# On Almost Open Sets In Metric Spaces

حول المجموعات المفتوحة تقريبا في الفضاءات المترية

Mohammed Yahya Abid Basim Karim Al-Saltani

Computers Dept.,/Science College,/Kerbalaa University/

**Abstract**

In this paper we present a new notion the almost open sets, to define the almost dense and almost perfect sets in metric space X. furthermore we prove some theorems related by these concepts .

**الخلاصه**

في هذا البحث قدمنا مفهومين جديدين باستخدام المجموعة المفتوحة تقريبا، وهذين المفهومين هو المجموعة الكثيفة بنفسها تقريبا و المجموعة التامة تقريبا في الفضاءات المترية و أثبتنا بعض النظريات المتعلقة بهذين التعريفيين الجديدين

## 1. INTRODACTION

Before we present the almost dense-in-itself we give some definitions and remarks .

Definition(1-1) : Let (X,d) be a metric space and $E \subseteq X$ we say that E is *almost open* iff $E \subseteq (\bar{E})^{\circ}$ ,where $(\bar{E})^{\circ}$ is denoted to the interior of the closure of E [1],[2]

Remark (1-2) : Every open set in (X,d) is almost open .

Definition (1-3) :The complement of almost open set will be called *almost closed* .

Definition (1-4) : A point $y \in X$ is called *almost limit point* of $E \subseteq X$ if for each almost open set U of X with $y \in U$ ,$E \cap (U-\{y\}) \neq Ø$, we denote the set of all almost limit points of E by $E^{*}$ which we call the almost derived set of E .

Remark (1-5) : Since every open set in (X,d) is almost open set, so every almost limit point of $E \subseteq X$ is Limit point of E .That is $E^{*} \subseteq E'$, where $E'$ is the set of all limit points of E .

Definition (1-6) : let $E \subseteq X$ , a point $b \in X$ is called *almost interior point* of E if there is an almost open set U such that $b \in U$ and $U \subseteq E$ .

Definition (1-7): let (X,d) be a metric space and $A \subseteq X$ the ***almost closure*** of A denoted by acl A defined as the set $\text{acl } A = \cap\{ F \subseteq X : F \text{ almost closed } ; A \subseteq F \}$ .

Theorem (1-8) [1] ,[2] : let A, B are subsets of a metric space X. then

1) $\text{acl}A \subseteq \bar{A}$ where $\bar{A}$ is the closer of A .

2) $A \subseteq B$ implies $\text{acl}A \subseteq \text{acl}B$ .

3) $aclA \cup aclB \subseteq acl(A\cup B)$ .

4) $acl(A\cap B) \subseteq aclA \cap aclB$ .

5) $acl(aclA) = aclA$ .

6) aclA is almost closed set .

7) A is almost closed set iff $aclA = A$ .

8) $acl\ A = A\cup A^*$ .

## 2. ALMOST DENSE- IN- ITSELF

Definition (2-1) : A subset E of a metric space (X,d) is called ***almost dense- in-itself*** if $E \subseteq E^*$ that is every points of E is almost limit point of E .

Remark (2-2) : Every almost limit point is a limit point .

Theorem (2-3) : Every almost dense -in-itself set is dense - in - itself

Proof : let A be almost dense-in-itself set that is ( every point in A is almost limit point of A ) , since every almost limit point is a limit point, then each point of A is a limit point, therefore A is dense-in- itself ■

Theorem (2-4): If E is almost dense -in -itself set then aclE is almost dense-in-itself.

Proof: $acl\ E = E \cup E^*$ since E is almost dense -in -itself that is ( every point of E is almost limit point of E ) then $E \cup E^*=E$ hence $aclE = E$, therfore aclE is almost dense-in-itself ■

Theorem (2-5) : The union of any family of almost dense - in-itself sets is almost dense - in-itself .

Proof : let $\{E_i\}$, $i\in I$, be a family of almost dense-in-itself sets . so $E_i \subseteq E_i^*$ ,$\forall\ i\in I$.

Let $p\in\cup E_i$ then $p\in E_i$.for some $i\in I$.

Hence for each almost open set U with $p\in U$, $E_i\cap U-\{p\} \neq \emptyset$.

Thus $(\cup E_i) \cap U-\{p\} \neq\emptyset$ , hence $p \in (\cup E_i)^*$ therefore $\cup E_i \subseteq(\cup E_i)^*$ hence $\cup E_i$ is almost denes-in-itsef ■

## 3. ALMOST PERFECT

Definition (3-1) : A subset $E \subseteq X$ is called ***almost perfect*** if it is almost closed & almost dense -in-itself .

Theorem (3-2) : $E \subseteq X$ is almost perfect iff $E = E^*$.

Proof : since E almost perfect then E almost closed and almost dense- in-itself then by theorem (1-8) we have $aclE=E$ if and only if $E=E \cup E^*$ if and only if $E=E^*$ ■

## REFERENCES

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