Non Semipre-Denseness in Topological Spaces

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Abstract

In this paper we introduced the notion "non semi-pre dense sets" and obtained some of their properties and related theorems by using the concept of semi-pre open set .

الخلاصه

في هذا البحث نقدم مفهوم جديد هو
(non semipre-denseness in topological spaces)
واستنتجت بعض خواصه واثبت بعض النظريات المتعلقه به باستخدام مفهوم semi-pre open set.

List Of Symbols

Symbols	Mean
Cl(A)	Closure of A
Int(A)	Interior of A
sp	Semipre
spo	Semipre-open
spc	Semipre-closed
spcl	Semipre-closure
spint	Semipre-interior
spd	Semipre-drived
spf	Semipre- frontier
Sp(A)	Semipre-of A
Spo(A)	Semipre-open of A
spc(A)	Semipre-closed of A
spcl(A)	Semipre-closure of A
spint(A)	Semipre-interior of A
spd(A)	Semipre-drived of A

1-INTRODUCTION AND PRELIMINARIES

D.Anderijevie introduces the following definitions,

Theorems and Notions of semipre-open in 1986 .[2]

Definition(1-1): A subset A of a topological spaces (X,τ) is
Called ***semipre-open*** if A ⊂ cl (int (cl(A))).

The complement of semipre-open of X set is called ***semipre-closed*** and denoted by **spc (X).**

The family of all semipre-open sets of X is denoted by **spo(X)**.

Definition (1-2): Let (X, τ) be a topological space and A ⊆ X.
Then A is called a ***semipre-neighbourehood*** of a point x in X ,if there exists semipre-open set U in X such that x ∈ U ⊂ A.

Definition (1-3): The union of all semi pre - open sets
contained in a set A is called *semipre-interior* of A and denoted by spint(A) .

Definition (1-4): The intersection of all semipre-closed sets containing a set A is called *semipre-closures* of A and denoted by spcl (A).

Definition (1-5): A point x∈X is said to be a *semipre-limit Point* if and only if U ∈ spo(X) implies
U ∩ (A-{x}) ≠ ∅ .where ∅ is the empty set.

Definition (1-6): The set of all semipre-limit points of A ⊆ X, is called the *semipre -drived set* of A and is denoted by spd(A) .

Definition (1-7): The set spcl(A) – spint(A) is called *semipre-frontier* of A is denoted by spf(A) .

Theorem (1-8): Let A be a subset of X . then spcl(A) is closed and A ⊆ spcl(A) further A is closed if and only if A = spcl(A) .

Theorem (1-9): Let A be a subset of X.then spint(A)is open
spint(A) $\subseteq$ A further A is open if and only if
A= spint(A).

Theorem (1-10): Let A be a subset of a topological spaces
(X, τ) Then spint(A) = X - spcl(X-A) and
spcl(A) = X - spint(X-A) .

Proof: Since X-A $\subseteq$ spcl(X-A) we have
X - spcl(X-A) $\subseteq$ A .
But X - spcl(X-A) is open (by
theorem (1-8),so X - spcl(X-A) $\subseteq$ spint(A).
On the other hand,
X-spint(A) is closed by theorem(1-9),
and X-A $\subseteq$ X-spint(A) ,
so spcl(X-A) $\subseteq$ X-spintA .
And hence Spint(A) $\subseteq$ X-spcl(X-A).
This shows that
spint(A) = X-spcl(X-A) , and the other
Relation follows from replace A by X-A ■

Definition(1-11): Let A be a subsets of the topological space
(X, τ).Then A is said to be *semipre-dense* in X
if Spcl(A) =X .

2-NON SEMiPRE-DENSE SETS

Definition(2-1): A subset A of a topological space (X,τ) is
non semipre-dense set if spint(spcl(A))= $\varnothing$
that is the semipre - interior of the semipre-
closure of A is empty.

Theorem(2-2) :Let A be a subset of a topological space (X,τ).
Then the following statements are equivalent
i) A is non sp-dense in X.
ii) Spcl(A) contains no sp-nhd.

Proof : (i) $\leftrightarrow$ (ii) we have A is non sp-dense

$\leftrightarrow$ Spint (spcl(A)) = $\varnothing$

$\leftrightarrow$ No point of X is a spint point of spcl(A)

$\leftrightarrow$ Spcl (A) has not a sp-nhd of any of its Points

$\leftrightarrow$ Spcl (A) contains no sp-nhds ■

Theorem(2-3): Let A be a subset of topological spaces (X, τ) if A is non sp-dense, then spcl(A) is not the entire space X.

Proof: Since X is sp-closed then X=spcl(X).

Again since X is sp-open, we have

spin(spcl(X))=spin(X)=X.

Since A is non sp-dense in X=X,

and spint(spcl(A)) = $\varnothing$.

Thus spn(spcl(X))=X ,

and spint(spcl(a))= $\varnothing$.

It follows spcl(A) $\neq \varnothing$ ■

Theorem (2-4): The union of finite number of non sp-dense set is non sp-dense sets.

Proof : it suffices to prove that the theorem for the case of two non sp-dense sets ,say A and B For simplicity we put

G=spint(spcl(A $\cup$ B)) So that

G $\subset$ spcl(A $\cup$ B) =spcl(A) $\cup$ spcl(B).

It follows that

$$G \cap [spcl(B)]' \subset (spcl(A) \cup spcl(B)) \cap [spcl(B)]'$$

$$= [\, spcl(A) \cap (spcl(B))' \,] \cup [spcl(B) \cap (spcl(B))' \,]$$

$$= spcl(A) \cap (spcl(B))' \quad \text{Since } [\, spcl(B) \cap (spcl(B))' = \varnothing]$$

$$\subset spcl(A).$$

Then spint(G $\cap$ ((spcl(B))$'$) $\subset$ spin(spcl(A)) = $\varnothing$

since A is non sp-dense.

But spint [G $\cap$ (spcl(B))]$'$ = G $\cap$ spcl(B)$'$.

Since $G \cap (spcl(B))'$ is an sp-open set ,It follows
that $G \cap (spcl(B))' = \varnothing$, which implies
$G \subset spcl(B)$ then $spint(G) \subset spint(spcl(B)) = \varnothing$
Sinse [B is non sp-dense] .
But $spint(G) = spint[spint(spcl(A \cup B))]$
$= spint(spcl(A \cup B))$.
So that $spint(spcl(A \cup B)) = \varnothing$.
Hense $A \cup B$ is non sp-dense ■

Theorem(2-5) :Let A be a subset of the topological spaces (X, τ),Then A is non sp-dense in X if and only if X-spcl(A) is Sp-dense in X.

Proof: By theorem (1-10)
$Spint(A) = X - spcl(X-A)$ and
$Spcl(A) = X - spint(X-A)$
it follows that $spint(spcl(A)) = X-spcl(X-spcl(A))$.
Sinse A is non sp-dense
then $spint(spcl(A))=\varnothing$
then $X-spcl(X-spcl(A)) = \varnothing$
then $spcl(X-spcl(A)) = X$
then spcl(X-spcl(A)) is sp-dense■

References

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