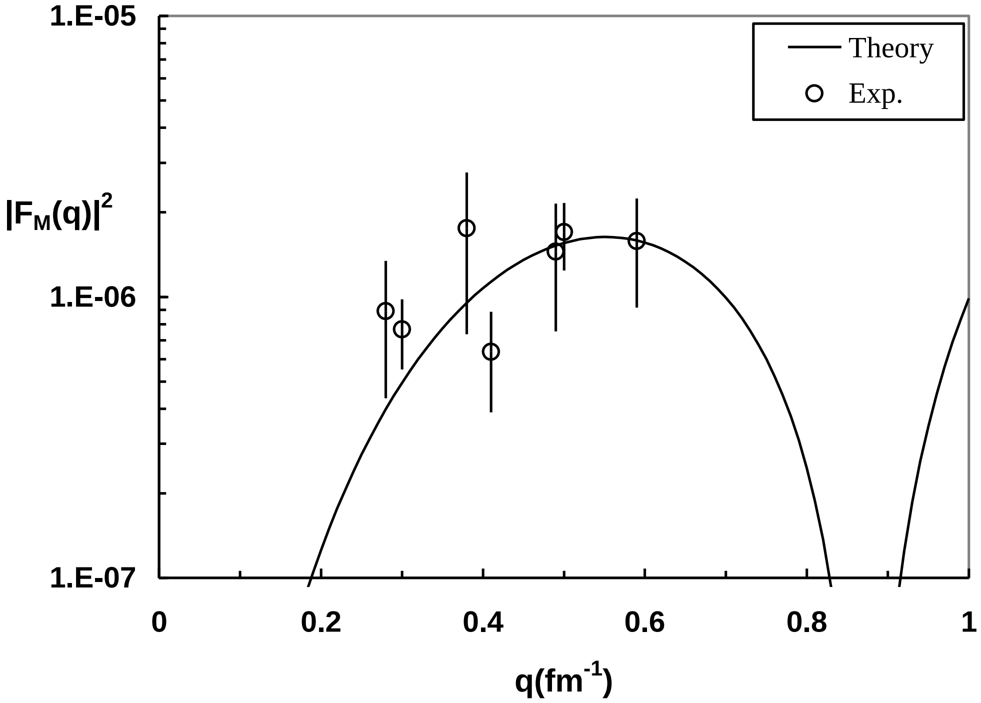

Fig.(1a):- M2 form factors for the isovector transition to the 2^- state at 9.96 MeV in ^{30}Si .

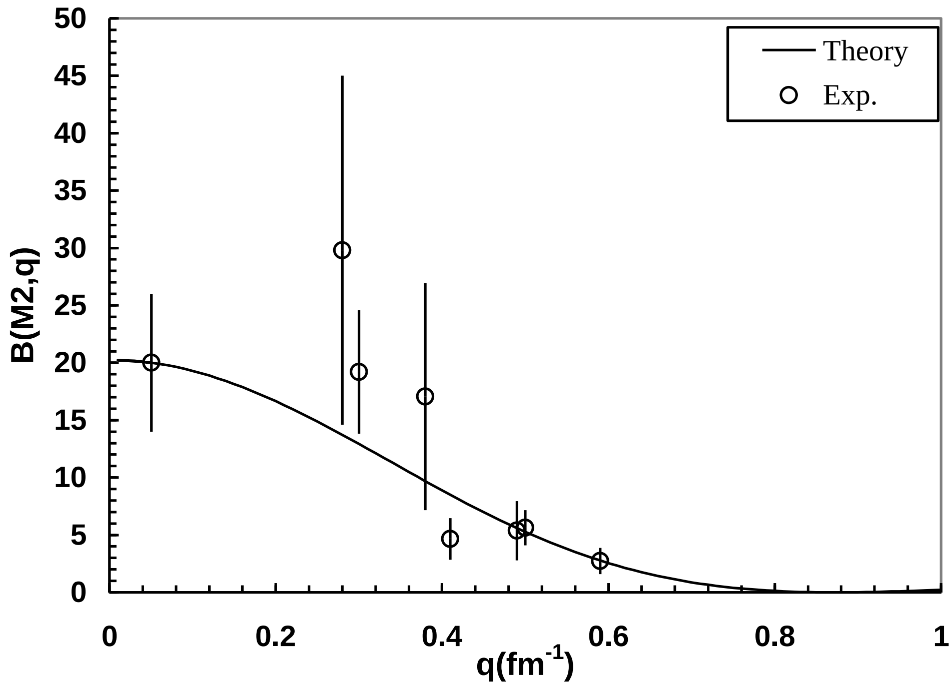

Fig.(1b):- Reduced transition probability (in units of $\mu_N^2\ fm^2$) for M2 isovector transition to the 2^- state at 9.96 MeV in ^{30}Si .

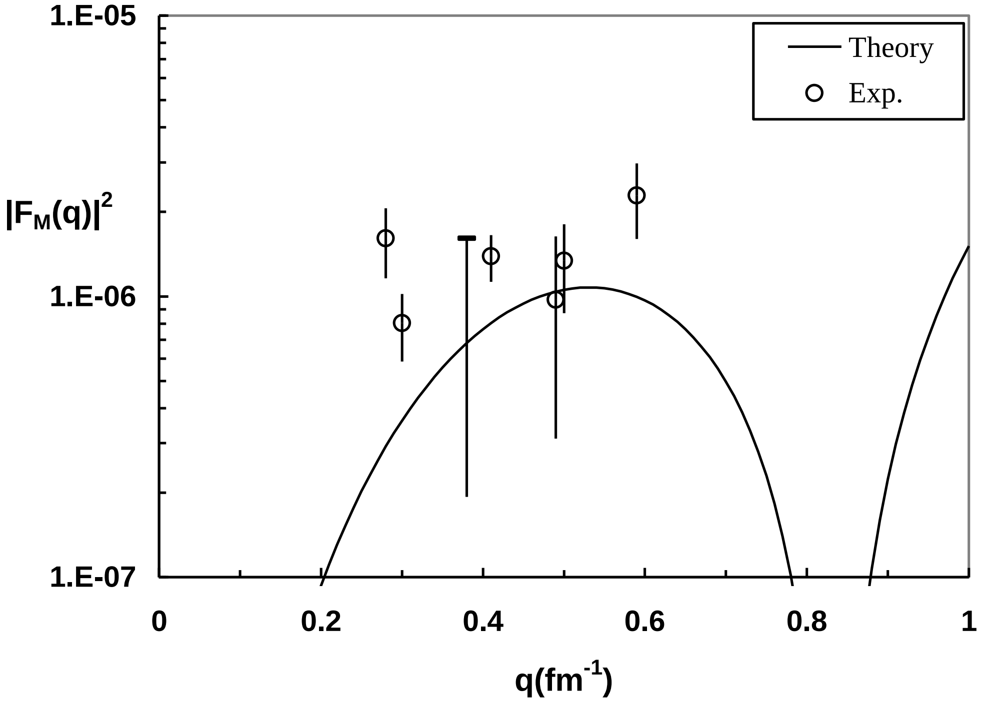

Fig.(2a):- M2 form factors for the isovector transition to the 2^- state at 10.27 MeV in ^{30}Si .

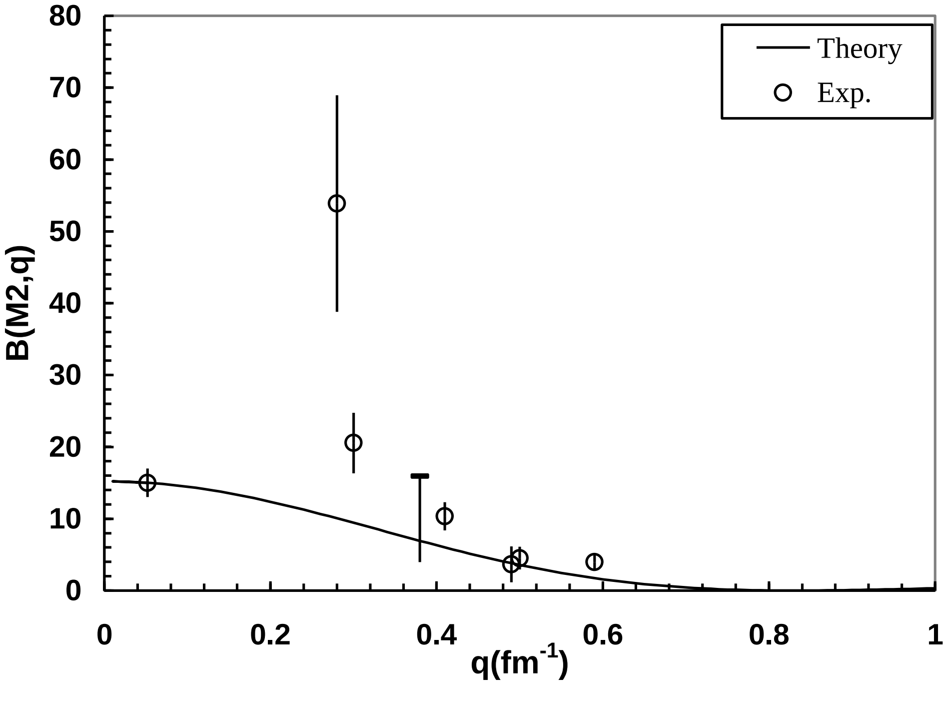

Fig.(2b):- Reduced transition probability (in units of $\mu_N^2\ fm^2$) for M2 isovector transition to the 2^- state at 10.27 MeV in ^{30}Si .

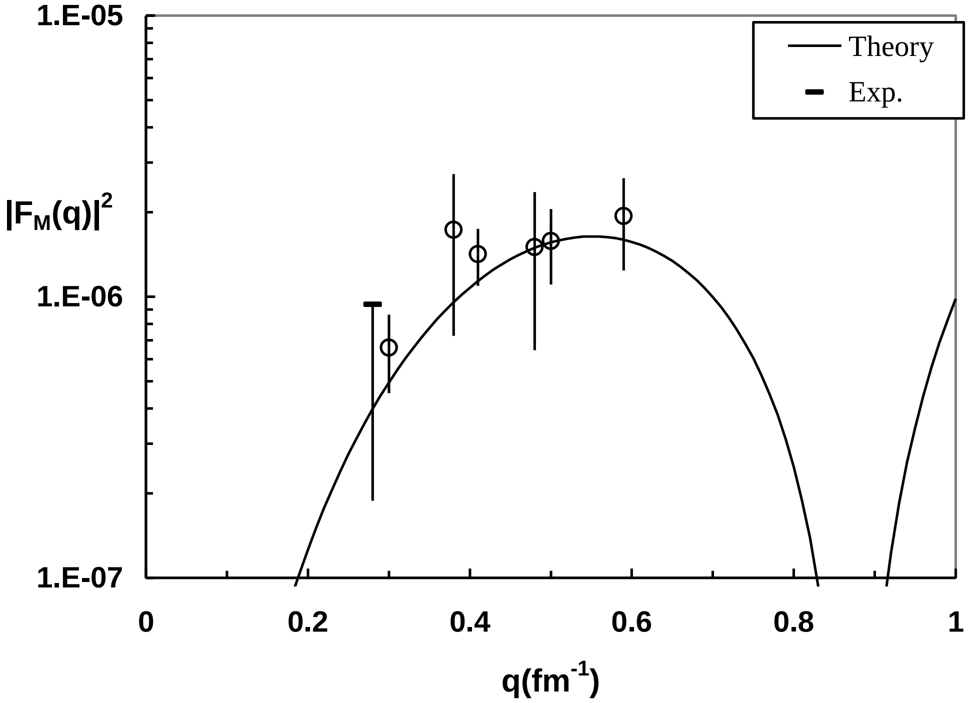

Fig.(3a):- M2 form factors for the isovector transition to the 2^- state at 10.93 MeV in ^{30}Si .

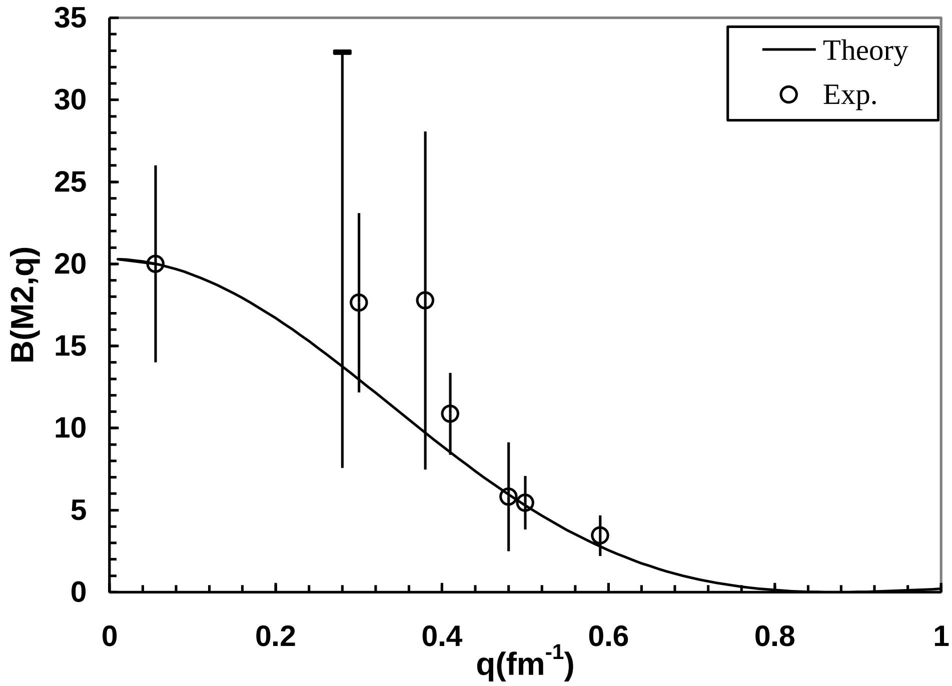

Fig.(3b):- Reduced transition probability (in units of $\mu_N^2\ fm^2$) for M2 isovector transition to the 2^- state at 10.93 MeV in ^{30}Si .

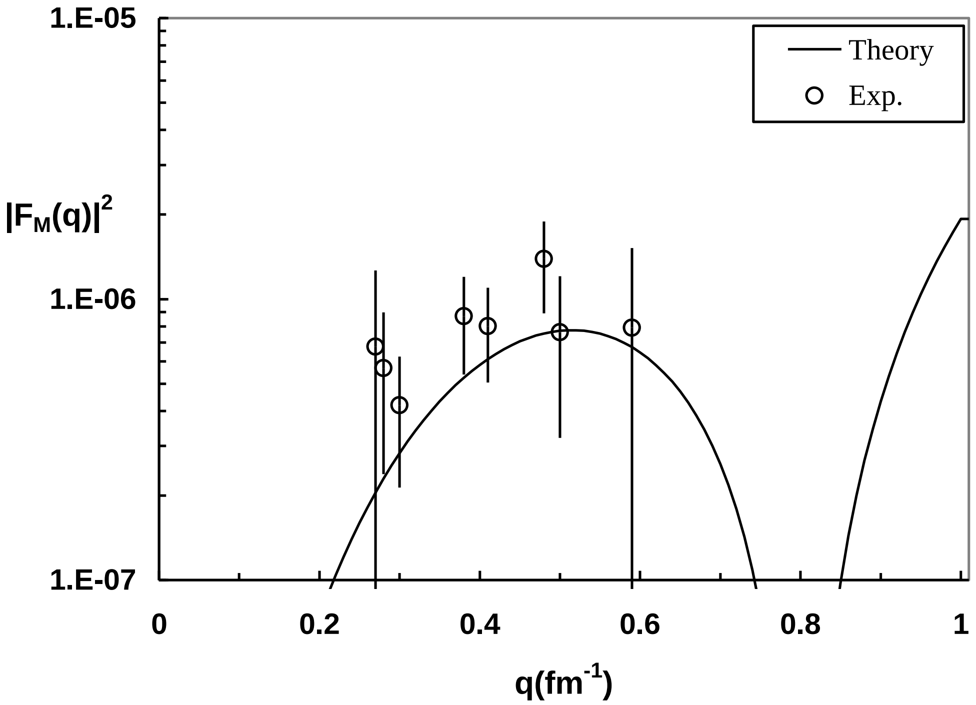

Fig.(4a):- M2 form factors for the isovector transition to the 2^- state at 11.18 MeV in ^{30}Si .

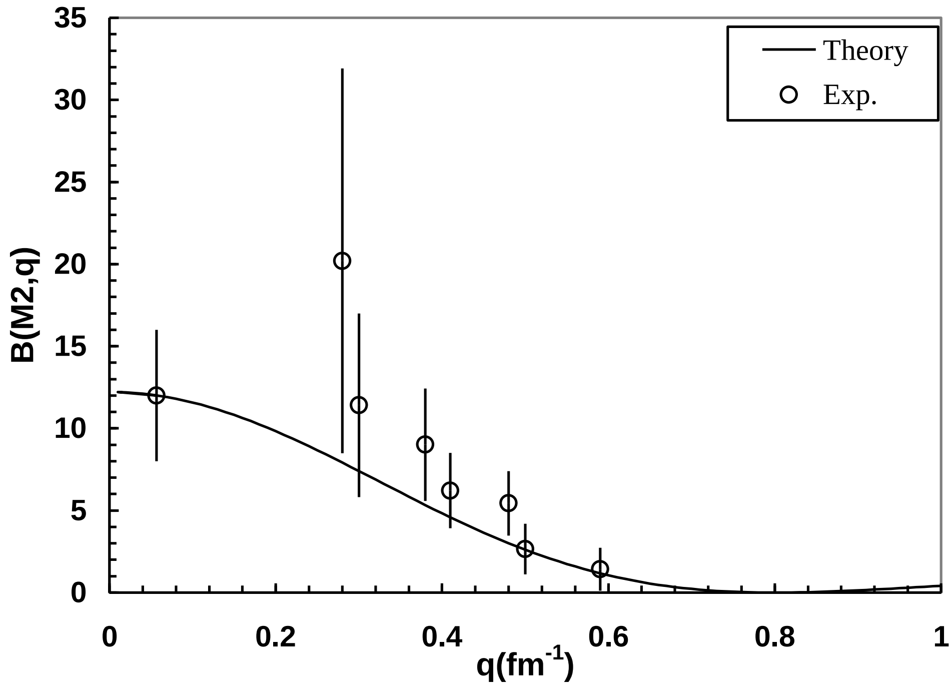

Fig.(4b):- Reduced transition probability (in units of $\mu_N^2 \; fm^2$) for M2 isovector transition to the 2^- state at 11.18 MeV in ^{30}Si .

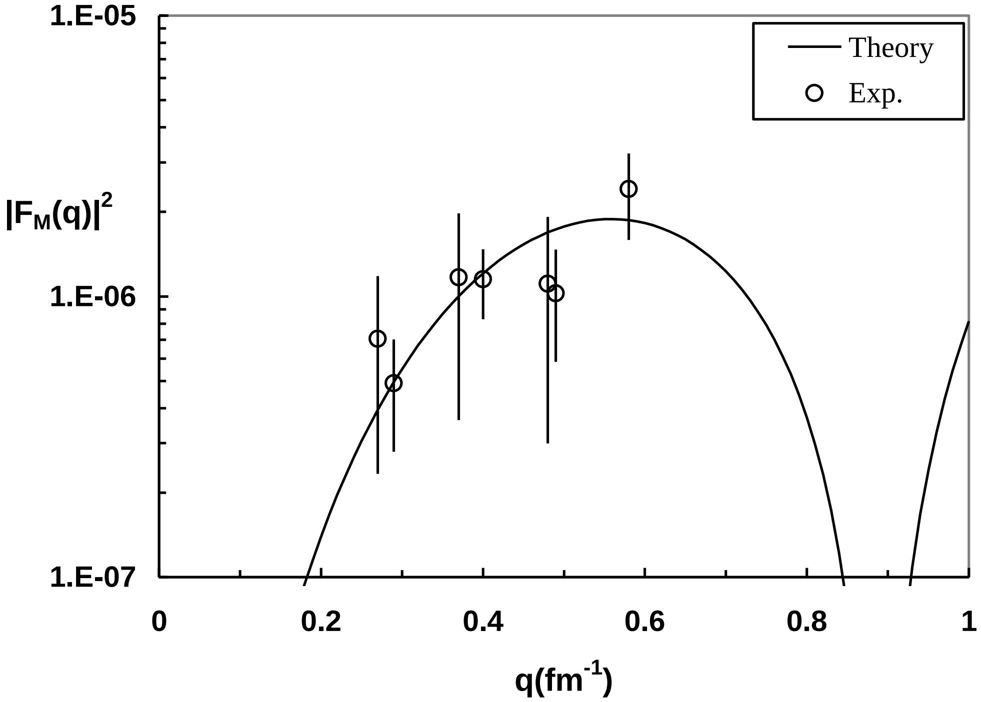

Fig.(5a):- M2 form factors for the isovector transition to the 2^- state at 11.84 MeV in ^{30}Si .

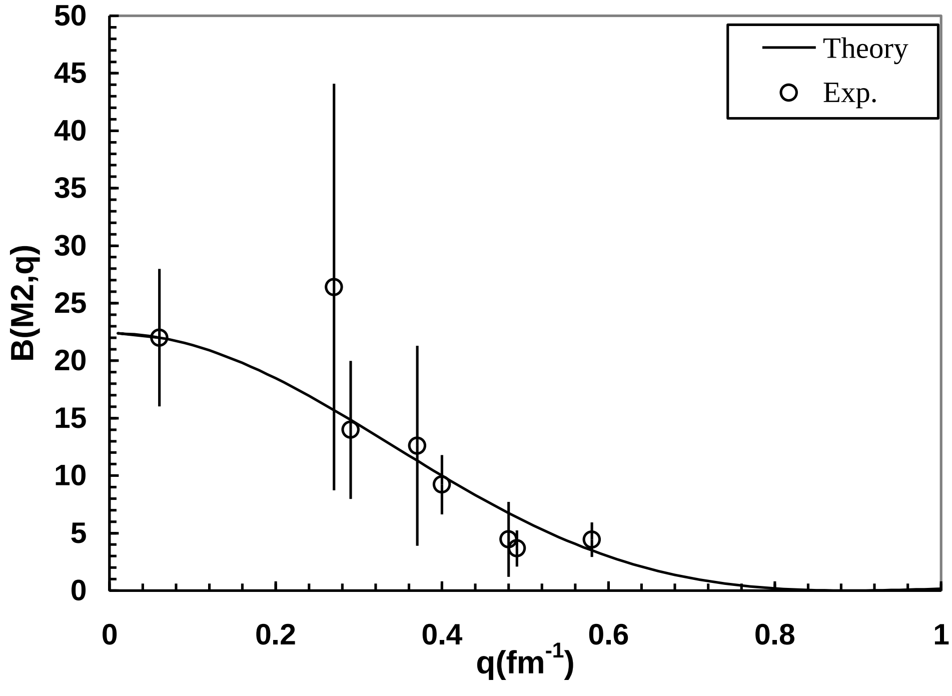

Fig.(5b):- Reduced transition probability (in units of $\mu_N^2\ fm^2$) for M2 isovector transition to the 2^- state at 11.84 MeV in ^{30}Si .

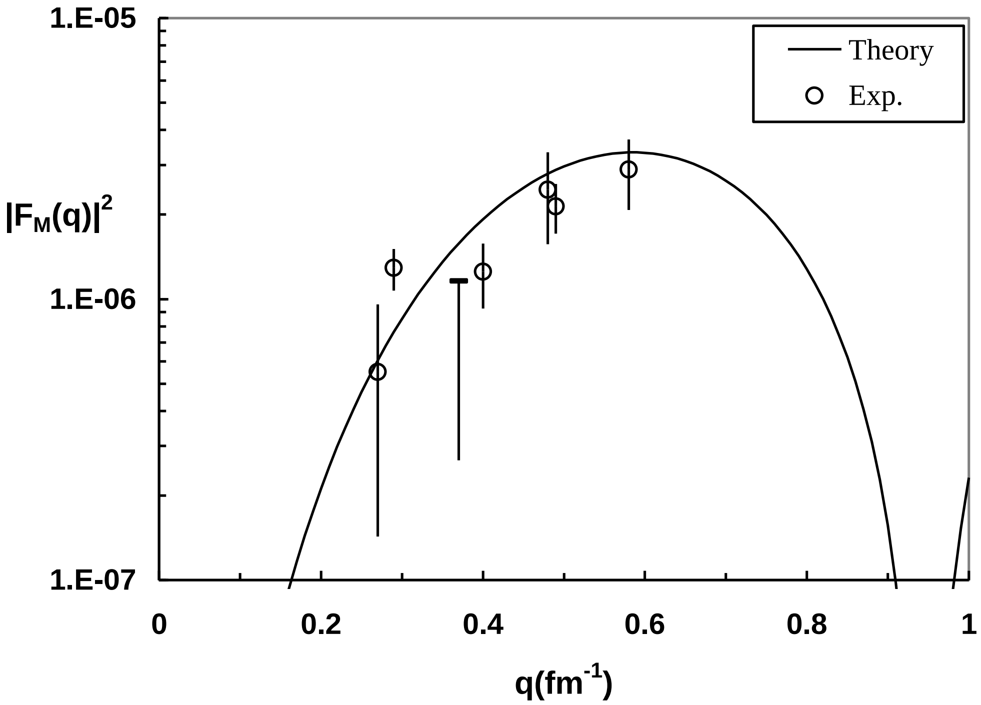

Fig.(6a):- M2 form factors for the isovector transition to the 2^- state at 12.02 MeV in ^{30}Si .

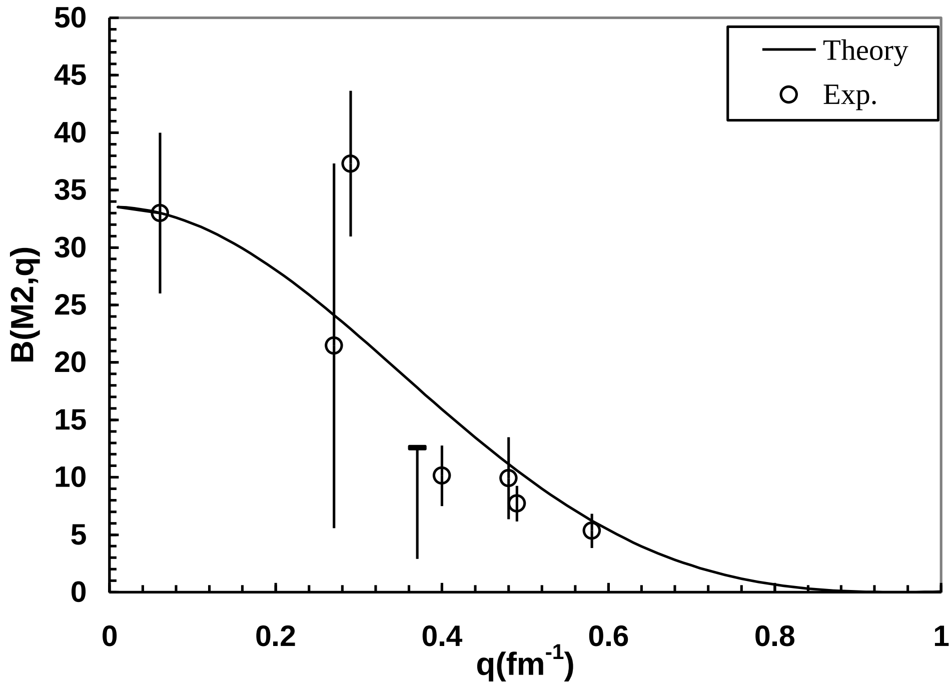

Fig.(6b):- Reduced transition probability (in units of $\mu_N^2\ fm^2$) for M2 isovector transition to the 2^- state at 12.02 MeV in ^{30}Si .

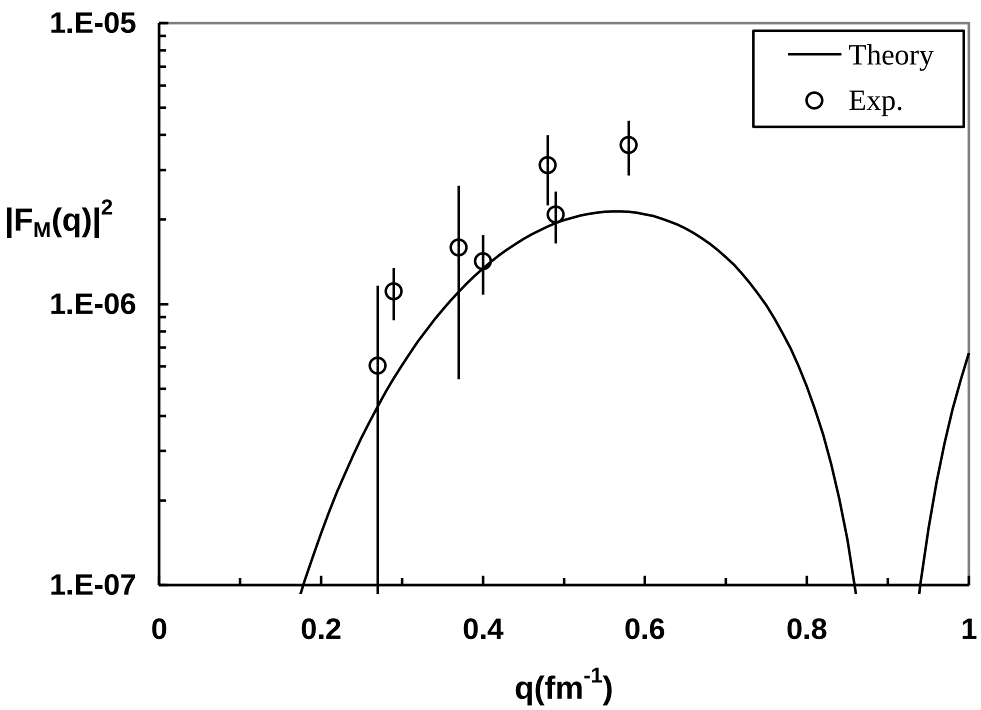

Fig.(7a):- M2 form factors for the isovector transition to the 2^- state at 12.4 MeV in ^{30}Si .

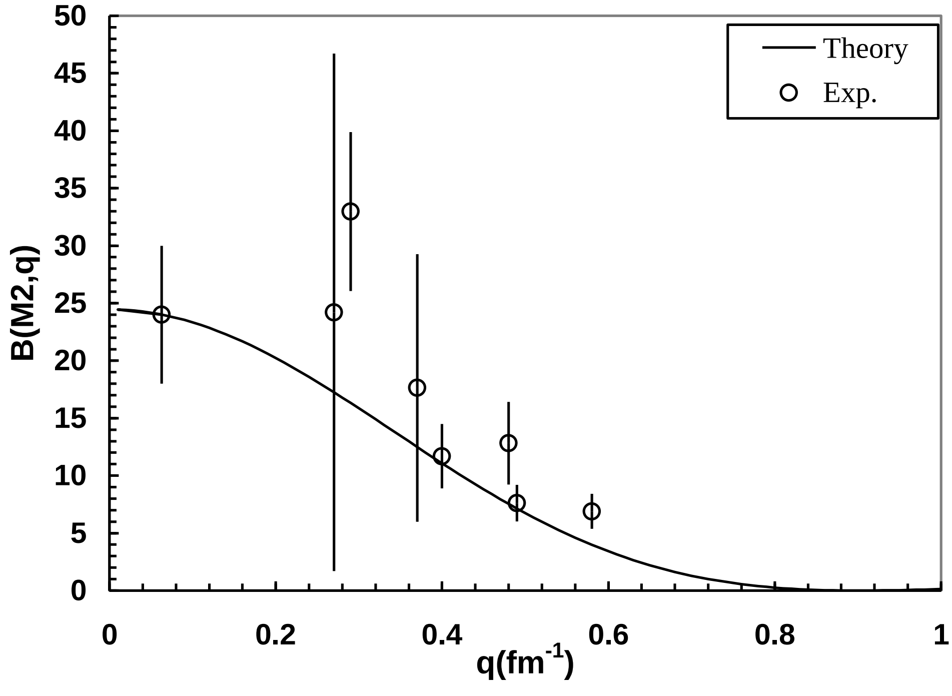

Fig.(7b):- Reduced transition probability (in units of $\mu_N^2\ fm^2$) for M2 isovector transition to the 2^- state at 12.4 MeV in ^{30}Si .

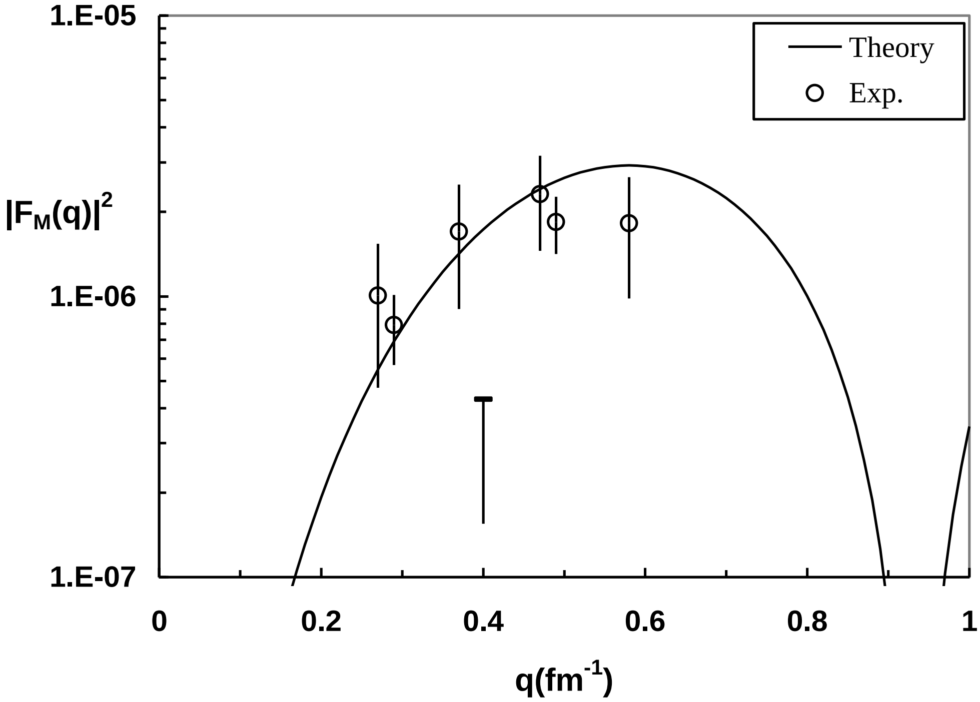

Fig.(8a):- M2 form factors for the isovector transition to the 2^- state at 12.7 MeV in ^{30}Si .

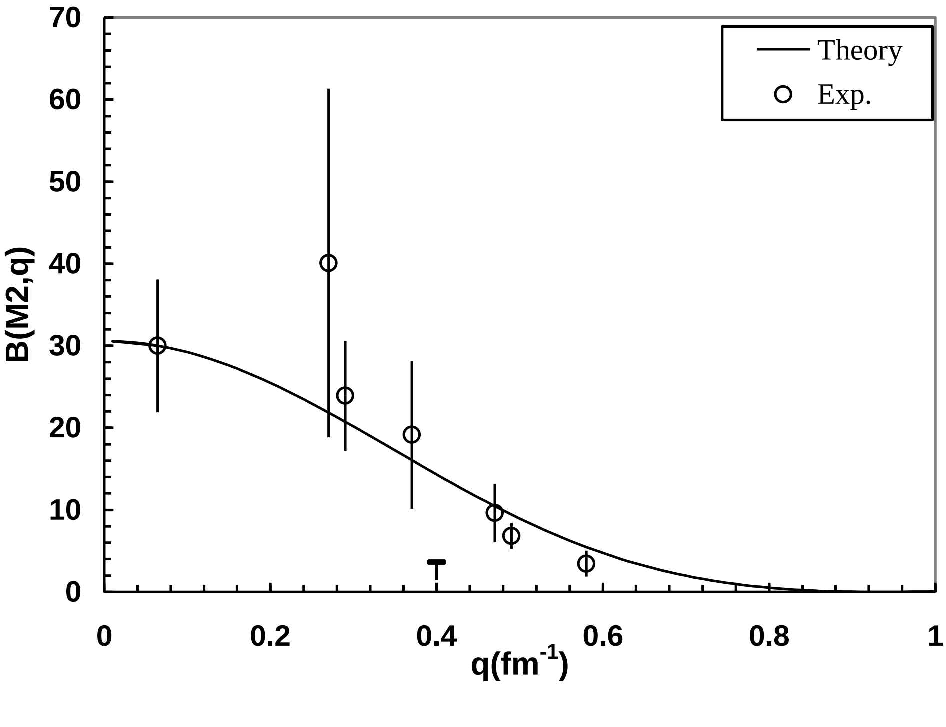

Fig.(8b):- Reduced transition probability (in units of $\mu_N^2\ fm^2$) for M2 isovector transition to the 2^- state at 12.7 MeV in ^{30}Si .

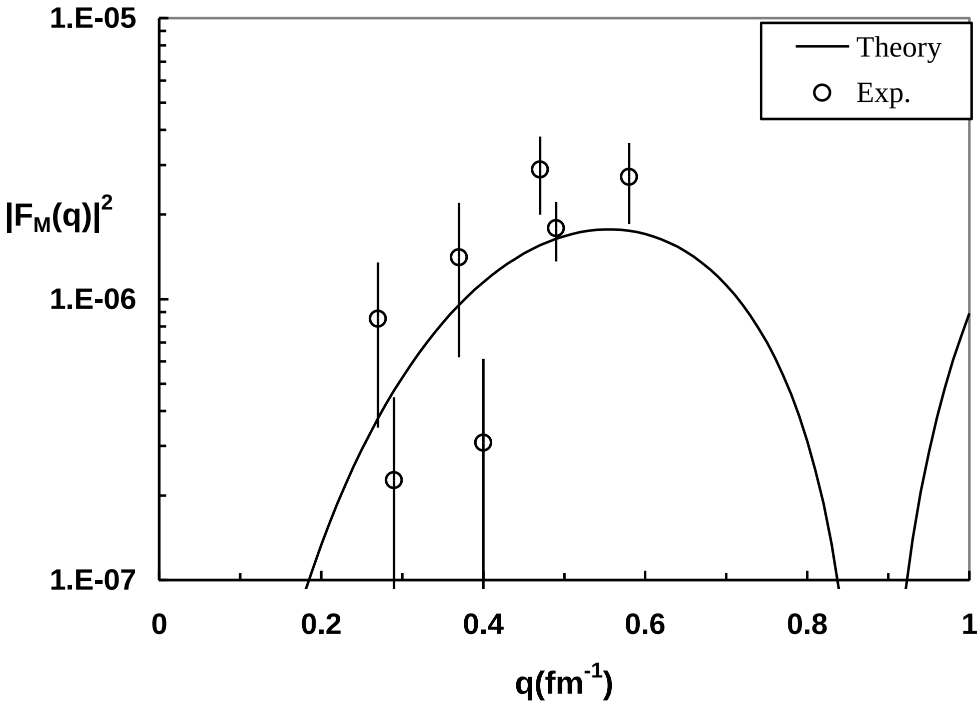

Fig.(9a):- M2 form factors for the isovector transition to the 2^- state at 12.83 MeV in ^{30}Si .

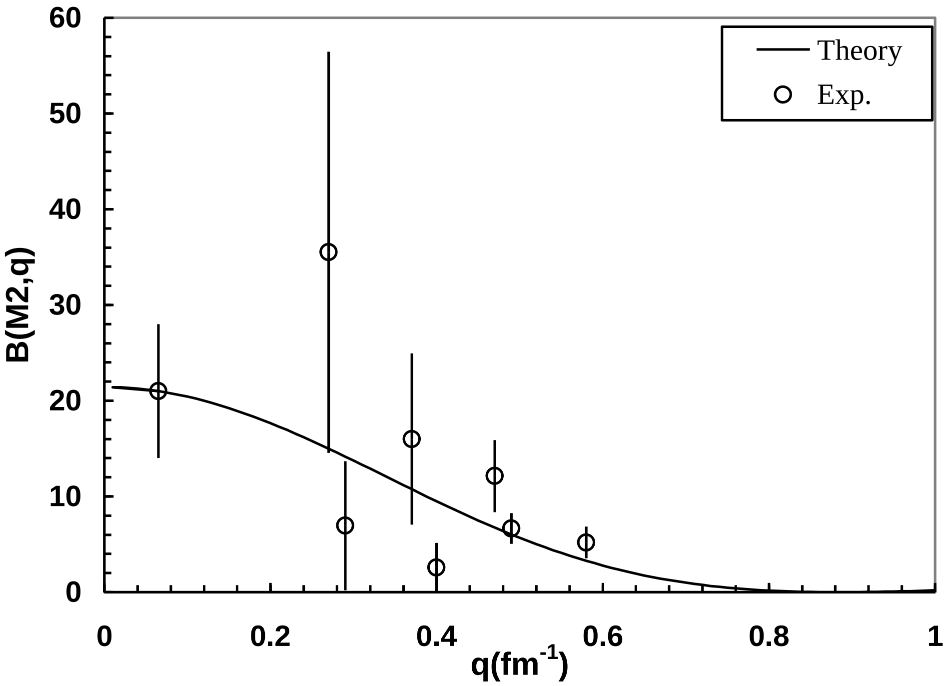

Fig.(9b):- Reduced transition probability (in units of $\mu_N^2\ fm^2$) for M2 isovector transition to the 2^- state at 12.83 MeV in ^{30}Si .

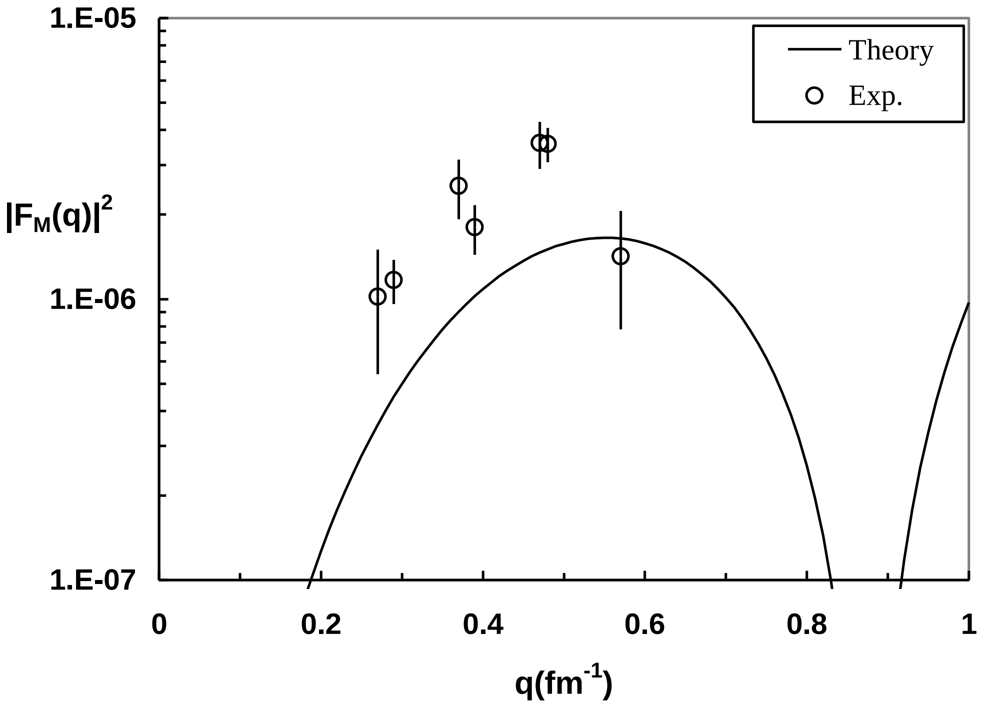

Fig.(10a):- M2 form factors for the isovector transition to the 2^- state at 13.14 MeV in ^{30}Si .

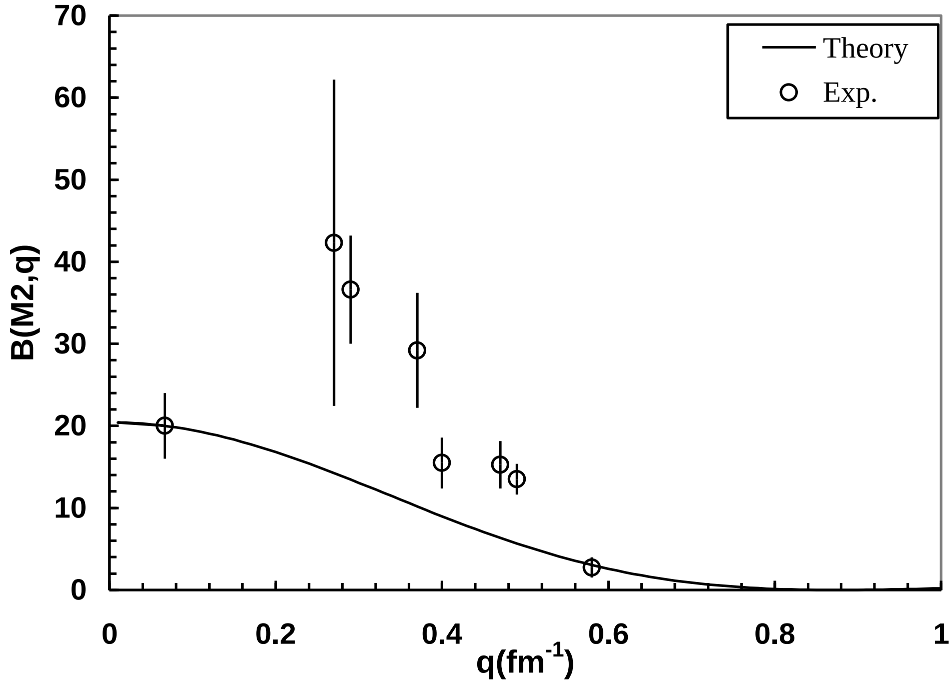

Fig.(10b):- Reduced transition probability (in units of $\mu_N^2\ fm^2$) for M2 isovector transition to the 2^- state at 13.14 MeV in ^{30}Si .

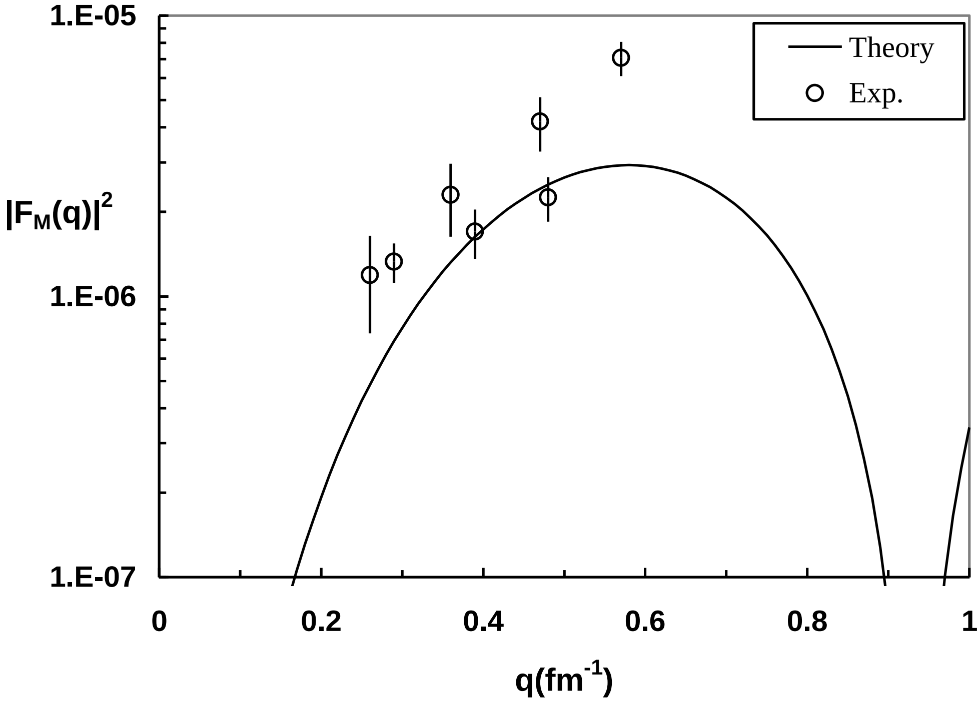

Fig.(11a):- M2 form factors for the isovector transition to the 2^- state at 13.4 MeV in ^{30}Si .

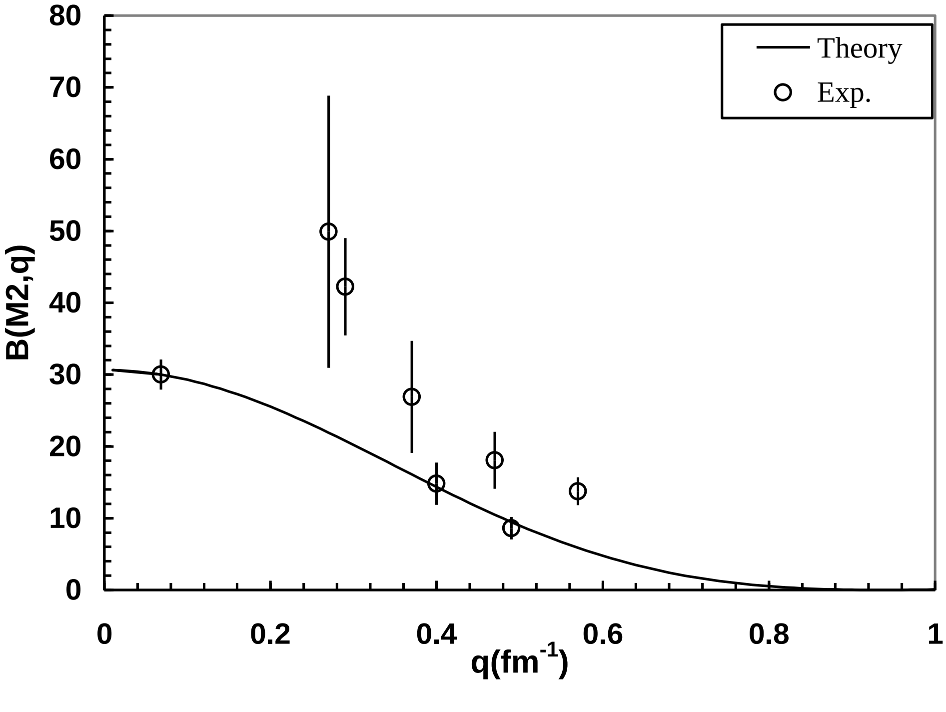

Fig.(11b):- Reduced transition probability (in units of $\mu_N^2\ fm^2$) for M2 isovector transition to the 2^- state at 13.4 MeV in ^{30}Si .

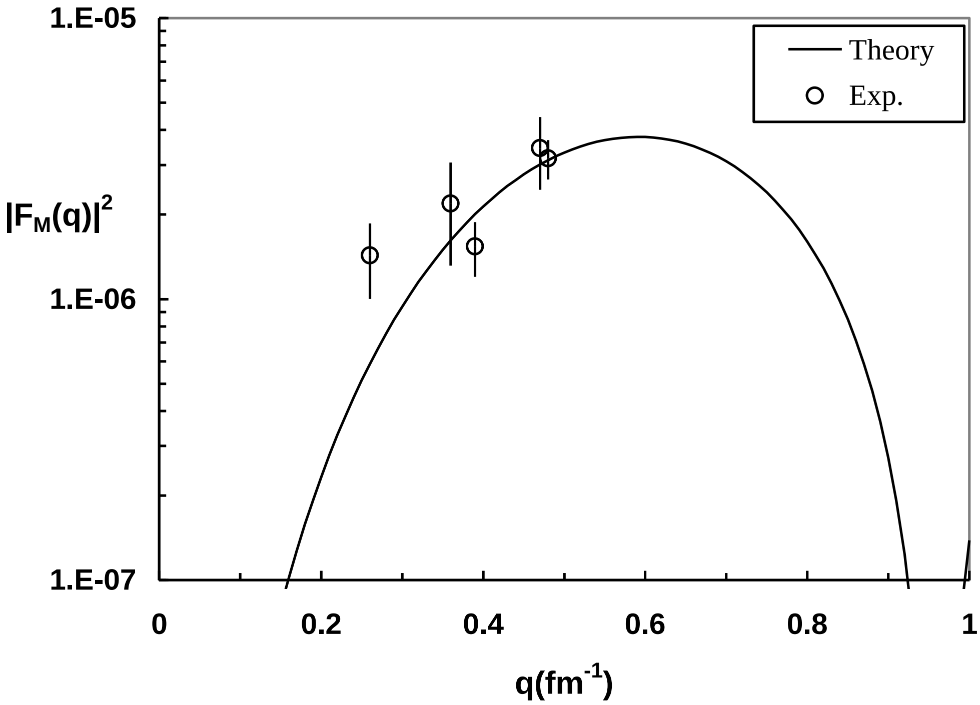

Fig.(12):- M2 form factors for the isovector transition to the 2^- state at 14.0 MeV in ^{30}Si .

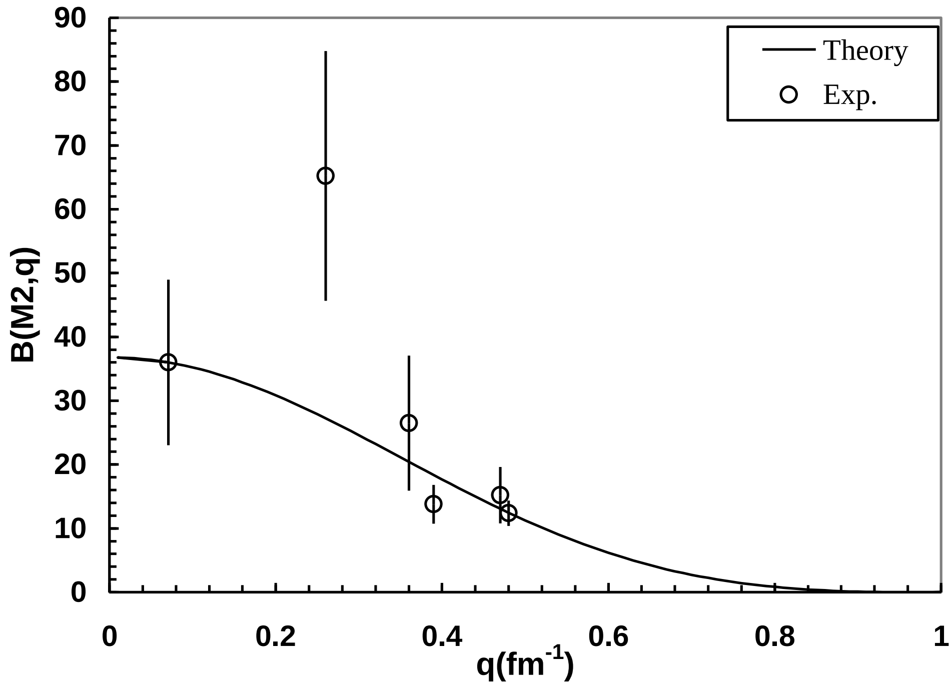

Fig.(12b):- Reduced transition probability (in units of $\mu_N^2\ fm^2$) for M2 isovector transition to the 2^- state at 14.0 MeV in ^{30}Si .

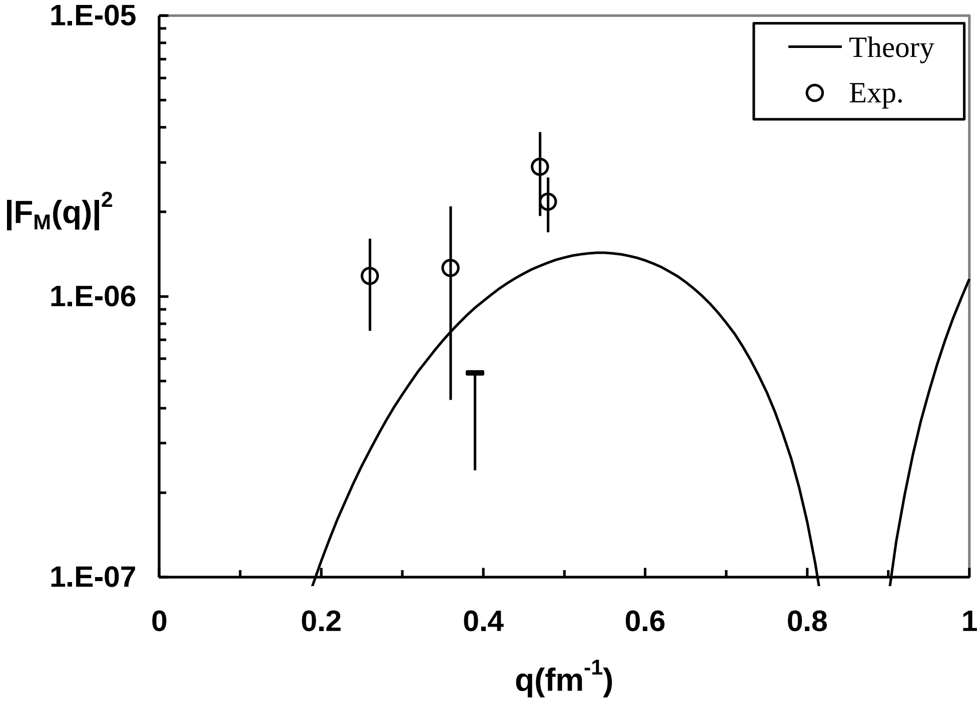

Fig.(13a):- M2 form factors for the isovector transition to the 2^- state at 14.19 MeV in ^{30}Si .

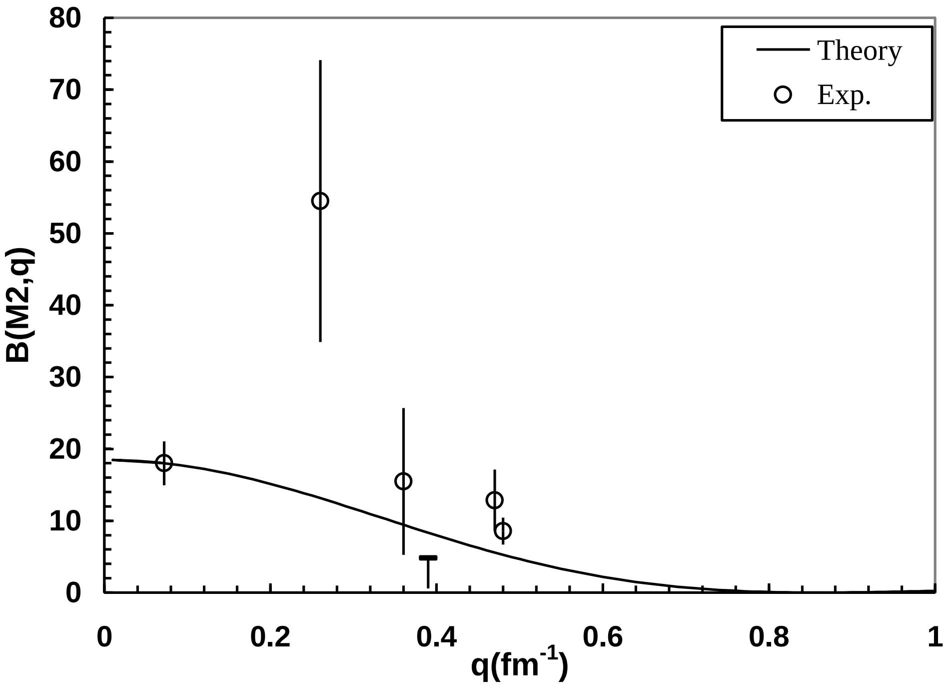

Fig.(13b):- Reduced transition probability (in units of $\mu_N^2 \ fm^2$) for M2 isovector transition to the 2^- state at 14.19 MeV in ^{30}Si .