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Diffractive structure of Electromagnetic form factors in ^{7}Li

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Abstract

The electron scattering form factors of ^{7}Li have been calculated in the limited 1P-shell model space as well as in the extended $6\hbar\omega$ model space through core-polarization effects. The predictions of longitudinal and transverse form factors which includes core-polarization effects to first order were compared with the experimental values. Core polarization matrix elements are calculated with MSDI and M3Y effective interaction respectively. The inclusion of core polarization leads to an enhancement of the calculated form factor, improving the agreement with experiment.

1. Introduction

At the last fifty years, different nuclear models have been adopted. One of these models, is the nuclear shell model, which has been very successful in describing the nuclear structure [1].

Shell model calculations are carried out within a model space in which the nucleons are restricted to occupy few orbits, which are sometime unable to reproduce the experimental data without scaling factors. Thus transition rates or electron scattering form factors in p-shell are not explained by the simple shell model, when a few nucleons are allowed to be distributed over the p-shell orbits outside a closed ^{4}He core[2]. For p-shell nuclei, Cohen-Kurth[3]model explains well the low–energy properties. Inadequacies in the shell model wave functions are revealed by the need to take into account higher configurations which are called core-polarization effects, which intermediate one-particle one-hole excitations outside the model space .These effects are found essential for obtaining a quantitative agreement with the experimental data [4,5].

A microscopic model [6,7] has been used in order to study the core polarization effect on the longitudinal form factors of p-shell nuclei. Such a microscopic model which adopted the first order core polarization was considered to calculate the C2 form factors of the light p-shell nuclei.

The purpose of the present work is to consider a first order core polarization through a microscopic theory which combines shell model wave functions and highly excited states. The effect of core polarization with the modified surface delta interaction (MSDI) [8]and Michigan three Yukawa's(M3Y)[9] residual interaction examined to reproduce the diffraction structure in ^{7}Li. With this model we will not introduce any adjustable parameters.

The strength of the MSDI A_T ,B and C are set equal to $A_0=A_1=B=(40/A)$MeV and C=0. The single particle wave function was those of the harmonic oscillator potential (HO) with size parameter *b* chosen to reproduce the measured ground state root mean square charge radii of these nuclei. The one-body density matrix (OBDM) elements ($\chi_{\Gamma_f\Gamma_i}{}^{\Lambda}(\alpha_f,\alpha_i)$) are calculated using the shell model code OXBASH[10] . The Cohen -Kurath interactions (CKI)[3] used in these calculation with three active nucleons distributed over the p-shell orbits.

2. Theory

The electron scattering form factor for a given multipolarity λ and momentum transfer q is expressed as [11],

$$\left|F_\lambda(q)\right|^2=\frac{1}{2j_i+1}\left(\frac{4\pi}{Z^2}\right)\sum(-1)^{T_f-T_z}\begin{pmatrix}T_f & T & T_I\\ -T_Z & 0 & T_Z\end{pmatrix}\left|\left\langle\Gamma_f \;\|\!\|\; \hat{T}^{\xi}_{\lambda} \;\|\!\|\; \Gamma_i\right\rangle\right|^2 \left| F_{f.s}F_{\ c.m}\ \right|^2 \qquad \text{…..(1)}$$

Where $F_{f.s}=e^{-0.43q^2/4}$ is the finite nucleon-size correction and $F_{c.m}=q^2b^2/4A$ is the center of mass correction, where A is the mass number and b is the harmonic oscillator size parameter.

The effect of the core polarization on the form factors is based on a microscopic theory, which combines shell-model wave functions and configurations with higher energy as particle-hole excitation. The reduced matrix element of the electron scattering operator $\hat{T}^{\xi}_{\lambda}$ is expressed as a sum of the p-shell model space (ms) contribution and the core-polarization (cp) contribution, as follows [6]

$$\left\langle\Gamma_f \;\|\!\|\; \hat{T}^{\xi}_{\lambda} \;\|\!\|\; \Gamma_i\right\rangle=\left\langle\Gamma_f \;\|\!\|\; \hat{T}^{\xi}_{\lambda} \;\|\!\|\; \Gamma_i\right\rangle_{ms}+\left\langle\Gamma_f \;\|\!\|\; \delta\hat{T}^{\xi}_{\lambda} \;\|\!\|\; \Gamma_i\right\rangle_{cp} \qquad \text{…..(2)}$$

with ξ selects the longitudinal (L), electric(E) and magnetic(M) transverse form factors, respectively. The Greek symbols were used to denote quantum numbers in coordinate space and isospace, i.e. $\Gamma_i\equiv J_iT_i$, $\Gamma_f\equiv J_fT_f$ and $\lambda\equiv JT$. The p-shell model space matrix element can be expressed as linear combination of the single-particle matrix element[12],

$$\left\langle\Gamma_f \;\|\!\|\; \hat{T}^{\xi}_{\lambda} \;\|\!\|\; \Gamma_i\right\rangle_{ms}=\sum_{\alpha_f\alpha_i}\chi_{\Gamma_f\Gamma_i}{}^{\lambda}(\alpha_f,\alpha_i)\left\langle\ \alpha_f \;\|\!\|\; \hat{T}^{\xi}_{\lambda} \;\|\!\|\; \alpha_i\ \right\rangle \qquad \text{…..(3)}$$

where $\chi_{\Gamma_f\Gamma_i}{}^{\lambda}(\alpha_f,\alpha_i)$ are the structure factors (one body density matrix element), given by,

$$\chi_{\Gamma_f\Gamma_i}{}^{\lambda}(\alpha_f,\alpha_i)=\frac{\left\langle\Gamma_f \;\|\!\|\; \left[a^+(\alpha_f)\otimes\tilde{a}(\alpha_i)\right]^{\lambda} \;\|\!\|\; \Gamma_i\ \right\rangle}{\sqrt{2\lambda+1}} \qquad \text{…..(4)}$$

α_f and α_i label single-particle states for the p-shell m odel space. Similarly, core-polarization matrix elements are given as follows [13]:

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$$\left\langle\Gamma_f \;\|\!\|\; \delta\hat{T}^{\xi}_{\lambda} \;\|\!\|\; \Gamma_i\right\rangle_{cp}=\sum_{\alpha_f\alpha_i}\chi^{\lambda}\Gamma_f\Gamma_i(\alpha_f,\alpha_i)\left\langle\ \alpha_f \;\|\!\|\; \delta T^{\xi}_{\lambda} \;\|\!\|\; \alpha_i\ \right\rangle \qquad \text{…..(5)}$$

Up to the first order perturbation theory, the single-particle matrix element for the higher-energy configuration is given by [14]

$$\langle\ \alpha_f\ \|\|\ \delta T_\lambda^\xi\ \|\|\ \alpha_i\ \rangle = \langle\ \alpha_f\ \|\|\ \hat{T}_\lambda^\xi \frac{Q}{E_i - H_o} V_{res}\ \|\|\ \alpha_i\ \rangle + \langle\ \alpha\Gamma_f\ \|\|\ V_{res} \frac{Q}{E_f - H_o} \hat{T}_\lambda^\xi\ \|\|\ \alpha\Gamma_i\ \rangle \quad \text{.....(6)}$$

The operator Q is the projection operator on the space outside the model space. E_i and E_f are the energies of initial and final states. For the residual interaction V_{res} the MSDI and M3Y were adopted.

The two terms of Eq.(6)can be written as [8]

$$\sum_{\alpha_1\alpha_2\Gamma} \frac{(-1)^{\alpha_i+\alpha_2+\Gamma}}{e_{\alpha_i} - e_{\alpha_f} - e_{\alpha_1} + e_{\alpha_2}} (2\Gamma+1) \begin{Bmatrix} \alpha_f & \alpha_i & \lambda \\ \alpha_2 & \alpha_1 & \Gamma \end{Bmatrix} \sqrt{(1+\delta_{\alpha_1\alpha_f})(1+\delta_{\alpha_2\alpha_i})} \langle \alpha_2 \|\|T_\lambda\|\| \alpha_1 \rangle \times \langle \alpha_f \alpha_1 | V_{res} | \alpha_i \alpha_2 \rangle_\Gamma + \text{Terms with } \alpha_1 \text{ and } \alpha_2 \text{ exchanged with an over all minus sign,} \quad \text{.....(7)}$$

Where α_1 runs over particle states and α_2 over hole state and e is the single-particle energy . The hole states cover $1s$ core orbit as well as 1p shells.

$$e_{nlj} = (2n + l - \frac{1}{2})\hbar\omega + \begin{cases} -\frac{1}{2}(l+1)\langle f(r)\rangle_{nl} & for \quad j = l - \frac{1}{2}, \\ \frac{1}{2} l \langle f(r)\rangle_{nl} & for \quad j = l + \frac{1}{2}, \end{cases} \quad \text{.....(8)}$$

with $\langle f(r)\rangle_{nl} \approx -20A^{-2/3}$ and $\hbar\omega = 45A^{-1/3} - 25A^{-2/3}$

The electric transition strength is given by,

$$B(C\lambda, k) = \frac{|(2\lambda+1)!!|^2}{4\pi} \frac{Z^2}{k^\lambda} |F_\lambda(k)|^2$$

Where k = E_x /ħc .

3. Results and Discussion

Calculations are presented for ground state 3/2$^-$ 1/2 , 0.478MeV 1/2$^-$ 1/2 and 4.63MeV 7/2$^-$ 1/2 in ^{7}Li.The reduce transition probabilities B(C2↑) values for 1p-shell model and 1p+cp model were tabulated in table 1 as well as the experimental values. The harmonic-oscillator single - particle wave functions with size parameter b=1.65 fm was employed [15]. Fig. 1 shows the calculations for the C2 transition from ground state ($J^\pi = \frac{3}{2}^-, T = \frac{1}{2}$) to the ($J^\pi = \frac{1}{2}^-, T = \frac{1}{2}$) ,($J^\pi = \frac{7}{2}^-, T = \frac{1}{2}$) states at E_x =0.468 and at E_x =4.63 MeV, respectively. The longitudinal C2 form factors for these states are shown in Fig.1(a,b). The calculations with MSDI as a

residual interaction excited 1p-1h up to $4\hbar\omega$ for core polarization effects which include excitations from the core 1s orbits to the higher allowed orbit were reproduced the data very well up to the momentum transfer of 2.5 fm^{-1}. The calculations show diffraction structure at q=3.2fm^{-1}, Till all previous calculations [16,17,18,19] failed to produce this structure in all model spaces. The minimum diffraction is very well reproduced but the calculated second maximum clearly underestimates the data in both transition.

The calculations with M3Y as a residual interaction with the core polarization effects that includes excitations from the core 1s and 1p orbits to the higher ($6\hbar\omega$) allowed orbits shown in Fig.1(c,d). The present work calculation reproduces the data very well up to a momentum transfer of q= 3.3fm^{-1} and shows diffractive structure at q=3.8fm^{-1} . The location of the diffraction minimum are slightly displaced to higher values to those of the experimental data.

The calculation for transverse multipole form factor for the $J^{\pi} = 3/2^{-}$ ground state and the transition to the $J^{\pi} = 1/2^{-}$ first excited state at 478 KeV are shown in Fig.2. The contribution multipoles for the transverse elastic form factor are M1 and M3, and for the 478 KeV states they are M1 and E2. Fig.2(a,b) shows the individual contributions of each multipole form factor to the total form factor which was done within MSDI as a residual interaction. The data are very well reproduced with core polarization effects that includes excitations from the core 1s orbits to the higher allowed orbits up to$4\hbar\omega$. Fig.2(c,d) shows the calculation of the total form factors which achieved with the M3Y as a residual interaction. The core polarization effects that include excitations from the core 1s and 1p orbits to the higher allowed orbits excited up to $6\hbar\omega$. The data are well described for q< 1 fm^{-1} and begin to deviate significantly from data at large-momentum transfers q>1.

3. Conclusions

The core polarization effects are essential in obtaining a reasonable description of the electron scattering data. The core polarization calculations presented in the present work succeeded in describing the electron scattering data and the locations of the diffraction minimum. The results of the present work show that in general the core polarization effects up to first order improve the agreement with experiment. These calculation can be extended to cover the entire p-shell region, and also can be carried out by using an others suitable residual interaction.

Table(1): Experimental [20] and calculated values of the reduced transition probabilities B(C2↑) (in units of e^2fm^4)

NUCLEUS	J^{π}_f	T_f	E_x(MeV)	b_{rms}(*fm*)	1p	1p+1st order cp(M3Y)	1p+1st order cp(MSDI)	Exp. Data
^{7}Li	$1/2^-$	1/2	0.478	1.65	1.727	6.43	8.02	8.3±0.5
^{7}Li	$7/2^-$	1/2	4.63	1.65	2.67	10.15	13.21	15.0±1.9

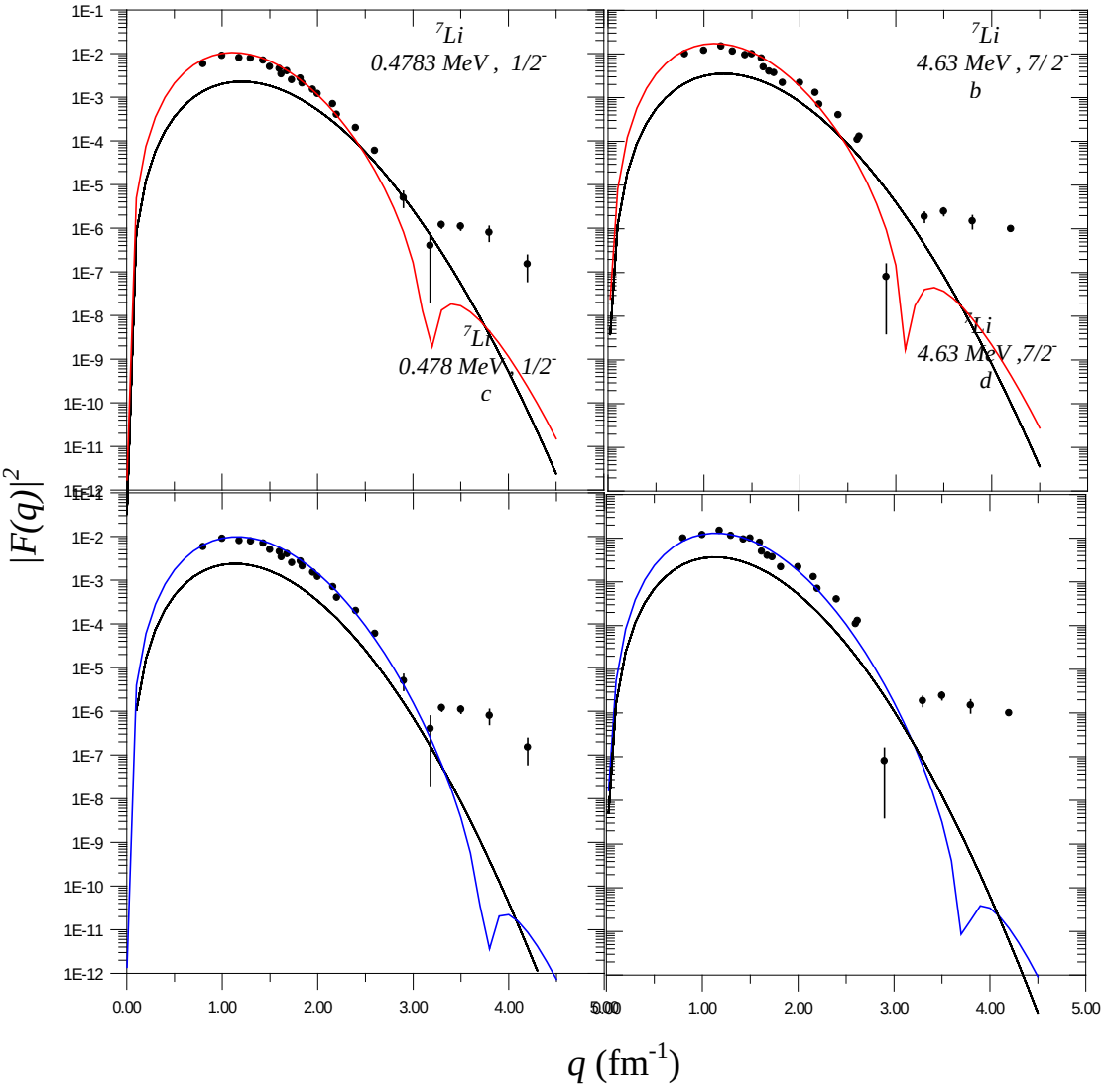

Fig. 1.The Coulomb form factor of quadrupole transition to the 1/2$^-$ 1/2 , 7/2$^-$ 1/2 states in ^{7}Li including the core polarization effects up to first order(solid curves) and those without core polarization effects (dotted curves). MSDI used M3Y are used as a residual interaction in(a, b) and(c, d) respectively. The data are taken from Ref. [21] and [22] .

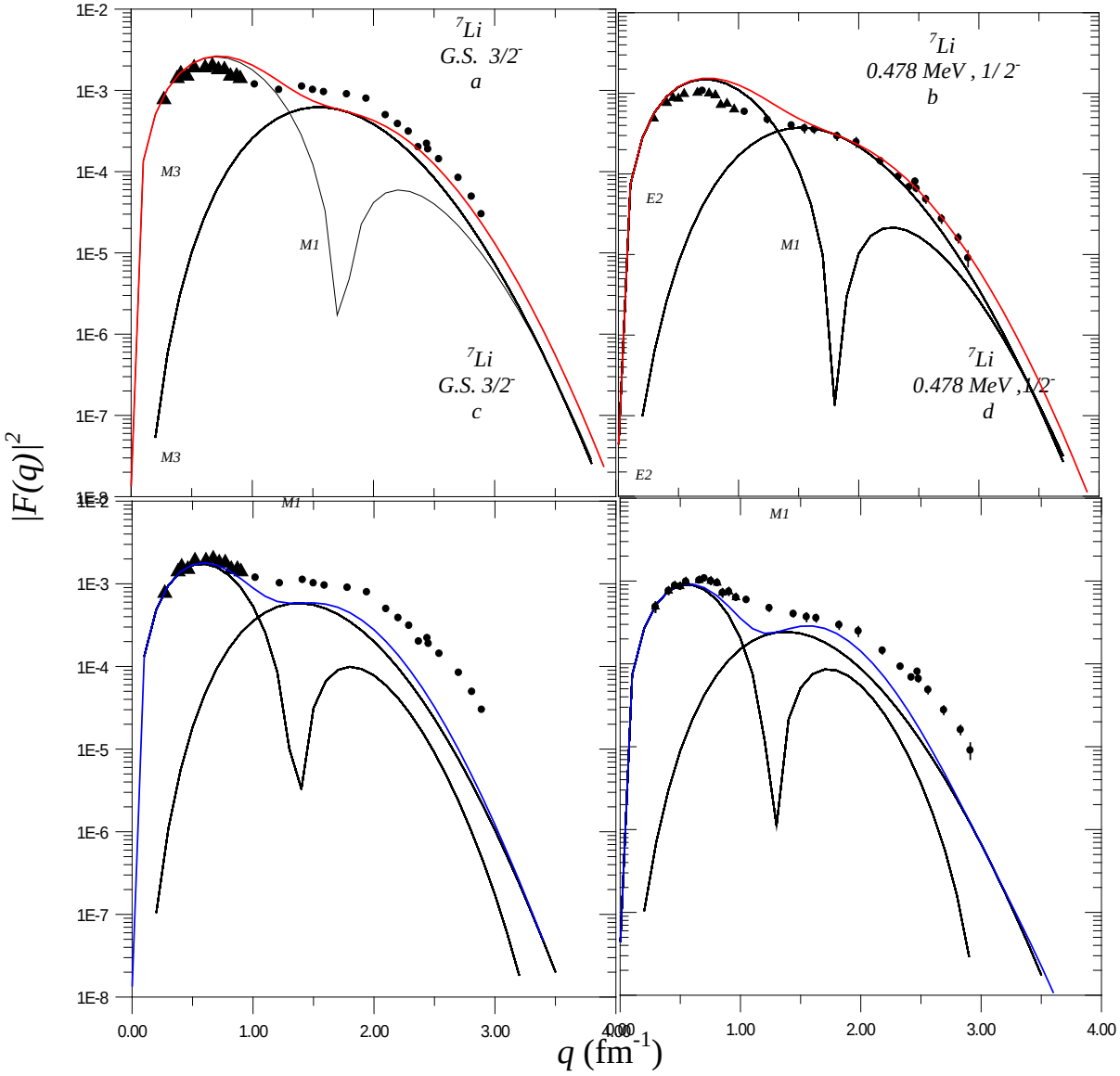

Fig. 2.The transverse form factor of ground state and excited state at 478 KeV in 7Li including the core polarization effects up to first order(solid curves).Also the individual contributions of the M1, M3 and E2 multipoles are shown for each form factor. MSDI and M3Y are used as a residual interaction in(a, b) and(c, d) respectively. The data represented as circles are taken from Ref. [15] **, while triangles are from Ref.** [23].

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