Adomian Decomposition Method with different polynomials for nonlinear Klein Gordon equation and a system of nonlinear partial differential equations

Mohanad Riyadh Saad
Department of Mathematics
College of Science
University of Basra
Iraq

Abstract

In this paper the Adomian Decomposition Method (ADM) with modified polynomial [1] is applied for nonlinear models, first we apply it for solving nonlinear partial differential equation (Klein Gordon equation with a quadratic non-linear term), then we discussed the solution of system of nonlinear partial differential equations with this modification. The numerical results obtained by this polynomial have been compared with numerical results in [2] and [3] to show the efficiency in applications.

Keywords: Nonlinear Klein Gordon equation, System of nonlinear partial differential equations, ADM.

تطبيق ADM مع متعددات حدود مختلفة لحل معادله Klein Gordon غير الخطيه ولحل نظام معادلات تفاضليه جزئيه غير خطيه

مهند رياض سعد
قسم الرياضيات
كليه العلوم
جامعه البصرة
العراق

المستخلص

في هده البحث استخدمنا متعددة حدود مطورة في (ADM) [1] لحل معادله تفاضليه جزئية غير خطيه (Klein Gordon equation with a quadratic non-linear term) ولحل نظام معادلات تفاضلية جزئيه غير خطيه، النتائج العدديه التي حصلنا عليها بهده الطريقه تم مقارنتها مع نتائج المصدرين [2] و [3] لبيان كفاءة الطريقة.

1. Introduction

Since the beginning of the 1980s, the ADM has been applied for a wide class of functional equations [4]. Adomian gives the solution as infinite series usually converging to an accurate solution. The procedure of ADM need a polynomial in applications, many researchers use Adomian polynomial in the implementation of ADM. Kalla [1] discussed a new polynomial for ADM as we see in section 2. In this paper we use ADM with Kalla polynomial for finding the solution of nonlinear Klein Gordon equation with a quadratic nonlinear term and the solution of a system of nonlinear partial differential equation. Test problems are discussed [2, 3], we use Maple 13 software for this purpose, the obtained results suggest that Kalla polynomials introduces a promising tool and powerful improvement for solving nonlinear partial differential equations and systems of nonlinear partial differential equations.

2. The Adomian decomposition method (ADM)

Let us consider the following equation

$$Lu + Nu + Ru = f(x) \qquad (1)$$

where L is an invertible linear operator, N represents the nonlinear operator and R is the remaining linear part, from equation (1) we have $Lu = f(x) - Nu - Ru$, now applying the inverse operator L^{-1} to both sides of equation (1) then use the initial conditions we find $u = g(x) - L^{-1}Nu - L^{-1}Ru$, where $L^{-1} = \int_0^x (\cdot)\, ds$ and $g(x)$ represents the terms having from integrating the remaining term $f(x)$ and from using the given initial or boundary conditions. The ADM assumes that the nonlinear operator $N(u)$ can be decomposed by an infinite series of polynomials given by $N(u) = \sum_{n=0}^{\infty} A_n(u_0, u_1, \ldots, u_n)$ where A_n are the Adomian's polynomials [1] defined as:

$$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[N\left(\sum_{n=0}^{\infty} \lambda^i h_i \right) \right]_{\lambda=0} \quad , \quad n = 0, 1, 2, \ldots \qquad (2)$$

So the A_n cane be given as:

$$A_0 = N(u_0)$$

$$A_1 = \frac{d}{dx}\big(N(u_0)\big)\,u_1$$

$$A_2 = \frac{1}{2}\,\frac{d^2}{dx^2}\big(N(u_0)\big)\,u_1^2 + \frac{d}{dx}\big(N(u_0)\big)\,u_2$$

$$A_3 = \frac{1}{6}\,\frac{d^3}{dx^3}\big(N(u_0)\big)\,u_1^3 + \frac{d^2}{dx^2}\big(N(u_0)\big)\,u_1\,u_2 + \frac{d}{dx}\big(N(u_0)\big)\,u_3$$

$$A_4 = \frac{1}{24}\,\frac{d^4}{dx^4}\big(N(u_0)\big)\,u_1^4 + \frac{1}{2}\,\frac{d^3}{dx^3}\big(N(u_0)\big)\,u_1^2\,u_2 + \frac{1}{2}\,\frac{d^2}{dx^2}\big(N(u_0)\big)\,u_2^2 + \frac{d^2}{dx^2}\big(N(u_0)\big)\,u_1\,u_3 + \frac{d}{dx}\big(N(u_0)\big)\,u_4$$

$$\vdots$$

Also $u(x)$ cane be expressed by an infinite series of the form $u(x) = \sum_{n=0}^{\infty} u_n$, identifying u_0 the remaining components for $n = 1, 2, \ldots$ can be determined by using recurrence relations

$$u_0(x) = g(x)$$

$$u_n(x) = -L^{-1}(A_n) - L^{-1}[R(u_n)] \qquad , \qquad n = 1, 2, \ldots$$

The other polynomials can be generated in a similar way. The solution will be the approximations $\varphi_k = \sum_{n=0}^{k-1} u_k$ with $\lim_{k\to\infty} \varphi_k = u(x)$.

Ibrahim L. El-Kalla [1] introduce a new formula for Adomian polynomials, he claimed that the Adomian solution using this new formula converges faster than using Adomian polynomials (2). Kalla polynomial given in the following form:

$$\overline{A}_n = N(S_n) - \sum_{i=0}^{n-1} \overline{A}_i \qquad (3)$$

Where $S_n = u_0 + u_1 + \ldots + u_n$. For example, if $N(u) = u^4$ the first three polynomials using formulas (2) and (3) are computed to be:

Using formula (2):

$$A_0 = u_0^4$$

$$A_1 = 4\,u_0^3\,u_1$$

$$A_2 = 6u_0^2 u_1^2 + 4u_0^3 u_2$$

$$A_3 = 12u_0^2 u_1 u_2 + 4u_0 u_1^3 + 4u_0^3 u_3$$

$$\vdots$$

Using formula (3):

$$\overline{A}_0 = u_0^4$$

$$\overline{A}_1 = 4u_0^3 u_1 + 6u_0^2 u_1^2 + 4u_0 u_1^3 + u_1^4$$

$$\overline{A}_2 = 4u_0^3 u_2 + 6u_0^2 u_2^2 + 4u_0 u_2^3 + 4u_1^3 u_2 + 6u_1^2 u_2^2 + 4u_1 u_2^3 + u_2^4 + 12u_0^2 u_1 u_2 + 12u_0 u_1^2 u_2 + 12u_0 u_1 u_2^2$$

$$\begin{aligned}\overline{A}_3 = {} & 4u_0^3 u_3 + 6u_0^2 u_3^2 + 4u_0 u_3^3 + 4u_1^3 u_3 + 6u_1^2 u_3^2 + 4u_1 u_3^3 + 4u_2^3 u_3 + 6u_2^2 u_3^2 + 4u_2 u_3^3 + 12u_0 u_2 u_3^2 \\ & + 12u_0 u_2^2 u_3 + 12u_0 u_1^2 u_3 + 12u_1 u_2 u_3^2 + 12u_0^2 u_1 u_3 + 12u_0^2 u_2 u_3 + 12u_1 u_2^2 u_3 + 12u_0 u_1 u_3^2 \\ & + 12u_1^2 u_2 u_3 + 24\, u_0 u_1 u_2 u_3 + u_3^4\end{aligned}$$

And so on. These formulas are easy to compute by using Maple 13 software.

3. Solution of nonlinear Klein Gordon equation by ADM

Let us consider the nonlinear Klein Gordon equation

$$\frac{\partial^2 u}{\partial t^2} - \nabla^2 u + au + N(u) = f(\overline{x}, t) \qquad (4)$$

where a is real constant $u = u(\overline{x}, t)$, $N(u)$ is a given nonlinear function and $\nabla^2 = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}$ and $\overline{x} = x + y + z$. The initial conditions are $u(\overline{x}, 0) = b_0(\overline{x})$ and $\left.\frac{\partial}{\partial t} u(\overline{x}, t)\right|_{t=0} = b_1(\overline{x})$, by ADM we suppose that $u = \sum_{n=0}^{\infty} u_n$ and $N(u) = \sum_{n=0}^{\infty} \overline{A}_n$, where $\overline{A}_n$ defined as (3), so $u_0 = b_0(\overline{x}) + b_1(\overline{x})\, t + L^{-1}\{f(\overline{x}, t)\}$ and $u_{n+1} = L^{-1}(\nabla^2 u_n) - aL^{-1}(u_n) + L^{-1}\{N(u_n)\}$, $n \geq 1$. The term $\varphi_n = \sum_{r=0}^{n} u_r(x, t)$ converge to the solution $u(x, t)$ as $n \to \infty$.

4. Test problems

In this section we use ADM with polynomial (3) for examples (3.1) [2] and (3.2) [3].

Example 4.1

Consider the equation

$$\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} + \frac{\pi^2}{4} u + u^2 = x^2 \sin^2 \frac{\pi t}{2} \tag{5}$$

With the initial conditions $u(x,0) = 0$ and $u_t(x,0) = \frac{\pi x}{2}$. The exact solution of (5) is $u(x,t) = x \sin \frac{\pi t}{2}$. The Numerical and Exact solutions are shown in Figures 1.a, 1.b and 1.c. also table (1) shows the comparison between the results by ADM (D) with polynomial (2), ADM (M) with the polynomial (3) and the exact solution (E).

Example 4.2

Consider the coupled system of nonlinear physical equations

$$\frac{\partial u(x,t)}{\partial t} = u(1 - u^2 - v) + \frac{\partial^2 u}{\partial x^2}, \qquad t > 0,$$
$$\frac{\partial v(x,t)}{\partial t} = v(1 - u - v) + \frac{\partial^2 v}{\partial x^2}$$

With the initial conditions $u(x,0) = \frac{e^{kx}}{\left(1 + e^{kx}\right)}$, $v(x,0) = \frac{1 + \frac{3}{4} e^{kx}}{\left(1 + e^{kx}\right)^2}$, where k is constants.

The exact solutions are $u(z) = \frac{e^{k(x+ct)}}{\left(1 + e^{k(x+ct)}\right)}$ and $v(z) = \frac{1 + \frac{3}{4} e^{k(x+ct)}}{\left(1 + e^{k(x+ct)}\right)^2}$.

With a fixed values of $k = 1$ and $c = 1$ and for different values of time t the numerical solutions of $u(x,t)$ and $v(x,t)$ by ADM with polynomial (2) are shown in Figures. 2.a and 2.b, while the numerical solutions of $u(x,t)$ and $v(x,t)$ by ADM with polynomial (3) are shown in Figures 3.a and 3.b. The behavior of the exact solutions of $u(x,t)$ and $v(x,t)$ are shown in Figures 4.a and 4.b. Table (2) shows the comparison between the results by ADM (*Du* and *Dv*) with polynomial (2), ADM (*Mu* and *Mv*) with the polynomial (3) and the exact solution (*Eu* and *Ev*).

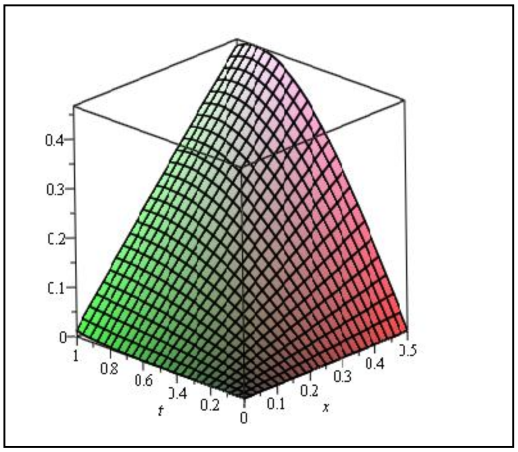

Fig. 1.a The solution of u with polynomial (2)

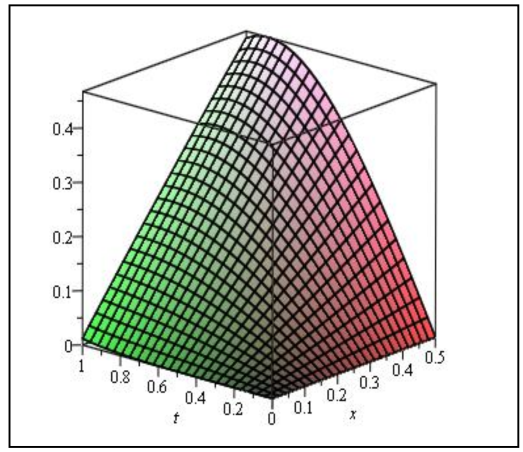

Fig. 1.b The solution of u with polynomial (3)

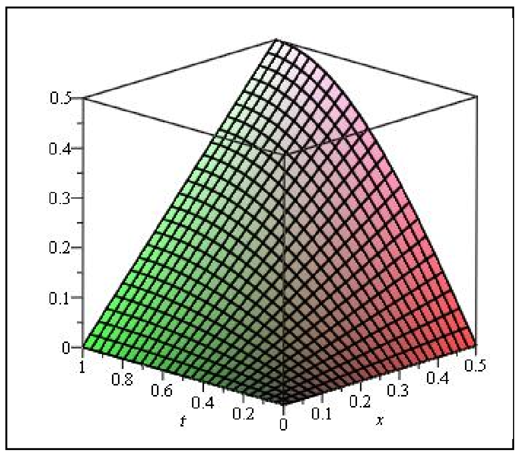

Fig. 1.c The Exact solution of u

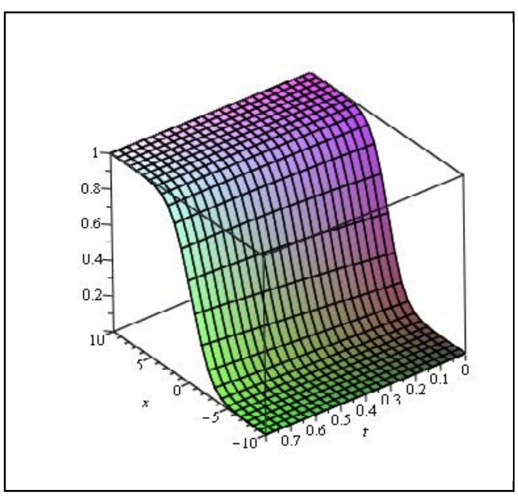

Fig. 2.a the solution of $u(x,t)$ by ADM with polynomial (2)

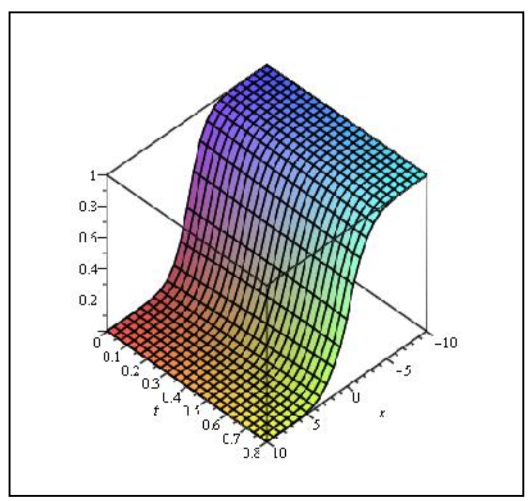

Fig. 2.b the solution of $v(x,t)$ by ADM with polynomial (2)

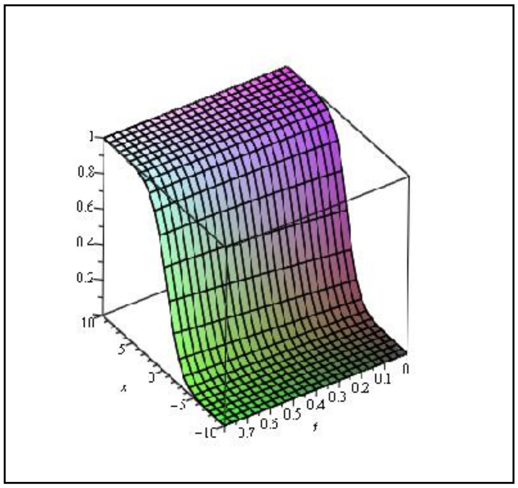

Fig. 3.a the solution of $u(x,t)$ by ADM with polynomial (3)

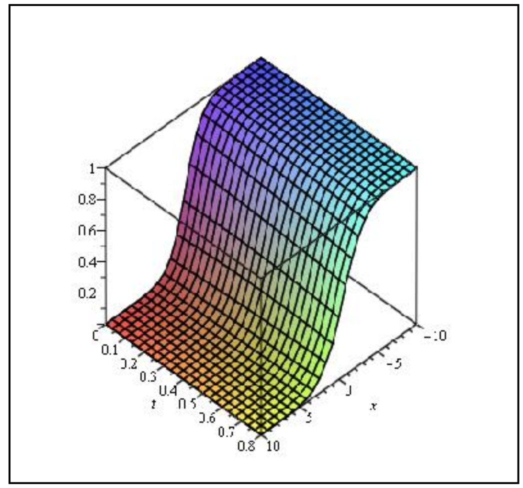

Fig. 3.b the solution of $v(x,t)$ by ADM with polynomial (3)

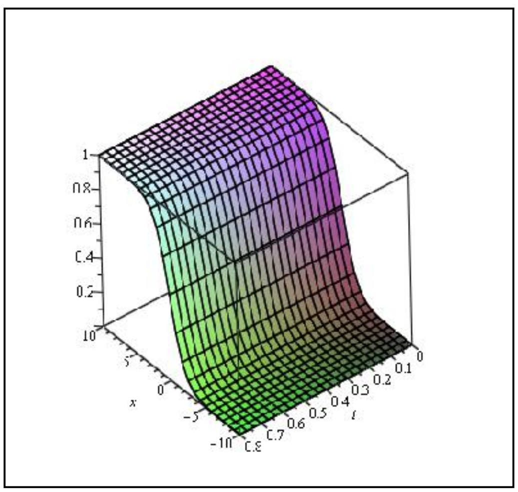

Fig. 4.a the Exact solution of $u(x,t)$

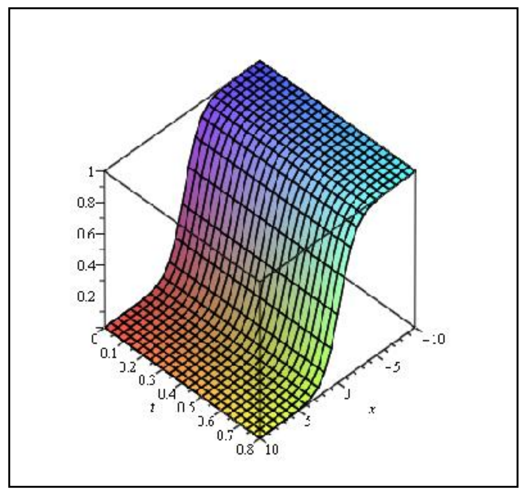

Fig. 4.b the Exact solution of $v(x,t)$

Table (1)

	x = 0.1	**x = 0.2**	**x = 0.3**	**x = 0.4**	**x = 0.5**
t = 0.1	**0.015643(E)**	**0.031287(E)**	**0.046930(E)**	**0.062574(E)**	**0.078217(E)**
	0.015643(D)	**0.031287(D)**	**0.046930(D)**	**0.062574(D)**	**0.078217(D)**
	0.015643(M)	**0.031287(M)**	**0.046930(M)**	**0.062574(M)**	**0.078217(M)**
t = 0.2	**0.030902(E)**	**0.061803(E)**	**0.092705(E)**	**0.123607(E)**	**0.154508(E)**
	0.030902(D)	**0.061803(D)**	**0.092705(D)**	**0.123607(D)**	**0.154508(D)**
	0.030901(M)	**0.061803(M)**	**0.092705(M)**	**0.123607(M)**	**0.154509(M)**

	x = 0.1	x = 0.2	x = 0.3	x = 0.4	x = 0.5
t = 0.3	0.045399(E)	0.090798(E)	0.136197(E)	0.181596(E)	0.226995(E)
	0.045399(D)	0.090798(D)	0.136197(D)	0.181596(D)	0.226995(D)
	0.045399(M)	0.090798(M)	0.136197(M)	0.181596(M)	0.226996(M)
t = 0.4	0.058779(E)	0.117557(E)	0.176336(E)	0.235114(E)	0.293893(E)
	0.058779(D)	0.117557(D)	0.176336(D)	0.235114(D)	0.293893(D)
	0.058778(M)	0.117557(M)	0.176336(M)	0.235114(M)	0.293893(M)
t = 0.5	0.070711(E)	0.141422(E)	0.212132(E)	0.282843(E)	0.353553(E)
	0.070711(D)	0.141422(D)	0.212132(D)	0.282843(D)	0.353553(D)
	0.070710(M)	0.141422(M)	0.212132(M)	0.282843(M)	0.353554(M)
t = 0.6	0.080902(E)	0.161803(E)	0.242705(E)	0.323607(E)	0.404508(E)
	0.080904(D)	0.161806(D)	0.242707(D)	0.323608(D)	0.404508(D)
	0.080901(M)	0.161804(M)	0.242705(M)	0.323607(M)	0.404509(M)
t = 0.7	0.089101(E)	0.178201(E)	0.267302(E)	0.356403(E)	0.445503(E)
	0.089113(D)	0.178213(D)	0.267311(D)	0.356408(D)	0.445503(D)
	0.089100(M)	0.178202(M)	0.267302(M)	0.356403(M)	0.445504(M)
t = 0.8	0.095106(E)	0.190211(E)	0.285317(E)	0.380423(E)	0.475528(E)
	0.095153(D)	0.190257(D)	0.285355(D)	0.380446(D)	0.475530(D)
	0.095105(M)	0.190211(M)	0.285317(M)	0.380423(M)	0.475528(M)
t = 0.9	0.098769(E)	0.197538(E)	0.296307(E)	0.395075(E)	0.493844(E)
	0.098925(D)	0.197689(D)	0.296436(D)	0.395160(D)	0.493858(D)
	0.098768(M)	0.197538(M)	0.296307(M)	0.395076(M)	0.493844(M)
t=1	0.100090(E)	0.200000(E)	0.300000(E)	0.400000(E)	0.500000(E)
	0.100448(D)	0.200443(D)	0.300386(D)	0.400264(D)	0.500060(D)
	0.100000(M)	0.200000(M)	0.300000(M)	0.400000(M)	0.500000(M)

Table(2)

	x = 0	*x* = 0.1	*x* = 0.2	*x* = 0.3	*x* = 0.4	*x* = 0.5
t = 0	0.500000 (Eu)	0.524979 (Eu)	0.549833 (Eu)	0.574443 (Eu)	0.598687 (Eu)	0.622459 (Eu)
	0.500000 (Du)	0.524979 (Du)	0.549833 (Du)	0.574443 (Du)	0.598687 (Du)	0.622459 (Du)
	0.500000(Mu)	0.524979(Mu)	0.549833(Mu)	0.574443(Mu)	0.598687(Mu)	0.622459(Mu)
	0.437500 (Ev)	0.412678 (Ev)	0.388288 (Ev)	0.364442 (Ev)	0.341248 (Ev)	0.318790 (Ev)
	0.437500 (Dv)	0.412678 (Dv)	0.388288 (Dv)	0.364442 (Dv)	0.341248 (Dv)	0.318790 (Dv)
	0.437500(Mv)	0.412678(Mv)	0.388288(Mv)	0.364442(Mv)	0.341248(Mv)	0.318790(Mv)
t= 0.1	0.500000 (Eu)	0.524979 (Eu)	0.549833 (Eu)	0.574443 (Eu)	0.598687 (Eu)	0.622459 (Eu)
	0.514941 (Du)	0.539395 (Du)	0.563663 (Du)	0.587633 (Du)	0.611203 (Du)	0.634269 (Du)
	0.514941(Mu)	0.539395(Mu)	0.563663(Mu)	0.587633(Mu)	0.611203(Mu)	0.634269(Mu)
	0.437500 (Ev)	0.412678 (Ev)	0.388288 (Ev)	0.364442 (Ev)	0.341248 (Ev)	0.318790 (Ev)
	0.442012 (Dv)	0.418177 (Dv)	0.394696 (Dv)	0.371676 (Dv)	0.349203 (Dv)	0.327362 (Dv)
	0.442532(Mv)	0.418680(Mv)	0.395184(Mv)	0.372150(Mv)	0.349667(Mv)	0.327814(Mv)
t= 0.2	0.500000 (Eu)	0.524979 (Eu)	0.549833 (Eu)	0.574443 (Eu)	0.598687 (Eu)	0.622459 (Eu)
	0.528516 (Du)	0.552407 (Du)	0.576063 (Du)	0.599391 (Du)	0.622276 (Du)	0.644648 (Du)
	0.528516(Mu)	0.552407(Mu)	0.576063(Mu)	0.599391(Mu)	0.622276(Mu)	0.644648(Mu)
	0.437500 (Ev)	0.412678 (Ev)	0.388288 (Ev)	0.364442 (Ev)	0.341248 (Ev)	0.318790 (Ev)
	0.443828 (Dv)	0.420846 (Dv)	0.398188 (Dv)	0.375938 (Dv)	0.354184 (Dv)	0.333001 (Dv)
	0.445894(Mv)	0.422840(Mv)	0.400118(Mv)	0.377812(Mv)	0.356008(Mv)	0.334781(Mv)
t= 0.3	0.500000 (Eu)	0.524979 (Eu)	0.549833 (Eu)	0.574443 (Eu)	0.598687 (Eu)	0.622459 (Eu)
	0.540723 (Du)	0.564017 (Du)	0.587029 (Du)	0.609688 (Du)	0.631899 (Du)	0.653580 (Du)
	0.540723(Mu)	0.564017(Mu)	0.587029(Mu)	0.609688(Mu)	0.631899(Mu)	0.653580(Mu)
	0.437500 (Ev)	0.412678 (Ev)	0.388288 (Ev)	0.364442 (Ev)	0.341248 (Ev)	0.318790 (Ev)
	0.442949 (Dv)	0.420688 (Dv)	0.398751 (Dv)	0.377228 (Dv)	0.356192 (Dv)	0.335711 (Dv)
	0.447560(Mv)	0.425128(Mv)	0.403039(Mv)	0.381385(Mv)	0.360231(Mv)	0.339646(Mv)
t= 0.4	0.500000 (Eu)	0.524979 (Eu)	0.549833 (Eu)	0.574443 (Eu)	0.598687 (Eu)	0.622459 (Eu)
	0.551562 (Du)	0.574218 (Du)	0.596570 (Du)	0.618546 (Du)	0.640086 (Du)	0.661077 (Du)
	0.551562(Mu)	0.574218(Mu)	0.596570(Mu)	0.618546(Mu)	0.640086(Mu)	0.661077(Mu)
	0.437500 (Ev)	0.412678 (Ev)	0.388288 (Ev)	0.364442 (Ev)	0.341248 (Ev)	0.318790 (Ev)
	0.439375 (Dv)	0.417702 (Dv)	0.396397 (Dv)	0.375547 (Dv)	0.355222 (Dv)	0.335483 (Dv)
	0.447505(Mv)	0.425513(Mv)	0.403926(Mv)	0.382831(Mv)	0.362289(Mv)	0.342360(Mv)
t= 0.5	0.500000 (Eu)	0.524979 (Eu)	0.549833 (Eu)	0.574443 (Eu)	0.598687 (Eu)	0.622459 (Eu)
	0.561035 (Du)	0.583011 (Du)	0.604668 (Du)	0.625976 (Du)	0.646819 (Du)	0.667139 (Du)
	0.561035(Mu)	0.583011(Mu)	0.604668(Mu)	0.625976(Mu)	0.646819(Mu)	0.667139(Mu)
	0.437500 (Ev)	0.412678 (Ev)	0.388288 (Ev)	0.364442 (Ev)	0.341248 (Ev)	0.318790 (Ev)
	0.433105 (Dv)	0.411887 (Dv)	0.391123 (Dv)	0.370894 (Dv)	0.351278 (Dv)	0.332332 (Dv)
	0.445704(Mv)	0.423964(Mv)	0.402741(Mv)	0.382112(Mv)	0.362137(Mv)	0.342888(Mv)

	x = 0	*x* = 0.1	*x* = 0.2	*x* = 0.3	*x* = 0.4	*x* = 0.5
	0.500000 (Eu)	**0.524979 (Eu)**	**0.549833 (Eu)**	**0.574443 (Eu)**	**0.598687 (Eu)**	**0.622459 (Eu)**
	0.569141 (Du)	**0.590398 (Du)**	**0.611357 (Du)**	**0.631943 (Du)**	**0.652113 (Du)**	**0.671747(Du)**
	0.569141(Mu)	**0.590398(Mu)**	**0.611357(Mu)**	**0.631943(Mu)**	**0.652113(Mu)**	**0.671747(Mu)**
***t*=0.6**	**0.437500 (Ev)**	**0.412678 (Ev)**	**0.388288 (Ev)**	**0.364442 (Ev)**	**0.341248 (Ev)**	**0.318790 (Ev)**
	0.424141 (Dv)	**0.403240 (Dv)**	**0.382919 (Dv)**	**0.363272 (Dv)**	**0.344352 (Dv)**	**0.32624 (Dv)**
	0.442131(Mv)	**0.420449(Mv)**	**0.399440(Mv)**	**0.379185(Mv)**	**0.359731(Mv)**	**0.34117 (Mv)**
	0.500000 (Eu)	**0.524979 (Eu)**	**0.549833 (Eu)**	**0.574443 (Eu)**	**0.598687 (Eu)**	**0.622459 (Eu)**
	0.575879 (Du)	**0.596383 (Du)**	**0.616608 (Du)**	**0.636487 (Du)**	**0.655966 (Du)**	**0.674930(Du)**
t=0.7	**0.575879(Mu)**	**0.596383(Mu)**	**0.616608(Mu)**	**0.636487(Mu)**	**0.655966(Mu)**	**0.674930(Mu)**
	0.437500 (Ev)	**0.412678 (Ev)**	**0.388288 (Ev)**	**0.364442 (Ev)**	**0.341248 (Ev)**	**0.318790 (Ev)**
	0.412480(DV)	**0.391758(DV)**	**0.371804(DV)**	**0.352667(DV)**	**0.334457(DV)**	**0.31721 (DV)**
	0.436761(MV)	**0.414938(MV)**	**0.393994(MV)**	**0.374012(MV)**	**0.355051(MV)**	**0.33716 (MV)**
	0.500000 (Eu)	**0.524979 (Eu)**	**0.549833 (Eu)**	**0.574443 (Eu)**	**0.598687 (Eu)**	**0.622459 (Eu)**
	0.581250 (Du)	**0.600964 (Du)**	**0.620422 (Du)**	**0.639561 (Du)**	**0.658374 (Du)**	**0.676693(Du)**
	0.581250(Mu)	**0.600964(Mu)**	**0.620422(Mu)**	**0.639561(Mu)**	**0.658374(Mu)**	**0.676693(Mu)**
t=0.8	**0.437500 (Ev)**	**0.412678 (Ev)**	**0.388288 (Ev)**	**0.364442 (Ev)**	**0.341248 (Ev)**	**0.318790 (Ev)**
	0.398125(DV)	**0.377469(DV)**	**0.357771(DV)**	**0.339113(DV)**	**0.321617(DV)**	**0.30525 (DV)**
	0.429570(MV)	**0.407419(MV)**	**0.386381(MV)**	**0.366570(MV)**	**0.348059(MV)**	**0.33083 (MV)**

5. Conclusions

In this paper, the ADM was applied for finding the solutions of the nonlinear partial differential equations and the system of nonlinear partial differential equations. It’s clear that ADM with different polynomials give different solutions. The ADM with polynomial (3) is better than ADM with polynomial (2). Also polynomial (3) are more simpler than polynomial (2) in calculations. It may be concluded that ADM with polynomial (3) is very powerful and efficient in application for wide classes of problems.

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