# Determination of 3-dimentional coordinates of Objects from Satellite Images 

Dr. Loay E. George Dr. Najem A. Najem Duaa Deyaa


#### Abstract

: Mathematical framework of the inverse transform from image coordinates to geographic coordinates was developed and investigated. This transform was utilized to estimate the unknown parameters of the satellite imaging system using some of the data extracted from the raw images that produced by the imaging system. A sequence of determinations based on some geometrical relationships between the angular separation of the satellite projection point (nadir points) and a sequence of uniformly distributed reference points were used for autonomous assignment of the initial values of the imaging system parameters. Also, an estimation procedure is given to assess the value of the satellite distance, and an equation for estimating the value of image scale parameter was derived and implemented. Optimization used to improve the determination accuracy. Through each optimization step the value of one of the imaging parameters is adjusted leads to minimize the overall error. The optimization methods have been implemented as two layers. The suggested system was implemented on two sets of Meteosat and Noaa images, and the results justified the suitability of the proposed system.


| تم تطوير الإطار الرياضي للتحويلات العكسية من إحداثيات الصورة إلى الاحداثيات الجغرافيـة، وقد تم التحقق منـه. واستعـتا <br>  التي أنتجت بنظام التصوير. إن إستعمال بعض العلاقات الهندسية بين الإفتزاق الزاوي لنقطة تقنير القمر الصناعي (نقاطنظير سمت) و مجمو عة النقاط المرجية الُموز عة بشكل متجانس على الصورة كان الغرض منها تخمين قيم معاملات نظّام التصوير بصورة تقرييبة، وإجراء تققير يعطى تَقيبيم قيمة مسافة القمر الصناعي، ومعادلة لتخمين قيمة معامل مقياس الصورة. لتحسين دقة <br>  التصوير نؤدي الى تقليل الخطأ العام. إن النظام المقتر حطبق على صور تين من مجمو عـات صور Meteosat وNoaa، ولا و هذه <br>  الصناعي، كما إن النقاط القريبة من مركز الصـورة تعطي دقة أفضل عند حساب إحداثيات موقع نقطة النظير ، بصورة عامة عدد <br> النقاط يجب أن لا يقل عن ثمانية نقاط موز عة على كل أجزاء الصورة. |
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## Introduction

Translation and scaling transformations in three dimensions can be expressed in the same manner as in two dimensions, but for rotations, we must choose an axis. The formulas for transforming the coordinates after a rotation around an arbitrary axis are rather cumber, but it is possible to give them in concise form with rather simple proofs if we introduce certain concepts of vector algebra. [1,2]
This transform called forward transform, we find the inverse that necessary to estimate the geographic coordinates (Longitude, Latitude) of the assigned image points (in terms of it is corresponding image projection coordinates, i.e., x and y ). Also, a numerical model based on Hooks \& Jeeves search method is proposed to assess the values of the pan ( $\beta$ ) angle, Tilt ( $\alpha$ ) angle, and the position coordinates of the satellite imaging system.
During the last two decades different researches were published concerned with geometrical transforms such as:
1.Hayden (1997)[4]: he had investigated the three-dimensional perspective projection, he studied the projection done according to a camera.
2.Orhan, et. al., (2000)[5]: they describes the development and animation of a system for threedimensional imaging.

## Processing and Theoretical Model

Let us denote that the Cartesian world coordinates: ( $\mathrm{X}, \mathrm{Y}, \mathrm{Z}$ ) where the center of the coordinate system is the Earth's center. At first, let us translate the coordinates to a new position, that is the satellite coordinates ( $\mathrm{X}_{\mathrm{s}}, \mathrm{Y}_{\mathrm{s}}, \mathrm{Z}_{\mathrm{s}}$ ). So that we have the following sequence of 3D transforms:
we can find the relationship between the coordinates of any points $(\varphi, \Theta, h)$ and the coordinates of the corresponding projection point $\left(\mathrm{X}_{\mathrm{p}}, \mathrm{Y}_{\mathrm{p}}\right)$ :

$$
\begin{align*}
& X_{P}=S \frac{\left[e_{1} \cos (\varphi) \cos (\theta)+e_{2} \sin (\varphi) \cos (\theta)+e_{3}\right]}{d_{1} \cos (\varphi) \cos (\theta)+\mathrm{d}_{2} \sin (\varphi) \cos (\theta)+\mathrm{d}_{3} \sin (\theta)+\mathrm{d}_{4}},  \tag{1}\\
& Y_{p}=S \frac{\left[\mathrm{f}_{1} \cos (\varphi) \cos (\theta)+\mathrm{f}_{2} \sin (\varphi) \cos (\theta)+\mathrm{f}_{3} \sin (\theta)+\mathrm{f}_{4}\right]}{\mathrm{d}_{1} \cos (\varphi) \cos (\theta)+\mathrm{d}_{2} \sin (\varphi) \cos (\theta)+\mathrm{d}_{3} \sin (\theta)+\mathrm{d}_{4}}, \tag{2}
\end{align*}
$$

The projection coordinate depend on the following parameters:

1. The coordinates of the projected point $(\Theta, \varphi, h)$.
2. The coordinates of the satellite imaging system $\left(\Theta_{s}, \varphi_{s}, \mathrm{D}\right)$.
3. The camera alignment angles ( $\mathrm{pan}=\alpha$, tilt $=\beta$ ).
4. The imaging scaling factors (S).

In order to align the imaging system toward certain area on the Earth surface, the angle pan and tilt should be put at certain values to get a coverage for the required area. The values of pan and tilt angles are determined by using the criteria: "the projection coordinates of the nadir point should be $(0,0)$ ", therefore, in order to determine the required pan $(\alpha)$ and tilt $(\beta)$ angles in terms of nadir point coordinates $\left(\varphi_{n}, \Theta_{n}, \mathrm{~h}_{\mathrm{n}}\right)$ and satellite coordinates $\left(\varphi_{s}, \Theta_{\mathrm{s}}, \mathrm{D}\right)$, the projection coordinates of the nadir point are taken ( $\mathrm{X}_{\mathrm{p}}=0 ; \mathrm{Y}_{\mathrm{p}}=0$ ), applying these values in equations (1) and (2) will lead to:
$\mathrm{e}_{1} \cos \left(\varphi_{\mathrm{n}}\right) \cos \left(\Theta_{\mathrm{n}}\right)+\mathrm{e}_{2} \sin \left(\varphi_{\mathrm{n}}\right) \cos \left(\Theta_{\mathrm{n}}\right)+\mathrm{e}_{3}=0$
and, $\mathrm{t}_{1} \cos \left(\varphi_{\mathrm{n}}\right) \cos \left(\Theta_{\mathrm{n}}\right)+\mathrm{t}_{2} \sin \left(\varphi_{\mathrm{n}}\right) \cos \left(\Theta_{\mathrm{n}}\right)+\mathrm{t}_{3} \sin \left(\Theta_{\mathrm{n}}\right)+\mathrm{t}_{4}=0$
Now, determine the required $\operatorname{Pan}(\alpha)$ and $\operatorname{Tilt}(\beta)$ angles from equations:

$$
\begin{equation*}
\tan (\alpha)=\frac{\Delta y}{\Delta x} \tag{5}
\end{equation*}
$$

$$
\begin{equation*}
\tan (\beta)=\frac{\Delta z}{\Delta x \frac{\Delta x}{\sqrt{(\Delta x)^{2}+(\Delta y)^{2}}}+\Delta y \frac{\Delta y}{\sqrt{(\Delta x)^{2}+(\Delta y)^{2}}}}, \tag{6}
\end{equation*}
$$

## Inverse Transform

As a first step let us start with the final transform equations (i.e., determination of image coordinates X, Y from the geocentric coordinates) (1) and (2), Now, by many of the mathematical representations, could be the last equations arranged in the following form:

$$
\begin{equation*}
\theta=\tan ^{-1}\left[\frac{\mathrm{p}_{1} \cos (\varphi)+\mathrm{p}_{2} \sin (\varphi)}{\mathrm{p}_{3}}\right] \tag{7}
\end{equation*}
$$

$\varphi=\frac{1}{2}\left[\cos ^{-1}\left(\frac{2 \mathrm{r}_{4}-\mathrm{r}_{3}-\mathrm{r}_{1}}{\sqrt{\left(\mathrm{r}_{1}-\mathrm{r}_{3}\right)^{2}+4 \mathrm{r}_{2}^{2}}}-\delta\right)\right]$,
Where,
$\theta$ : is the longitude value.
$\varphi$ : is the latitude value.

## Least square error

The method of the least squares assumes that the best-fit approximation for a given set of data is that mathematical representation leads to the minimal sum of the deviations (squared) between it's determined (predicated or assessed) values from the given set of data.
For the considered problem in this research work the sum of difference (or deflection) squared could be determined by using the following equation [5]:
$\chi^{2}=\sum\left[\mathrm{X}_{\mathrm{P}}(\mathrm{i})-\mathrm{X}_{\mathrm{d}}(\mathrm{i})\right]^{2}+\left[\mathrm{Y}_{\mathrm{P}}(\mathrm{i})-\mathrm{Y}_{\mathrm{d}}(\mathrm{i})\right]^{2}$
Where:
$\mathrm{X}_{\mathrm{P}}(\mathrm{i}), \mathrm{Y}_{\mathrm{P}}(\mathrm{i})$ : are the $\mathrm{X}, \mathrm{Y}$ coordinates of the i th point extracted from the image.
$X_{d}(i), Y_{d}(i)$ : are the corresponding coordinates determined by using the forward transform equations.
Since, the determined values of the coordinates $\left\{\mathrm{X}_{\mathrm{P}}(\mathrm{i}), \mathrm{Y}_{\mathrm{P}}(\mathrm{i})\right\}$ are strongly depends on the parameters:

1. Satellite coordinates $\left(\varphi_{\mathrm{S}}, \theta_{\mathrm{S}}, \mathrm{D}\right)$
2. Camera alignment angles ( $\alpha, \beta$ )
3. Scaling factor (S)

## The Optimization Problem

The need for the optimization methods arises due to the mathematical complexity that is required to handle the computations related to the theories of systems, processes, equipments, and devices which occur in practice. Sometime, even quite simple system represented by a theory, contain some approximations, parameters which vary with time, or parameters that vary in a random manner. For many reasons the developed theoretical approach might be imperfect, yet it must be used to predict the optimum operating conditions of the system such that some performance criterion is needed to be satisfied [5].
At best such theory can predict only that the system is near to the desired optimum. Optimization methods are, then, used to explore the local region of operation, and predict the way that the system parameters should be adjusted to bring the system to optimum [5,6].

## Hooks \& Jeeves

This method date back to 1961, but still it is considered a very efficient and ingenious optimization procedure. The search consists of a sequence of exploration steps about a base point which if successful is followed by a pattern of moves [6].
There is a figure (see fig. (1)) shown the flow chart of Hookes \& Jeeves in source [5].


Fig.(1) Flow Chart for Hooks and Jeeves [5]

## The Proposed System (Results)

The layout of the proposed system to assess the parameters of the satellite imaging system is presented. The established system is called Satellite Imaging Parameters Determination System (SIPDS). Also, the steps of implemented stages of the suggested system are clarified, and some of the corresponding algorithms are given.
The established program to perform all the required computations has been developed by using Visual Basic (version-6) programming language. The two optimization methods (Hooks and Jeeves) have been adopted to control the search for best (optimal) values of the parameters of the satellite imaging system.
Some of the test results are given to illustrate the capability of the proposed system to assess the parameters of the imaging system.

## System Layout

Figure (2) illustrates the stages of the proposed system, it consists of the following major modules:

1. Load satellite image.
2. Automatic setting of the initial parameters.
3. Checking the parameters validity.
4. Optimization Search.
5. Latitude-Longitude grid generation.


Fig. (2) The general steps of the proposed (SIPDS)

## Conclusions and Suggestions:

From the considered theoretical framework, and the results of the conducted tests, to evaluate the performance of the suggested system (SIPDS), the following conclusions and suggestions were conducted:

## Conclusions:

1. When the angular distance between the satellite projection point and the nadir point is small, then the relationship between image coordinate and corresponding geographic coordinates will tend to be isotropic.
2. The effect of the reference point heights on the accuracy of the determination of the imaging parameters tends to be negligible, especially when the satellite altitude is large.

## Suggestions:

1. Develop a more sophisticated method for estimating the initial values of the satellite coordinates from the behavior of some selected reference points to assess the initial values of the satellite coordinates.
2. Extend the established system capabilities to directly utilize a library of reference digital maps. This added will help the user to directly assign the geographic coordinates of the selected reference points.

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