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Covariates In One –Way Multivariate Repeated Measurements Analysis Of Covariance (Ancova) Model

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Abstract

This research is devoted to the study theOne -Way Multivariate repeated measurements analysis of covariance model (MRM ANCOVA), which contains one between-units factor (Group with q levels) incorporating one random effect, one within-units factor (Time with p levels) and two covariates (Z_1, Z_2). For this model the covariates are time-independent, that is measured only once. The test statistics of various hypotheses on between-unites factor, within-units factor and the interaction between them are given, and including the application to study the assessment of water quality of the northern part of shatt AL-Arab river at Basra,Iraq and determine its suitability for aquatic environment ,drinking and irrigation, through studying some variables including the chemical ,physical and biological quality variables that have the greatest effect on water quality in order to find the index of water quality,which is available in the experiment during one year through twelve months.

Key words :One-Way Multivariate Repeated Measures Model,Wilks distribution, Wishart distribution,(ANCOVA) analysis of covariance , covariates.

1. Introduction

Repeated measurements analysis is widely used in many fields, for example, the health and life sciences, epidemiology, biomedical, agricultural, industrial, psychological, psychology and education and so on. (see, Huynh and Feldt (1970)[7], and Vonesh and Chinchilli(1997)[11]). “Repeated measurements” is a term used to describe data in which the response variable for each experimental unit is observed on multiple occasions and possibly under different experimental conditions (see, Vonesh and Chinchilli (1997)[11]. Repeated measurements analysis of variance, often referred to as randomized block and split-plot designs (see Bennett and Franklin (1954)[4], Sendeecor and Cochran (1967)[10])." Repeated measures designs involving two or more independent groups are among the most common experimental designs in a variety of research settings. Various statistical procedures have been suggested for analyzing data from split-plot designs when parametric model assumptions are violated (see, Brunner and Langer (2000)[5])

AL-Mouel (2004) [1] studied a one-way multivariate repeated measurement analysis of variance (MRM ANOVA) model for complete data.AL-Moueland Wang (2004)[2] studied the sphericity test for one-way multivariate repeated measurements analysis of variance model.Jassim (2006)[8] studied a two-way multivariate repeated measurement analysis of variance (MRM ANOVA) model for complete data ,with two within –units factors which he labeled as "Time "and "day" ,and two between-units factors (factor A factor B).Falhy (2008) [6] studied a one-way multivariate repeated measurement analysis of variance (MRM ANOVA) model for complete data. Also she studied the problems of the testing hypotheses of a one-way multivariate repeated measurements analysis of variance (MRM ANCOVA) model for complete data and the test statistics of various hypotheses on between-units factors and within-units factors by using test statistic namely the multivariate Wilks test.AL-Moueland Fakhir(2011)[3] studied a two-way multivariate repeated measurement analysis of covariance (MRM ANOVA) which contains two between-unites factors (factor A and factor B), two within-units factors (Time and Day) and two covariates (Z_1, Z_2) .For this model the two covariates are time-independent, that is measured only once. Also they studied the test statistics of various hypotheses on between-unites factors, within-units factors and the interaction between them is given, and including the application to study the effect of some internal and external factors on blood parameters, which is available in the experiment during one year through nine months.Naji (2011) [9] studied a one –way multivariat repeated measurement analysis of covariance model (MRM ANCOVA),in this model the covariate is time independent,that is measured only once.Also she studied the test statistics of various hypotheses on between-units factor,within-units factor and the interaction between them is given.

The focusof this paper is to study the One-way multivariate repeated measurements analysis of covariance (MRM ANCOVA) model with two covariates. The test statistics of various hypotheses on between-units factors, within-units factors, and the interaction between them are given respectively by using test statistic namely the multivariate Wilks test.We can conclude from the applications study that the main effect for factor into units (one year and twelve months),the effects of each of the between-units factors , the covariate(Z_1) , the error of covariate (Z_1), the within-units factors and effect to interaction between-unit factor (Group) and within-units factors (Time)are high incorporeal.Used for the extraction of the results MATLAB program.

2.The One –Way MRM with two covariates :

There is a variety of possibilities for the between units factors in a one-way design. In a randomized one-way MRM experiment, the experimental units are randomized to one between units factor (Groups with q levels), one within-units factor(Time with p levels) and two covariates(Z_1, Z_2) . For convenience ,we define the following linear model and the parameterization for the one-way repeated measurements design with one between units factors incorporation two covariates:-

$$y_{ijk} = \mu + \tau_j + \delta_{i(j)} + \gamma_k + (\tau\gamma)_{jk} + (z_{1ij} - \bar{z}_{1..})\beta_1 + (z_{2ij} - \bar{z}_{2..})\beta_2 + e_{ijk} \quad \dots (1)$$

Where

$i = 1, \cdots, n_j$ is an index for experimental unit within group j

$j = 1, \cdots, q$ is an index for levels of the between-units factor (Group).

$k = 1, \cdots, p$ is an index for levels of the within-units factor (Time)

$Y_{ijk} = [Y_{ijk1}, \cdots, Y_{ijkr}]'$ is the response measurement at time k for unit i within group j.

$\mu = [\mu_1, \cdots, \mu_r]'$ is the over all mean.

$\tau_j = [\tau_{j1}, \cdots, \tau_{jr}]'$ is the added effect for group j

$\delta_{i(j)} = [\delta_{i(j)1}, \cdots, \delta_{i(j)r}]'$ is the random effect due to experimental unit i within treatment group j.

$\gamma_k = [\gamma_{k1}, \cdots, \gamma_{kr}]'$ is the added effect for time k .

$(\tau\gamma)_{jk} = [(\tau\gamma)_{jk1}, \cdots, (\tau\gamma)_{jkr}]'$ is the added effect for the group $j \times$ time k interaction,

$Z_{1ij} = [\ Z_{1ij1}, \cdots, Z_{1ijr}]'$ is the value of covariate Z_1 for unit i within groups j ,
$\bar{Z}_{1..} = [\bar{Z}_{1\cdots1}, \cdots, \bar{Z}_{1\cdots r}]$ is the mean of covariate Z_1 over all experimental units,
$\beta_1 = [\beta_{11}, \cdots, \beta_{1r}]'$ is the slope corresponding to covariate Z_1,
$Z_{2ij} = [Z_{2ij1}, \cdots, Z_{2ijr}]'$ is the random value of covariate Z_2 for unit i within groups j,
$\bar{Z}_{2\cdots} = [\bar{Z}_{2\cdots1}, \cdots, \bar{Z}_{2\cdots r}]'$ is the mean of covariate Z_2 over all experimental units,
$\beta_2 = [\beta_{21}, \cdots, \beta_{2r}]'$ is the slope corresponding to covariate Z_2,
$e_{ijk} = [e_{ijk1}, \cdots, e_{ijkr}]'$ is the random error on time k for unit i within group j.
For the parameterization to be of full rank, we imposed the following set of conditions :

$$\sum_{j=1}^{q} \tau_j = 0 \sum_{k=1}^{p} \gamma_k = 0$$

$$\sum_{j=1}^{q} (\tau\gamma)_{jk} = 0\, foreachk = 1, \cdots, p$$

$$\sum_{k=1}^{p} (\tau\gamma)_{jk} = 0\, foreachj = 1, \cdots, q \quad \dots (2)$$

$$\sum_{i=1}^{n_j} z_{1ij} = 0\, foreachj = 1, \cdots, q$$

$$\sum_{j=1}^{q} z_{1ij} = 0\, foreachi = 1, \cdots, n_j$$

And we assume that the $e_{ijk}^{'s}$,$\delta_{i(j)}^{'s}$ and$Z_{2ij}^{'s}$ are independent with

$$e_{ijk} = [e_{ijk1}, \cdots, e_{ijkr}]' \sim i.i.d.N_r(0, \Sigma_e),$$

$$\delta_{i(j)} = [\delta_{i(j)1}, \cdots, \delta_{i(j)r}]' \sim i.i.d.N_r(0, \Sigma_\delta) \quad \dots \quad (3)$$

$$Z_{2ij} = [Z_{2ij1}, \cdots, Z_{2ijr}]' \sim i.i.d.N_r(0, \Sigma_{z_2})$$

whereΣ_ϵ, Σ_δ and Σ_{z_2} are $r \times r$ positive definite matrices.
$LetY_{ij} = [Y_{ij1}, Y_{ij2}, \cdots, Y_{ijp}]$, $i.e.$

$$Y_{ij} = \begin{bmatrix} Y_{ij11} & Y_{ij21}\cdots & Y_{ijp1} \\ Y_{ij12} & Y_{ij22}\cdots & Y_{ijp2} \\ \vdots & \vdots\ddots & \vdots \\ Y_{ij1r} & Y_{ij2r}\cdots & Y_{ijpr} \end{bmatrix} \dots (4)$$

Let the variance- covariance matrix of $\vec{Y}_{ij}$ be denoted by $\sum$, where

$$\vec{Y}_{ij} = Vec(Y_{ij}).$$

The $Vec(\cdot)$ operator creates a column vector from the matrixY_{ij} by simply stacking the column vectors of Y_{ij} one below another .
Where the variance- covariance matrix $\sum$ of the model (2.1) satisfies the assumption of compound symmetry, i.e.

$$\Sigma = I_p \otimes \Sigma_e + J_P \otimes \Sigma_\delta + J_p \otimes \Sigma_{Z_2}$$

$$= \begin{bmatrix} \Sigma_e + \Sigma_\delta + \Sigma_{z_2} & \Sigma_\delta + \Sigma_{z_2} & \cdots & \Sigma_\delta + \Sigma_{z_2} \\ \Sigma_\delta + \Sigma_{z_2} & \Sigma_e + \Sigma_\delta + \Sigma_{z_2} & \cdots & \Sigma_\delta + \Sigma_{z_2} \\ \vdots & \vdots & \ddots & \vdots \\ \Sigma_\delta + \Sigma_{z_2} & \Sigma_\delta + \Sigma_{z_2} & \cdots & \Sigma_e + \Sigma_\delta + \Sigma_{z_2} \end{bmatrix} \quad \dots(5)$$

where
I_p denotes the $p \times p$ identity matrix and J_p is a matrix of $1's$.
$\otimes$ is the Kronecker product operation of two matrices.
So

$$e_{ij} = \left[e_{ij1}, \cdots, e_{ijr}\right]' \sim i.i.d.\, N_{p\times r}\left(0, I_p \otimes \Sigma_e\right) \qquad \ldots \quad (6)$$

3. Transforming the One-Way Multivariate Repeated Measurements Analysis of Covariance (ANCOVA) Model

In this section, we use an orthogonal matrix to transform the observations Y_{ijk} for $i = 1, \cdots, n_j$, $j = 1, \cdots, q$, $k = 1, \cdots, p$. Let U^* be any $p \times p$ orthogonal matrix. It is partitioned as follows:

$$U^* = \left(p^{\frac{-1}{2}} j_p U\right) \ldots (7)$$

where j_p denotes the $p \times 1$ vector of one's , U is $p \times (p-1)$ matrix, $U' j_p = 0$ and $U'U = I_{p-1}$

let $Y_{ij}^* = Y_{ij} U^*$

$$\left[Y_{ij1}^*, Y_{ij2}^*, \cdots, Y_{ijp}^*\right]' = \left[Y_{ij1}, Y_{ij2}, \cdots, Y_{ijp}\right]' U^*$$

$$\begin{bmatrix} Y_{ij11}^* & \cdots & Y_{ijp1}^* \\ \vdots & \ddots & \vdots \\ Y_{ij1r}^* & \cdots & Y_{ijpr}^* \end{bmatrix} = \begin{bmatrix} Y_{ij11} & \cdots & Y_{ijp1} \\ \vdots & \ddots & \vdots \\ Y_{ij1r} & \cdots & Y_{ijpr} \end{bmatrix} \left[p^{\frac{-1}{2}} j_p U\right] \ldots (8)$$

So

$$Cov\left(\vec{Y}_{ij}^*\right) = Cov\left(\vec{Y}_{ij} U^*\right) = Cov\left(\left(U^{*'} \otimes I_r\right)\vec{Y}_{ij}\right)$$

$$= \left(U^{*'} \otimes I_r\right)\Sigma\left(U^* \otimes I_r\right) \qquad \ldots (9)$$

$$= \left(U^{*'} \otimes I_r\right)\left(I_p \otimes \Sigma_e + J_p \otimes \Sigma_\delta + J_p \otimes \Sigma_{Z_2}\right)\left(U^* \otimes I_r\right)$$

We can write the above in the following matrix form:

$$COV\left(\vec{Y}_{ij}^*\right) = \begin{bmatrix} \Sigma_e + p(\Sigma_\delta + \Sigma_{Z_2}) & 0 & \cdots & 0 \\ 0 & \Sigma_e & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \Sigma_e \end{bmatrix} \qquad \ldots (10)$$

4. Analysis of covariance (ANCOVA) for the One-Way Multivariate Repeated Measurements Model

In this section, we study the ANCOVA for the effects of between-units factors and within-units factors for the one-way RM model (1.1). Also we give the null hypotheses which is concerned with these effects and the interaction between them, and the test statistics for them.

Now

$$Y_{ij}^* = Y_{ij} U^*$$

$$Y_{ij1}^* = \left(Y_{ij}\right)\left(P^{-\frac{1}{2}} j_p\right)$$

$$\therefore Y_{ij1}^* = \begin{bmatrix} Y_{ij11}^* \\ Y_{ij12}^* \\ \vdots \\ Y_{ij1r}^* \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{p}} \sum_{k=1}^{p} Y_{ijk1} \\ \frac{1}{\sqrt{p}} \sum_{k=1}^{p} Y_{ijk2} \\ \vdots \\ \frac{1}{\sqrt{p}} \sum_{k=1}^{p} Y_{ijkr} \end{bmatrix}$$

From (1.1), we obtain

$$Y_{ij1}^* = p^{\frac{-1}{2}} \sum_{k=1}^{p} Y_{ijk}$$

$$= p^{\frac{-1}{2}} \sum_{k=1}^{p} \left(\mu + \tau_j + \delta_{i(j)} + \gamma_k + (\tau\gamma)_{jk} + \left(z_{1ij} - \bar{z}_{1..}\right)\beta_1 + \left(z_{2ij} - \bar{z}_{2..}\right)\beta_2 + e_{ijk}\right)$$

$$= P^{\frac{1}{2}}\mu + P^{\frac{1}{2}}\tau_j + P^{\frac{1}{2}}\delta_{i(j)} + P^{\frac{1}{2}}(z_{1ij} - \bar{z}_{1..})\beta_1 + P^{\frac{1}{2}}(z_{2ij} - \bar{z}_{2..})\beta_2 + P^{\frac{1}{2}}\sum_{k=1}^{p} e_{ijk}$$

$$= \mu^* + \tau_j^* + \delta_{i(j)}^* + (z_{1ij}^* - \bar{z}_{1..}^*)\beta_1^* + (z_{2ij}^* - \bar{z}_{2..}^*)\beta_2^* + e_{ij1}^*$$

Then the set of vectors

$(\boldsymbol{Y}_{\boldsymbol{111}}^*, \cdots, \boldsymbol{Y}_{\boldsymbol{n_1 11}}^*)', (\boldsymbol{Y}_{\boldsymbol{121}}^*, \cdots, \boldsymbol{Y}_{\boldsymbol{n_1 21}}^*)', \cdots, (\boldsymbol{Y}_{\boldsymbol{1q1}}^*, \cdots \boldsymbol{Y}_{\boldsymbol{n_q q1}}^*)'$

Have mean vectors

$X_1 = \sqrt{P}\mu + \sqrt{P}\tau_1 + \sqrt{P}(Z_{1ij} - \bar{Z}_{1..}^*)\beta_1$

$X_2 = \sqrt{P}\mu + \sqrt{P}\tau_2 + \sqrt{P}(Z_{1ij} - \bar{Z}_{1..}^*)\beta_1$

$\vdots$

$X_q = \sqrt{P}\mu + \sqrt{P}\tau_q + \sqrt{P}(Z_{1iq} - \bar{Z}_{1..}^*)\beta_1$

Respectively, and each of them has covariance matrix $\Sigma_e + P\Sigma_\delta + P\Sigma_{Z_2}$.

So, the null hypotheses of the same treatment effects are:

$$H_{01} = \tau_1^* + (z_{1ij}^* - \bar{z}_{1..}^*) = \cdots = \tau_q^* + (z_{1ij}^* - \bar{z}_{1..}^*) = 0$$

And are equivalent to the null hypothesis for the same average vector

$$H_{01} = X_1 = X_2 = \cdots = X_q = 0$$

The ANCOVA based on the set of transformed observations above the $Y_{ijk1}^{*,s}$ provides the ANCOVA for between-units effects. This leads to the following form for the sum squares terms:

$SS_G, SS_{Z_1}, SS_{u(Group\ Z_1)}$.

$$SS_G = \sum_{j=1}^{q} n_j (\bar{Y}_{j1}^* - \bar{Y}_1^*)(\bar{Y}_{j1}^* - \bar{Y}_1^*)'$$

$$SS_{Z_1} = \sum_{j=1}^{q}\sum_{i=1}^{n_j} \frac{1}{\beta_1}[(\bar{Y}_{ij1}^* - \bar{Y}_1^*)(\bar{Y}_{ij1}^* - \bar{Y}_1^*)']$$

$$SS_{u(Group\ Z_1)} = \sum_{j=1}^{q}\sum_{i=1}^{n_j} (\bar{Y}_{ij1}^* - \bar{Y}_{ij}^* - \bar{Y}_{j1}^* + \bar{Y}_j^*)(\bar{Y}_{ij1}^* - \bar{Y}_{ij}^* - \bar{Y}_{j1}^* + \bar{Y}_j^*)'$$

Where

$\bar{Y}_{j1}^* = \frac{\sum_{i=1}^{n_j} Y_{ij1}^*}{n_j}$, $\bar{Y}_1^* = \frac{\sum_{j=1}^{q}\sum_{i=1}^{n_j} Y_{ij1}^*}{n}$, $\bar{Y}_{ij1}^* = \frac{\sum_{j=1}^{q}\sum_{i=1}^{n_j} Y_{ij1}}{n_j q}$

Thus

$$SS_G \sim W_r(q-1, \Sigma_e + p(\Sigma_\delta + \Sigma_{Z_2}))$$

$$SS_{Z_1} \sim W_r(1, \Sigma_e + p(\Sigma_\delta + \Sigma_{Z_2}))$$

$$SS_{U(Group\ Z_1)} \sim W_r(n-q-1, \Sigma_e + p(\Sigma_\delta + \Sigma_{Z_2}))$$

where W_r denotes the multivariate-Wishart distribution.

The multivariate Wilks test [12].

$T_{w_1} = \frac{|SS_{U(Group\ Z_1)}|}{|SS_{U(Group\ Z_1)} + SS_G|}$, when H_{01} is true , $T_{w_1} \sim \Lambda_r(n-q-1, q-1)$

$T_{w_2} = \frac{|SS_{U(Group\ Z_1)}|}{|SS_{U(Group\ Z_1)} + SS_{Z_1}|}$, when H_{01} is true , $T_{w_2} \sim \Lambda_r(n-q-1, 1)$

The ANCOVA based on the set of transformed observations the $Y_{ijk}^{*,s}$ for each

$k = 2, \dots, p$

$$\text{i.e}Y^*_{ijk} = \begin{bmatrix} Y^*_{112} & Y^*_{113} & \cdots & Y^*_{11P} \\ Y^*_{212} & Y^*_{213} & \cdots & Y^*_{21P} \\ \vdots & \vdots & & \vdots \\ Y^*_{n_1 12} & Y^*_{n_1 13} & \dots & Y^*_{n_1 1p} \\ Y^*_{122} & Y^*_{123} & \cdots & Y^*_{12p} \\ Y^*_{222} & Y^*_{223} & \dots & Y^*_{22p} \\ \vdots & \vdots & \cdots & \vdots \\ Y^*_{n_2 22} & Y^*_{n_2 23} & \cdots & Y^*_{n_2 2p} \\ \vdots & \vdots & \cdots & \vdots \\ Y^*_{1q2} & Y^*_{1q3} & \cdots & Y^*_{1qp} \\ \vdots & \vdots & \cdots & \vdots \\ Y^*_{n_q q2} & Y^*_{n_q q3} & & Y^*_{n_q qp} \end{bmatrix}$$

has the model

$$Y^*_{ijk} = \sum_{k'=1}^{p} U_{kk'} Y_{ijk}$$

$$= \sum_{k'}^{p} U_{kk'}\left(\mu + \tau_j + \delta_{i(j)} + \gamma_{k'} + (\tau\gamma)_{jk'} + \left(Z_{1ij} - \bar{Z}_{1..}\right)\beta_1 + \left(Z_{2ij} - \bar{Z}_{2..}\right)\beta_2 + e_{ijk'}\right) \dots (12)$$

Because the components of $\left(\boldsymbol{u_{k1}}, \cdots, \boldsymbol{u_{kp}}\right)$sum to zero for each

$$k = 2, \cdots, p$$

Accordingly, the model k^{th} in (2.12) is equivalent to

$$Y^*_{ijk} = \gamma^*_k + (\tau\gamma)^*_{jk} + \left(z^*_{1ij} - \bar{z}^*_{1..}\right)\beta^*_1 + +e^*_{ij1} \dots \quad (13)$$

where

$[\gamma^*_2 + \left(z^*_{1ij} - \bar{z}^*_{1..}\right)\beta^*_1, \dots, \gamma^*_p + \left(z^*_{1ij} - \bar{z}^*_{1..}\right)\beta^*_1] =$

$$[\gamma_1 + \left(z_{1ij} - \bar{z}_{1..}\right)\beta_1, \dots, \gamma_p + \left(z_{1ij} - \bar{z}_{1..}\right)\beta_1]U^*$$

It is clear that when

$$\gamma_1 + \left(z_{1ij} - \bar{z}_{1..}\right) = \cdots = \gamma_p + \left(z_{1ij} - \bar{z}_{1..}\right) = 0$$

then

$$\gamma^*_2 + \left(z^*_{1ij} - \bar{z}^*_{1..}\right), \dots, \gamma^*_p + \left(z^*_{1ij} - \bar{z}^*_{1..}\right) = 0$$

inanother way$\sum_{k=1}^{p} \gamma_k = 0$

$[0, \gamma^*_2 + \left(z^*_{1ij} - \bar{z}^*_{1..}\right), \dots, \gamma^*_p + \left(z^*_{1ij} - \bar{z}^*_{1..}\right)] =$

$$[\gamma_1 + \left(z_{1ij} - \bar{z}_{1..}\right), \dots, \gamma_p + \left(z_{1ij} - \bar{z}_{1..}\right)]U^*$$

Then

$$\left[\gamma_1 + \left(z_{1ij} - \bar{z}_{1..}\right), \dots, \gamma_p + \left(z_{1ij} - \bar{z}_{1..}\right)\right] =$$
$$\left[0, \gamma^*_2 + \left(z^*_{1ij} - \bar{z}^*_{1..}\right), \dots, \gamma^*_p + \left(z^*_{1ij} - \bar{z}^*_{1..}\right)\right]U^{*\prime}$$

Which implies

$$\gamma^*_2 + \left(z^*_{1ij} - \bar{z}^*_{1..}\right), \dots, \gamma^*_p + \left(z^*_{1ij} - \bar{z}^*_{1..}\right) = 0$$

Therefore, the hypothesis

$$H_{02}: \gamma_1 + \left(z_{1ij} - \bar{z}_{1..}\right) = \cdots = \gamma_p + \left(z_{1ij} - \bar{z}_{1..}\right) = 0$$

That are equivalent to the hypothesis

$\mathrm{H}_{02}: \gamma^*_2 + \left(z^*_{1ij} - \bar{z}^*_{1..}\right), \dots, \gamma^*_p + \left(z^*_{1ij} - \bar{z}^*_{1..}\right) = 0$

Similarity of each j

$$[(\tau\gamma)^*_{j2} + (\bar{Z}^*_{1ij} - \bar{Z}^*_{1..}), \dots, (\tau\gamma)^*_{jp} + (\bar{Z}^*_{1ij} - \bar{Z}^*_{1..})] = [(\tau\gamma)_{j1} + (Z_{1ij} - \bar{Z}_{1..}), \dots, (\tau\gamma)_{jp} + (Z_{1ij} - \bar{Z}_{1..})]U^*$$

Andhypothesis

$$H_{03}: (\tau\gamma)_{j1} + (Z_{1ij} - \bar{Z}_{1..}) = \cdots = (\tau\gamma)_{jp} + (Z_{1ij} - \bar{Z}_{1..}) = 0$$

Be equivalent to the hypothesis

$$H_{03}: (\tau\gamma)^*_{j2} + (\bar{Z}^*_{1ij} - \bar{Z}^*_{1..}), \dots, (\tau\gamma)^*_{jp} + (\bar{Z}^*_{1ij} - \bar{Z}^*_{1..}) = 0$$

The ANCOVA based on the set of transformed observations above , the $[Y^*_{ij2}, \cdots, Y^*_{ijp}]$ provides the ANCOVA for within-units effects. This leads to the following forms for the sum squares terms :

$$SS_{Time} = \sum_{k=2}^{p} n(\bar{Y}^*_K(\bar{Y}^*_K)'$$

$$SS_{Time \times Group} = \sum_{k=2}^{p}\sum_{j=1}^{q} n_j (\bar{Y}^*_{jk} - \bar{Y}^*_k)(\bar{Y}^*_{jk} - \bar{Y}^*_k)'$$

$$SS_{Time \times Unit\,(Group\ Z_1)} = \sum_{k=2}^{p}\sum_{j=1}^{q}\sum_{i=}^{n_j} (\bar{Y}^*_{ijk} - \bar{Y}^*_{jk} - \bar{Y}^*_{ij} + \bar{Y}^*_j)(\bar{Y}^*_{ijk} - \bar{Y}^*_{jk} - \bar{Y}^*_{ij} + \bar{Y}^*_j)'$$

Where

$$\bar{Y}^*_k = \frac{\sum_{j=1}^{q} n_j Y^*_{jk}}{n} \qquad ,k = 2, \dots, p$$

$$\bar{Y}^*_{jk} = \frac{\sum_{i=1}^{n_j} Y^*_{ijk}}{n_j} \qquad ,k = 2, \dots, p$$

Then from the above sum squares terms , we have :

$$SS_{Time} \sim W_r(q-1, \Sigma_e)$$

$$SS_{Time \times Grup} \sim W_r((q-1)(q-1), \Sigma_e)$$

$$SS_{Time \times Unit\,(Grup\ Z_1)} \sim W_r((q-1)(n-q), \Sigma_e)$$

The multivariate Wilks test :

$$T_{w3} = \frac{|SS_{Time \times Unit\ (Group\ Z_1)}|}{|SS_{Time \times Unit\ (Group\ Z_1)} + SS_{Time}|} \quad \text{, when } H_{02} \text{ is true}$$

$$T_{w3} \sim \Lambda_r((p-1)(n-q), (p-1))$$

$$T_{w4} = \frac{|SS_{Time \times Unit\ (Group\ Z_1)}|}{|SS_{Time \times Unit\ (Group\ Z_1)} + SS_{Time \times Group}|} \quad \text{, when } H_{03} \text{ is true}$$

$$T_{w4} \sim \Lambda_r((p-1)(n-q), (p-1)(q-1))$$

The One-way MRM ANCOVA with one between-unit factor (Group) and two covariates (Z_1, Z_2) that are time-independent.

	Source	D.F	SS	Wilks Criterion
between	Group	$q-1$	SS_G	$T_{w_1} = \frac{\lvert SS_{U(Group\ Z_1)}\rvert}{\lvert SS_{U(Group\ Z_1)} + SS_G\rvert}$
	Z_1	1	SS_{Z_1}	$T_{w_2} = \frac{\lvert SS_{U(Group\ Z_1)}\rvert}{\lvert SS_{U(Group\ Z_1)} + SS_{Z_1}\rvert}$
	Unit($GroupZ_1$)	$n-q-1$	$SS_{u(Group\ Z_1)}$	
within	Time	$p-1$	SS_{Time}	$T_{w3} = \frac{\lvert SS_{Time\times Unit\ (Group\ Z_1)}\rvert}{\lvert SS_{Time\times Unit\ (Group\ Z_1)} + SS_{Time}\rvert}$
	Time×Group	$(p-1)(q-1$	$SS_{Time\times Group}$	$T_{w4} = \frac{\lvert SS_{Time\times Unit\ (Group\ Z_1)}\rvert}{\lvert SS_{Time\times Unit\ (Group\ Z_1)} + SS_{Time\times Group}\rvert}$
	Time×Unit(GroupZ_1)	$(p-1)(n-q)$	$SS_{Time\times Unit\ (Group\ Z_1)}$	

5. The Experiment

The experiment was carried out to study the assessment of water quality of the northern part of shatt AL-Arab river at Basra,Iraq and determine its suitability for aquatic environment ,drinking and irrigation. For the area located to the south of Quran up to the Abul-Khaseeb in the south through studying some variables including the chemical ,physical and biological quality variables that have the greatest effect on water quality in order to find the index of water quality.Among these variables, electrical conductivity, dissolved oxygen, biological oxygen demand and boron were chosen. Water samples from stations during the ebb period (extending from July 2009 to June 2010). The present study deals with the data that above mentioned(Biology department, College of science-University of Basra)study present. The design of the experiment was done according to the one-way multivariate repeated measurements model considering the effects of two independent factors. Multivariance analysis was done by fraction the total differences into two classes or groups:-

The First Class: refers to the differences of the essential elements between the measurement elements (between-units factors) which are out of control**.**

The Second Class: refers to the differences in within-units factors which are reated to the external factors.

According to the mathematical formula of the model study (1) and by applying the model to the experiment, we get the sum squares, of the effects between-units factors

, the covariate(Z_1) , the error of covariate (Z_1), the within-units factors, effect to interaction between-unit factor (Group) and within-units factors (Time), Of the Experiment as follows:

1- The sum squares of the effect of between-units factors to test the hypotheses $H_{01} = \tau_1^* + (z_{1i1}^* - \bar{z}_{1..}^*) = \cdots = \tau_4^* + (z_{1i4}^* - \bar{z}_{1..}^*) = 0$is SS_G which is 12×12 matrix.

The values of Wilks statisticto test the hypothesesH_{01} is

$$T_{w_1} = 0.0464$$

It is clear from the above that the Wilks statistic is distributed the Wilks distribution in the following :

$\Lambda_{01} \sim \Lambda_{4,11,3}$

$$\text{with } F = 3.9189$$

$F_t(12,21,0.05) = 2.2504$

At 0.05 level of significance, we found that the calculated F-value is greater than the statistical test value. This means that there are levels of significance found concentration of the elements among the four groups.

2- The sum squares of the covariate (Z_1) to test the hypotheses : $H_{01} = \tau_1^* + (z_{1i4}^* - \bar{z}_{1..}^*) = \cdots = \tau_4^* + (z_{1i4}^* - \bar{z}_{1..}^*) = 0$is SS_{Z_1}which is 12×12 matrix.

The values of Wilks statistic to test the hypothesesH_{01} is

$T_{w_2} = 0.9455$

the Wilks statistics distributed the Wilks distribution in the following :

$\Lambda_{02} \sim \Lambda_{4,11,1}$

$$\text{with } F = 0.1153$$

$F_t(4,8,0.05) = 3.8378$

0.05 level of significance, we found that the calculated F-value is less than the statistical test value. This means that there is no level of significance of concentration of the elements among the four groups. ,i.e. there are no level of significance of the covariate on concentration of elements

3-The sum squares of the within-units factor is to test the hypotheses

$H_{02}: \gamma_1 + \left(z_{1ij} - \bar{z}_{1..}\right) = \cdots = \gamma_4 + \left(z_{1ij} - \bar{z}_{1..}\right) = 0$is SS_T which is 12×12 matrix

The values of Wilks statistic to test the hypothesesH_{02} is

T_{w3} = 1.8844e-009

the Wilks statistics distributed the Wilks distribution in the following :

$$\Lambda_{03} \sim \Lambda_{4,36,3}$$

$$\text{with } F = 1.4481e + 004$$

$F_t(12,88,0.05) = 2.4961$

0.05 level of significance, we found that the calculated F-value is greater than the statistical test value. This means that there is a level of significance of concentration of the elements within the four groups.

4-Sum squares of the effect to the interaction between-unit factor (Group) and within-units factors (Time) to test the hypotheses

$H_{03}: (\tau\gamma^*)_{11} + (z_{1i1}^* - \bar{z}_{1..}^*) = \cdots = (\tau\gamma^*)_{14} + (z_{1i1}^* - \bar{z}_{1..}^*) = \cdots =$

$\tau\gamma*41+$

$(z_{1i4}^* - \bar{z}_{1..}^*) = \cdots = (\tau\gamma^*)_{44} + (z_{1i4}^* - \bar{z}_{1..}^*) = 0$is $SS_{T\times G}$ which is 12×12 matrix.

The values of Wilks statistics to test the hypothesesH_{03} is

T_{w4} =3.3094e-009

The Wilks statistics distributed the Wilks distribution in the following :

$$\Lambda_{04} \sim \Lambda_{4,36,9}$$

$$\text{with } F = 634.6899$$

$F_t(36,125,0.05) = 1.7628$

0.05 level of significance, we found that the calculated F-value is greater than the statistical test value . This means that there is a level of significance of concentration of the elements within the four groups.

Table 1: The table below shows the analysis of covariance of effects between-units factor and within-units factors to design a model of multivariate measurements one-way and with one factor in the experiment through all units of Wilks aspect:

	Source	**D.F**	**SS**	**Wilks**
Between	Group	3	SS_G	0.0464
	Z_1	1	SS_{Z_1}	0.9455
	Unit($GroupZ_1$)	11	$SS_{u(Group\ Z_1)}$	
Within	Time	3	SS_{Time}	1.8844e-009
	Time×Group	9	$SS_{Time\ \times Grup}$	3.3094e-009
	Time×Unit(GroupZ_1)	36	$SS_{Time\ \times Unit\ (Grup\ Z_1)}$	

Conclusion

The conclusions which are obtained throughout this work are given as follows:

1- The multivariate Wilks statistic for testing the null hypotheses for between-units effects and within-units effects are :

$$T_{w_1} = \frac{|SS_{U(Group\ Z_1)}|}{|SS_{U(Group\ Z_1)}+SS_G|}$$, when H_{01} is true

$$T_{w_1} \sim \Lambda_r(n-q-1, q-1)$$

$$T_{w_2} = \frac{|SS_{U(Group\ Z_1)}|}{|SS_{U(Group\ Z_1)}+SS_{Z_1}|}$$, when H_{01} is true

$$T_{w_2} \sim \Lambda_r(n-q-1, 1)$$

$$T_{w3} = \frac{|SS_{Time\ \times Unit\ (Grup\ Z_1)}|}{|SS_{Time\ \times Unit\ (Grup\ Z_1)}+SS_{Time}\ |}$$, when H_{02} is true

$$T_{w3} \sim \Lambda_r((p-1)(n-q), (p-1))$$

$$T_{w4} = \frac{|SS_{Time\ \times Unit\ (Grup\ Z_1)}|}{|SS_{Time\ \times Unit\ (Grup\ Z_1)}+SS_{Time\ \times Grup}\ |}$$, when H_{03} is true

$$T_{w4} \sim \Lambda_r((p-1)(n-q), (p-1)(q-1))$$

Recommendations

1- We recommend the use of repeated measurements model for ecological experiments because it takes into consideration the correlations between the measurements for the every experimental unit at each period under different experimental circumstances.

2- We recommend much attention should be paid and much fund should be devoted to the scientific research related to waters and predicting their future situation.

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