Relations among the separation axioms in Topological , Bitopological and Tritopological spaces

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ABSTRACT :-

In this work , We introduce a relations among separation axioms in Topological , Bitopological and Tritopological space , by using some theorems to make the one spaces implies the others .

الخلاصة :

في هذا العمل ، قدمنا علاقة بين بديهيات الفصل في الفضاء التبولوجي و بديهيات الفصل في الفضاء الثنائي التبولوجي و بديهيات الفصل في الفضاء الثلاثي التبولوجي ، و ذلك بالاعتماد على بعض النظريات السابقة حتى نتمكن من تحويل احد الفضاءات الى الآخر .

1-INTRODUCTION :-

Throughout this paper we adopt the notations and terminology of [5] , [3] and [2] , and the following conventions : (X,T), (X,T',Ω'), (X,T'',Ω'',ρ'') will always denoted Topological space , Bitopological space and Tritopological space respectively . And the following conventions (T_0,T_1,T_2,T_3,T_4) , $(\delta-T_0,\delta-T_1,\delta-T_2,\delta-T_3,\delta-T_4)$, $(\delta*-T_0,\delta*-T_1,\delta*-T_2,\delta*-T_3,\delta*-T_4)$ will always denoted the separation axioms in Topological space , Bitopological space and Tritopological space respectively.

Let (X,T',Ω') be a Bitopological space , and A be asubset of X , then A is said to be a δ-open set iff $A \subseteq T'-\text{int}(\Omega'-cl(T'-\text{int}(A)))$ [3] , and the family of all δ-open sets is denoted by $\delta.O(X)$. The complement of δ-open set is called δ-closed set .

Let (X,T'',Ω'',ρ'') be a Tritopological space , a subset A of X is said to be $\delta*$-open set iff $A \subseteq T''-\text{int}(\Omega''-cl(\rho''-\text{int}(A)))$ [2] , and the family of all $\delta*$-open sets is denoted by $\delta^*.O(X)$. The complement of $\delta*$-open set is called a $\delta*$-closed set .

We introduce a conditions to make us able to change the separation axioms in Topological space to the separation axioms in Bitopological space , the separation axioms in Bitopological space to the separation axioms in Tritopological space , and the separation axioms in Topological space to the separation axioms in Tritopological space ((T_0,T_1,T_2,T_3,T_4) to $(\delta-T_0,\delta-T_1,\delta-T_2,\delta-T_3,\delta-T_4)$, $(\delta-T_0,\delta-T_1,\delta-T_2,\delta-T_3,\delta-T_4)$ to $(\delta*-T_0,\delta*-T_1,\delta*-T_2,\delta*-T_3,\delta*-T_4)$ and (T_0,T_1,T_2,T_3,T_4) to ($\delta*-T_0,\delta*-T_1,\delta*-T_2,\delta*-T_3,\delta*-T_4$)) , and the convers is not hold .

***** This research is the complement of the last research [1] .

2- Some Important Theorems :

2.1 Theorem :

T_0-space implies $\delta - T_0$ space .

Proof :

Let (X,T) be a T_0-space , by definition of T_0-space ; for each distinct points x ,y $\in$ X ,there exists an open set G contains x but not y or an open set H contains y but not x . By using theorem **{ A topological space (X,T) becomes a bitopological space (X,T',Ω') by taking T' equal to the T , and taking Ω' equal to the complement of any open set in T . (i.e. every open set can be a δ-open set) [1] }.**

Then H and G can be a δ-open sets , and the definition of T_0-space becomes the definition of $\delta - T_0$ space . So that T_0-space implies $\delta - T_0$ space . ◘

2.2 Remark :

The convers of above theorem is not true , because the bitopological space not always represent a topology [3] . (i.e. any δ-open set not necessary to be an open set) .

2.3 Example :

Let X={a,b,c,d} , T' ={ **X** , φ, {c,d},{a,c,d},{a}}

, Ω' = { **X** , φ }

(X,T') and (X,Ω') are two topological spaces , then (X,T',Ω'))is a bitopological space , such that :

δ.O(X)={ **X** , φ,{a},{a,b},{a,c},{a,d}, {c,d},{a,c,d},{a,b,c},{a,b,d},{b,c,d}}

Clearly that δ.O(X) does not represent a topology on **X** .

Therefore not every a bitopological space can be a topological space .

2.4 Theorem :

$\delta - T_0$ space implies $\delta^* - T_0$-space .

Proof :

Let (X,T',Ω') be a bitopological , by definition of $\delta - T_0$-space ; for each distinct points x ,y $\in$ X , there exists a δ-open set G contains x but not y or a δ-open set H contains y but not x . And by using theorem **{ A bitopological space (X,T',Ω') becomes a tritopological (X,T'',Ω'',ρ'') by taking $T'' = T'$, $\Omega'' = \Omega'$ and ρ'' = T' . (i.e. every δ-open set can be a δ^*-open set) [1] } .**

Then H and G can be a δ^*-open sets , and the definition of $\delta - T_0$-space becomes the definition of $\delta^* - T_0$-space . So that $\delta - T_0$-space implies $\delta^* - T_0$-space .◘

2.5 Remark :

The convers of above theorem is not true because we must delete one topology of tritopological space to be a bitopological space and the definition will be not satisfaied . (i.e. any δ^*-open set not necessary to be δ-open set)

2.6 Example :

Let **X** ={a,b,c,d} , T'' ={ **X** , φ, {c,d} }

, Ω'' = { **X** , φ,{a,b,c},{a}}

and ρ'' ={ **X** , φ,{d}, {c,d},{a,d}}

(X,T''), (X,Ω'') and (X,ρ'') are three topological spaces , then (X,T'',Ω'',ρ'') is a tritopological space , such that :

$$\delta^*.O(X) = \{X,\varphi,\{a,d\},\{c,d\},\{a,b,d\},\{a,c,d\},\{b,c,d\}\}$$

If we want to make this tritopological space a bitopological space we must delete one topology , and the definition of δ -open set will not satisfy .

2.7 Theorem :

T_0 -space implies $\delta * - T_0$ -space .

Proof :

By using theorems (2.1) and (2.4) . ◘

2.8 Remark :

The convers of a bove theorem is not true , because the tritopological space not always represent a topology [2] . (i.e. any $\delta *$-open set not necessary to be an open set) .

2.9 Example :

Let $\mathbf{X} = \{a,b,c,d\}$, $T'' = \{\mathbf{X}, \varphi, \{c,d\}\}$

, $\Omega'' = \{\mathbf{X}, \varphi, \{a,b,c\}, \{a\}\}$

and $\rho'' = \{\mathbf{X}, \varphi, \{d\}, \{c,d\}, \{a,d\}\}$

(X,T''), (X,Ω'') and (X,ρ'') are three topological spaces , then (X,T'',Ω'',ρ'') is a tritopological space , such that :

$$\delta^{*}.O(X) = \{X, \varphi, \{a,d\}, \{c,d\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}\}$$

Clearly that $\delta^{*}.O(X)$ does not represent a topology on $\mathbf{X}$.

Therefore not every a tritopological space can be a topological space .

***In the same way the following theorems are satisfy :**

2.10 Theorem :

T_1 -space implies $\delta - T_1$ - space .

2.11 Theorem :

T_2 -space implies $\delta - T_2$ - space .

2.12 Theorem :

T_3 -space implies $\delta - T_3$ - space .

2.13 Theorem :

T_4 -space implies $\delta - T_4$ - space .

2.14 Theorem :

$\delta - T_1$ - space implies $\delta * - T_1$ -space .

2.15 Theorem :

$\delta - T_2$ - space implies $\delta * - T_2$ -space .

2.16 Theorem :

$\delta - T_3$ - space implies $\delta * - T_3$ -space .

2.17 Theorem :

$\delta - T_4$ - space implies $\delta * - T_4$ -space .

2.18 Theorem :

T_1-space implies $\delta * - T_1$-space .

2.19 Theorem :

T_2-space implies $\delta * - T_2$-space .

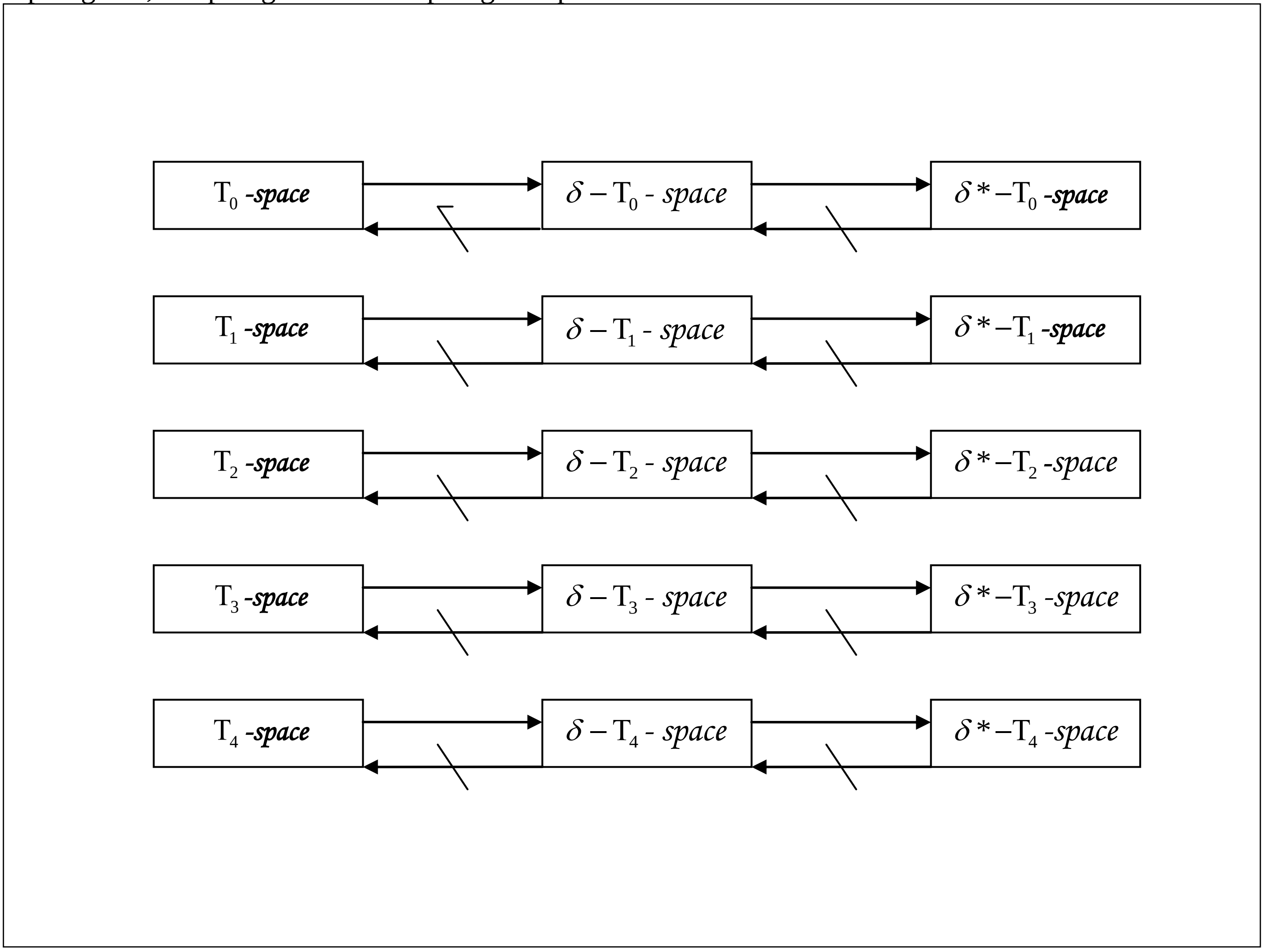
2.20 Theorem :

T_3-space implies $\delta * - T_3$-space .

2.21 Theorem :

T_4-space implies $\delta * - T_4$-space .

The following diagram illustrates the relationship among the separation axioms in topological , bitopological and tritopological spaces



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