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Shape Transition and Collective Excitations in $^{166-180}$Hf Isotopes

شكل الانتقال والتهيجات الجماعية في نظائر الهافنيوم ١٦٦-١٨٠

أ.م.د.عامر عبد الأمير محمد علي
قسم الفيزياء / كلية العلوم / جامعة كربلاء

Abstract

The low-lying collective levels in $^{166-180}$Hf isotopes are investigated in the frame work of the interacting boson model (IBM-1). The contour plot of the potential energy surfaces, $V(\beta,\gamma)$, shows two wells on the prolate and oblate sides which indicate that all hafnium nuclei are deformed and have rotational characters. The levels energy, electromagnetic transition rates B(E2) and electric quadrupole moment [$Q_{2_1^+}$] are calculated. All calculated values are compared with the available experimental data and show reasonable agreement.

Key Words: [IBM-1, deformation parameter, quadrupole moment, electromagnetic transitions.]

الخلاصة

المستويات الجماعية الدنيا في نظائر الهافنيوم ١٦٦-١٨٠ حققت باستخدام نموذج البوزونات المتفاعلة-١. مخطط كونتور لطاقة جهد السطح بين إن شطري الجهد للبروليت والابليت لنظائر الهافنيوم على أنها تمتلك خصائص التحديد ألدوراني. مستويات الطاقة، والقيمة المختزلة للانتقالات الكهربائية ورباعي القطب الكهربائي تم حسابها. جميع القيم المحسوبة قورنت مع القيم العملية المتوفرة وبينت تلك النتائج التطابق المعقول.

Introduction

The medium-to heavy-mass hafnium (Hf) isotopes are located in the rear-earth mass region were well-deformed nuclei can be populated to very high spin. Nuclei in the mass number A=150-190 region exhibit axially symmetric prolate deformation in their ground states. The low-laying excited states of these nuclei are therefore characterized by collective rotational bands. Furthermore, near their respective Fermi levels, both protons and neutrons have available high-j orbitals with large projections (Ω) along the symmetry axis [1]. This stimulates competition along the yarst line between collective angular momentum perpendicular to symmetry axis and particle angular momentum aligned along the symmetry axis. The interplay and changing dominance between collective and noncolective modes of excitations as a function of angular momentum remains a key focus of nuclear structure investigations. The stable isotopes of hafnium (Z=72) have fairly large deformation parameters ($\beta_2 \approx 0.26$) and relatively high γ vibrational bands (≈ 1.2 MeV) [2]. The hafnium nuclei are located in the region where β_2 starts to decrease [3, 4] and by implication the tendency toward γ softness begins. Hence it is of interest to investigate the shapes of the hafnium nuclei by determining with high precision the isotope shifts, quadrupole moments, and B(E2) values of their low lying states.

Interacting Boson Model (IBM)

The IBM has become one of the most intensively used nuclear models, due to its ability to describe of the changing low-lying collective properties of nuclei across an entire major shell with a simple Hamiltonian. In the IBM the spectroscopies of low-lying collective properties of even-even nuclei are described in terms of a system of interacting **s** bosons (**L=0**) and **d** bosons (**L=2**). Furthermore, the model assumes that the structure of low-lying levels is dominated by excitations among the valence partials outside major closed shells. In the particle space the number of proton bosons N_π, and neutron bosons N_ν, is counted from the nearest closed shell, and the resulting total boson number is a strictly conserved quantity.
The underlying structure of the six-dimensional unitary group SU(6) of the model leads to a simple Hamiltonian, capable of describing the three specific types of collective structure with classical geometrical analogues (vibrational [5], rotational [6], and γ-unstable[7]) and also the transitional nuclei [8] whose structure are intermediate.
The full Hamiltonian **H** contains six adjustable parameters, and can be written explicitly in terms of boson creation $\mathbf{d}^\dagger$ and annihilation **d** operators such that [9]

$$\mathbf{H} = \boldsymbol{\varepsilon} n_d + \mathbf{a}_0 \mathbf{P}^\dagger \cdot \mathbf{P} + \mathbf{a}_1 \mathbf{L}.\mathbf{L} + \mathbf{a}_2 \mathbf{Q}.\mathbf{Q} + \mathbf{a}_3 \mathbf{T}_3.\mathbf{T}_3 + \mathbf{a}_4 \mathbf{T}_4.\mathbf{T}_4 \qquad (1)$$

Where $\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}_d - \boldsymbol{\varepsilon}_s$ is the boson energy. The parameters a_0, a_1, a_2, a_3 and a_4 designated the strength of the pairing, angular momentum, quadrupole, octupole and hexdecupole interaction between the bosons.

Another advantage of the interacting d-boson model is that the matrix elements of the electric quadrupole operator. The reduced matrix elements of the E2 operator T(E2) has the form [5]

$$\mathbf{T}(E2) = \boldsymbol{\alpha}_2 [\mathbf{d}^\dagger \mathbf{s} + \mathbf{s}^\dagger \mathbf{d}]^{(2)} + \boldsymbol{\beta}_2 [\mathbf{d}^\dagger \mathbf{d}]^{(2)} \qquad (2)$$

Where ($s^\dagger$, $d^\dagger$) and (s, d) are creation and annihilation operators for s and d bosons, respectively, while α_2 and β_2 are two parameters. The quadropole moments for the $L=2^+$ state are defined as [10],

$$Q_{2_1^+} = -\alpha_2 \sqrt{\frac{16\pi}{40}} \frac{2}{7} (4N+3) \qquad (3)$$

Potential Energy Surface

In order to obtain a more intuitive insight into the problem of triaxial shapes, the classical limit of the Hamiltonian of Eq.(1) can be calculated. The technique described by Dieperink et al[9] allows one also to give an algebraic description of the nature of the transition between one phase and another. Simpler expressions, which display the essential dependence on β and γ, have been given[11],

$$E(N,\beta,\gamma) = \varepsilon_d N [\beta^2/(1+\beta^2)] + a_1 N (N-1) [\beta^4/(1+\beta^2)^2] , \ldots SU(5)$$

$$E(N,\beta,\gamma) = a_2 N(N-1) [(1+ \tfrac{3}{4} \beta^4 - \sqrt{2}\, \beta^3 \cos 3\gamma)/(1+\beta^2)^2] , \ldots SU(3) \ldots (4)$$

$$E(N,\beta,\gamma) = a_0 N(N-1) [(1-\beta^2)/(1+\beta^2)]^2 , \ldots\ldots\ldots\ldots\ldots O(6)$$

These expression give (for large N) $\beta_{min} = 0$, $\sqrt{2}$, and 1 for SU(5), SU(3), and O(6), respectively.

Results and Discussion

In the present work the rotational limit of the IBM-1 has been applied to $^{172-180}$Hf and breaking this limit for $^{166-170}$Hf by introduction of **P.P** term into the Hamiltonian, so that, for example, the β and γ bands are no longer degenerate, the wave function are still not greatly different from those of the SU (3) limit, and hence, in a geometrical description, the bands can still be thought of as having **K** quantum numbers. The levels energy, transitions probabilities B(E2)'s, electric quadropole moments for the $L=2_1^+$ state and the potential energy surfaces of the nucleus has been calculated and compared with the available experimental data. The calculations have been preformed with code IBM (IBM-1) and hence, no distinction made between neutron and proton bosons. For the analysis of excitation energies in hafnium isotopes it was tried to keep to minimum the number of free parameters in Hamiltonian. The explicit expression of Hamiltonian adopted in calculations is[9],

$$\mathbf{H = a_1\, L \cdot L + a_2\, Q \cdot Q + a_0\, P^{\dagger} \cdot P}$$

In the framework of the IBM-1, the isotopic chains of hafnium (Z=72) nuclei, having a number of proton bosons holes 5, a number of neutron bosons particles varies from (6 to 11) for $^{166-176}$Hf, and number of neutron boson holes for $^{178-180}$Hf are (10-9).

The coefficient values that give a good agreement with the experimental results are shown in Table (1). Fig's (1-8) show the calculated excitation pattern for positive parity states in $^{166-180}$Hf are compared to the experimental one [12-19].

Table 1. Adopted values for the parameters used for IBM-1 calculations. All parameters are given in MeV.

A	EPS	P.P	L.L	Q.Q	$T_3.T_3$	$T_4.T_4$	CHI
166	0.0	0.005	0.02	-0.013	0.0	0.0	-1.12
168	0.0	0.004	0.0175	-0.013	0.0	0.0	-1.12
170	0.0	0.003	0.0127	-0.0117	0.0	0.0	-1.30
172	0.0	0.0	0.0117	-0.0106	0.0	0.0	-1.30
174	0.0	0.0	0.0116	-0.0094	0.0	0.0	-1.32
176	0.0	0.0	0.0102	-0.0122	0.0	0.0	-1.31
178	0.0	0.0	0.0104	-0.0130	0.0	0.0	-1.31
180	0.0	0.0	0.0106	-0.0135	0.0	0.0	-1.32

For the calculations of the absolute B(E2) values two parameters α_2 and β_2 of (2) were adjusted according to the experimental $B(E2;2_1^+ \rightarrow 0_1^+)$. Table (2) shows the values of the parameters E2SD and E2DD, obtained in the present calculation.

Table 2. Parameters obtained from T(E2) operator in program IBMT to calculate the absolute B(E2) values.

A	E2SD (eb)	E2DD (eb)
166	0.113	-0.148
168	0.114	-0.152
170	0.116	-0.153
172	0.100	-0.133
174	0.094	-0.124
176	0.098	-0.129
178	0.098	-0.129
180	0.104	-0.137

The transition probabilities of B(E2)'s are calculated and normalized to the previous experimental value as well as electric quadrupole moment and presented in Table (3). Unfortunately there are no enough measurements of electromagnetic transition rates B(E2) for these series nuclei.
The calculated values reported in Table (3) are fairly good agreement with the experimental ones. Can be noted in particular that the magnitude and sign of quadrupole moments as well as E2 probabilities for transitions of vastly different intensity originating from the same level are satisfactorily reproduced. The agreement between experimental and computed data for $0_2^+ \rightarrow 2_1^+$ transition gives further support to our interpretation of the 0_2^+ state as a state belonging to the IBM space.

Table 3. Experimental and calculated values for electric quadrupole moment Q(in eb) and reduced transition probabilities B(E2) (in e^2b^2).

A	Transition	B(E2)	
		Exp.	IBM-1
^{166}Hf	$2_1^+ \rightarrow 0_1^+$	0.694 (04) [20]	0.696
	$2_3^+ \rightarrow 0_2^+$	------	0.529
	$3_1^+ \rightarrow 2_2^+$	------	0.941
	$3_1^+ \rightarrow 2_3^+$	------	0.529
	$3_1^+ \rightarrow 4_2^+$	------	0.904
	$4_1^+ \rightarrow 2_1^+$	1.095 (04) [20]	0.976
	$6_1^+ \rightarrow 4_1^+$	1.20 (07) [20]	1.040
	$8_1^+ \rightarrow 6_1^+$	1.52 (16) [20]	1.033
	$Q_{2_1^+}$	------	-2.233
^{168}Hf	$2_1^+ \rightarrow 0_1^+$	0.843 (04) [21]	0.843
	$2_3^+ \rightarrow 0_2^+$	------	0.655
	$3_1^+ \rightarrow 2_2^+$	------	1.167
	$3_1^+ \rightarrow 2_3^+$	------	0.016
	$3_1^+ \rightarrow 4_2^+$	------	1.121
	$4_1^+ \rightarrow 2_1^+$	1.146 (12) [21]	1.186
	$6_1^+ \rightarrow 4_1^+$	1.305 (13) [21]	1.269
	$8_1^+ \rightarrow 6_1^+$	1.377 (16) [21]	1.273
	$Q_{2_1^+}$	------	-2.459
^{170}Hf	$2_1^+ \rightarrow 0_1^+$	1.010 (28) [14]	1.012
	$2_3^+ \rightarrow 0_2^+$	------	0.803
	$3_1^+ \rightarrow 2_2^+$	------	1.434
	$3_1^+ \rightarrow 2_3^+$	------	0.015
	$3_1^+ \rightarrow 4_2^+$	------	1.372
	$4_1^+ \rightarrow 2_1^+$	1.455 (17) [14]	1.427

	$6_1^+ \rightarrow 4_1^+$	1.444 (17) [14]	1.534
	$8_1^+ \rightarrow 6_1^+$	1.680 (17) [14]	1.548
	$Q_{2_1^+}$	------	-2.693

A	Transition	B(E2)	
		Exp.	IBM-1
^{172}Hf	$2_1^+ \rightarrow 0_1^+$	0.875 (06) [15]	0.871
	$2_3^+ \rightarrow 0_2^+$	------	0.702
	$3_1^+ \rightarrow 2_2^+$	------	1.252
	$3_1^+ \rightarrow 2_3^+$	------	0.012
	$3_1^+ \rightarrow 4_2^+$	------	1.204
	$4_1^+ \rightarrow 2_1^+$	------	1.23
	$6_1^+ \rightarrow 4_1^+$	------	1.33
	$8_1^+ \rightarrow 6_1^+$	------	1.345
	$Q_{2_1^+}$	------	-2.498
^{174}Hf	$2_1^+ \rightarrow 0_1^+$	0.877 (05) [16]	0.873
	$2_3^+ \rightarrow 0_2^+$	------	0.714
	$3_1^+ \rightarrow 2_2^+$	------	1.274
	$3_1^+ \rightarrow 2_3^+$	------	0.011
	$3_1^+ \rightarrow 4_2^+$	------	1.224
	$4_1^+ \rightarrow 2_1^+$	------	1.235
	$6_1^+ \rightarrow 4_1^+$	------	1.335
	$8_1^+ \rightarrow 6_1^+$	------	1.360
	$Q_{2_1^+}$	------	-2.500
^{176}Hf	$2_1^+ \rightarrow 0_1^+$	1.073 (04) [17]	1.072
	$2_3^+ \rightarrow 0_2^+$	------	0.888
	$3_1^+ \rightarrow 2_2^+$	------	1.586
	$3_1^+ \rightarrow 2_3^+$	------	0.011
	$3_1^+ \rightarrow 4_2^+$	------	1.523
	$4_1^+ \rightarrow 2_1^+$	------	1.518
	$6_1^+ \rightarrow 4_1^+$	------	1.645
	$8_1^+ \rightarrow 6_1^+$	------	1.681
	$Q_{2_1^+}$	-2.10 9(2) [22]	-2.770

A	Transition	B(E2)	
		Exp.	IBM-1
^{178}Hf	$2_1^+ \rightarrow 0_1^+$	0.946 (03)[18]	0.948
	$2_3^+ \rightarrow 0_2^+$	------	0.775
	$3_1^+ \rightarrow 2_2^+$	------	1.384
	$3_1^+ \rightarrow 2_3^+$	------	0.012
	$3_1^+ \rightarrow 4_2^+$	------	1.329
	$4_1^+ \rightarrow 2_1^+$	------	1.340
	$6_1^+ \rightarrow 4_1^+$	1.303 (07) [18]	1.449
	$8_1^+ \rightarrow 6_1^+$	1.410 (04) [18]	1.476
	$Q_{2_1^+}$	-2.02(2)[22]	-2.605
^{180}Hf	$2_1^+ \rightarrow 0_1^+$	0.936(03)[19]	0.936
	$2_3^+ \rightarrow 0_2^+$	------	0.754
	$3_1^+ \rightarrow 2_2^+$	------	1.347
	$3_1^+ \rightarrow 2_3^+$	------	0.013
	$3_1^+ \rightarrow 4_2^+$	------	1.294
	$4_1^+ \rightarrow 2_1^+$	1.389(0.18) [19]	1.322
	$6_1^+ \rightarrow 4_1^+$	1.323(0.13) [19]	1.426
	$8_1^+ \rightarrow 6_1^+$	1.510(0.24) [19]	1.446
	$Q_{2_1^+}$	-2.00(2)[22]	-2.589

The calculated potential energy surfaces, V (β, γ), for hafnium series of isotopes are presented in Fig. 9. It shows that all nuclei are deformed and have rotational-like characters. The prolate deformation is deeper than oblate in all nuclei.

Conclusions

The IBM-1 model has been applied successfully to $^{166\text{-}180}$Hf isotopes and we have got:

1- The rotational limit has been used and the ground state and octupole bands are successfully reproduced;

2. The potential energy surfaces are calculated and show rotational behavior to $^{166\text{-}180}$Hf isotopes where they are prolate deformed nuclei;

3. Electromagnetic transition rates B(E2) and electric quadrupole moment [$Q_{2_1^+}$] are calculated and there is no enough experimental data for comparison;

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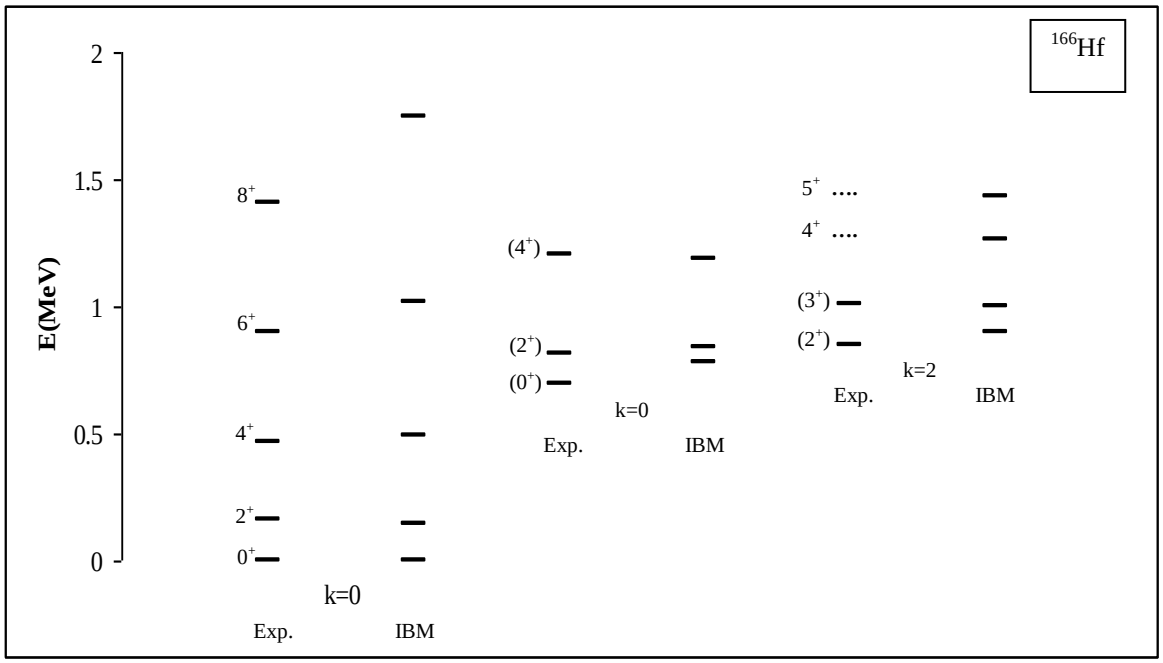

Fig. (1) Levels in ^{166}Hf compared with the results of IBM-1 calculations. Numbers in bracket refer to states whose spin and parity have been indefinitely assigned.

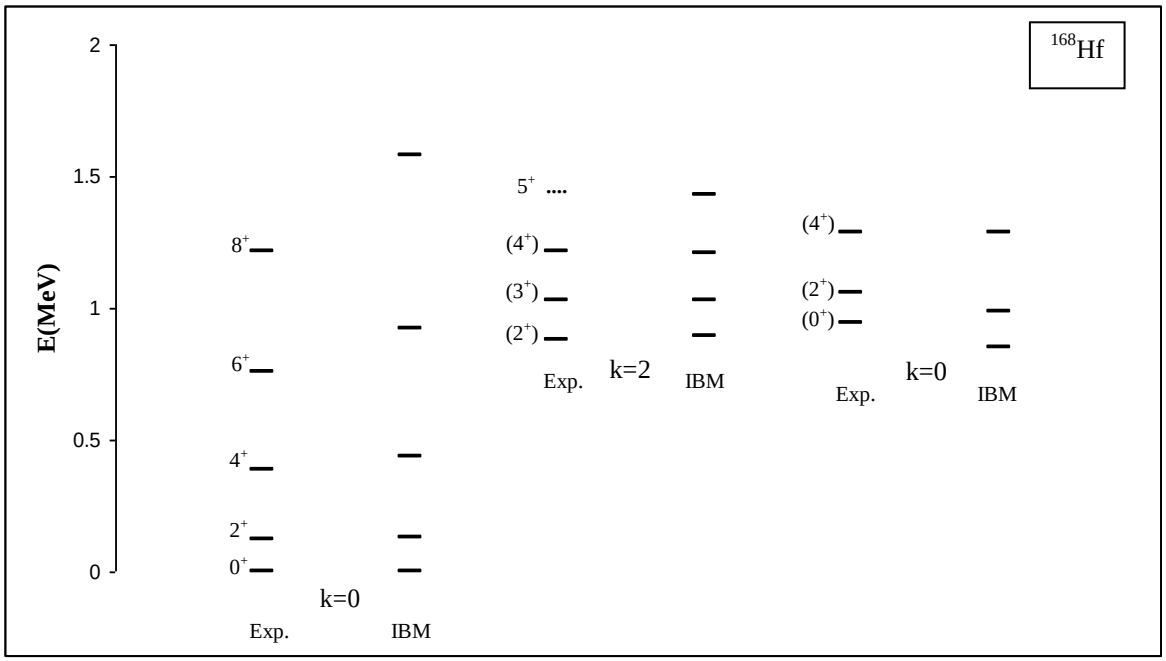

Fig. (2) Levels in ^{168}Hf compared with the results of IBM-1 calculations. Numbers in bracket refer to states whose spin and parity have been indefinitely assigned.

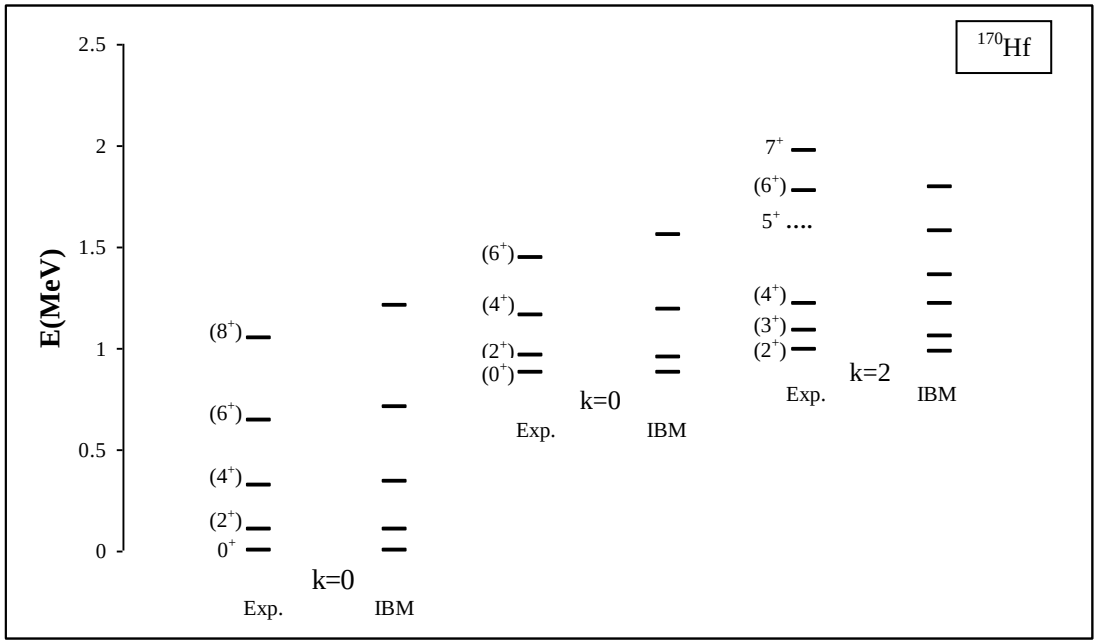

Fig. (3) Levels in ^{170}Hf compared with the results of IBM-1 calculations. Numbers in bracket refer to states whose spin and parity have been indefinitely assigned.

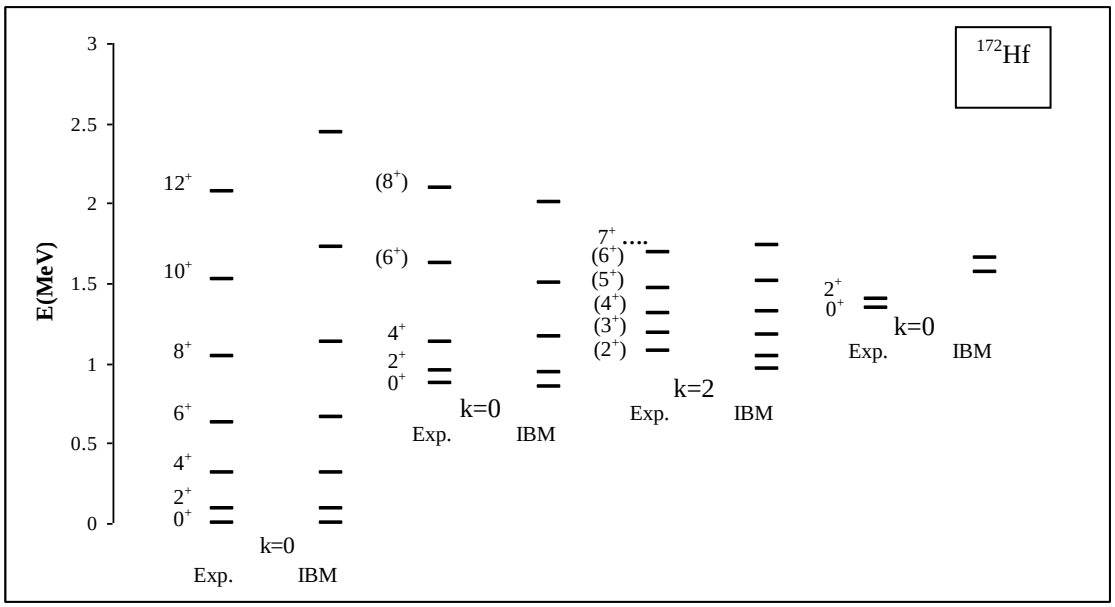

Fig. (4) Levels in ^{172}Hf compared with the results of IBM-1 calculations. Numbers in bracket refer to states whose spin and parity have been indefinitely assigned.

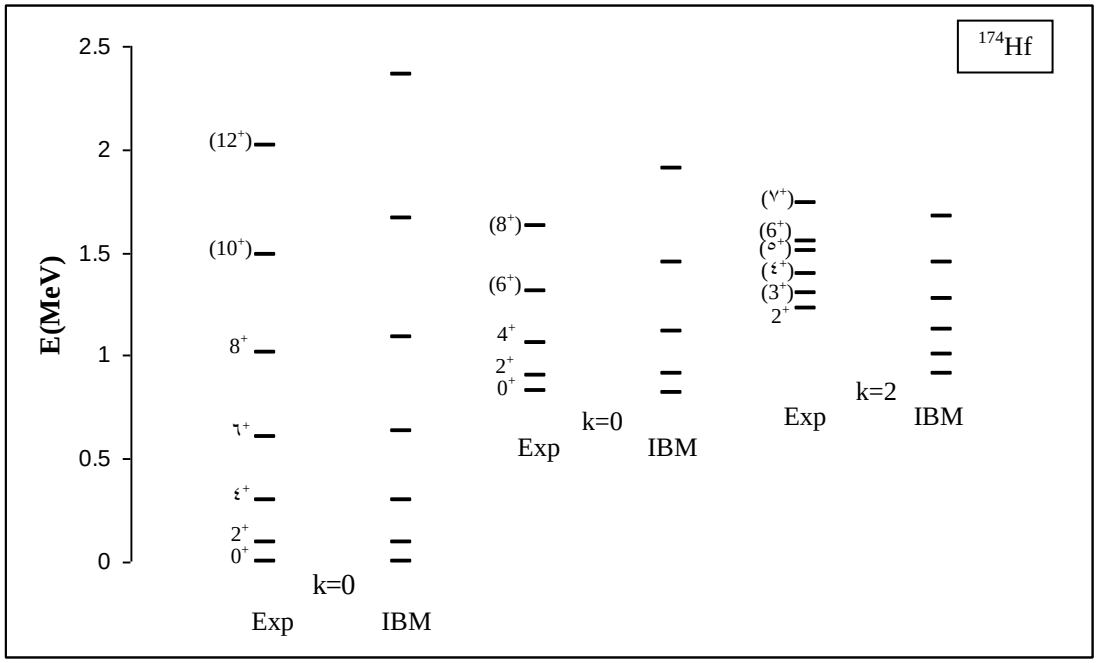

Fig. (5) Levels in ^{174}Hf compared with the results of IBM-1 calculations. Numbers in bracket refer to states whose spin and parity have been indefinitely assigned.

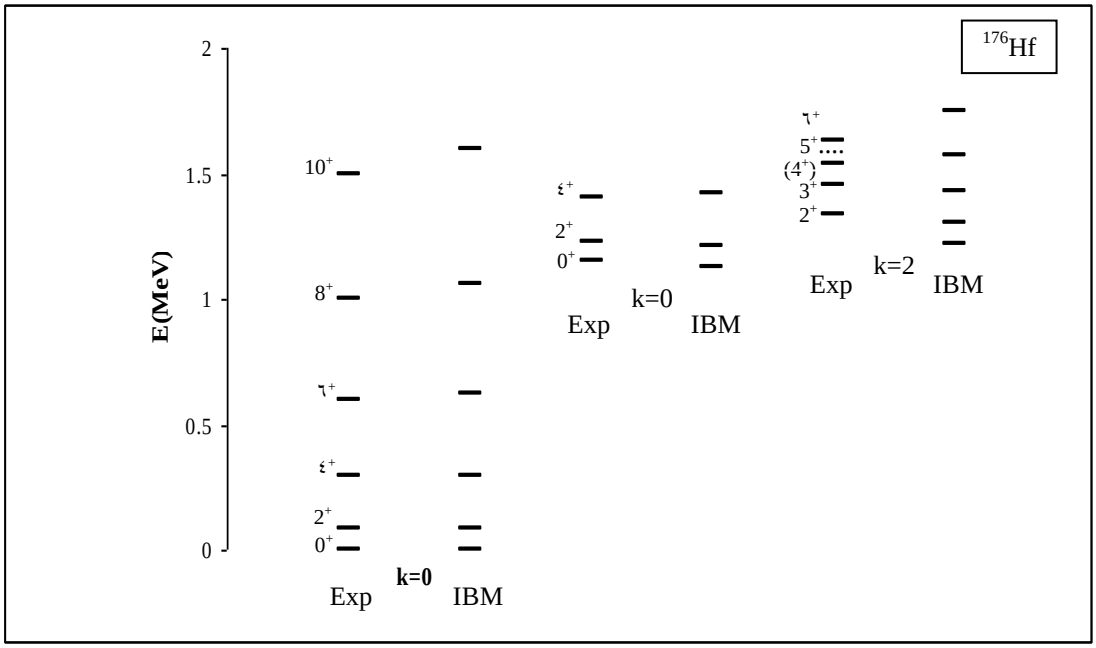

Fig. (6) Levels in ^{176}Hf compared with the results of IBM-1 calculations. Numbers in bracket refer to states whose spin and parity have been indefinitely assigned.

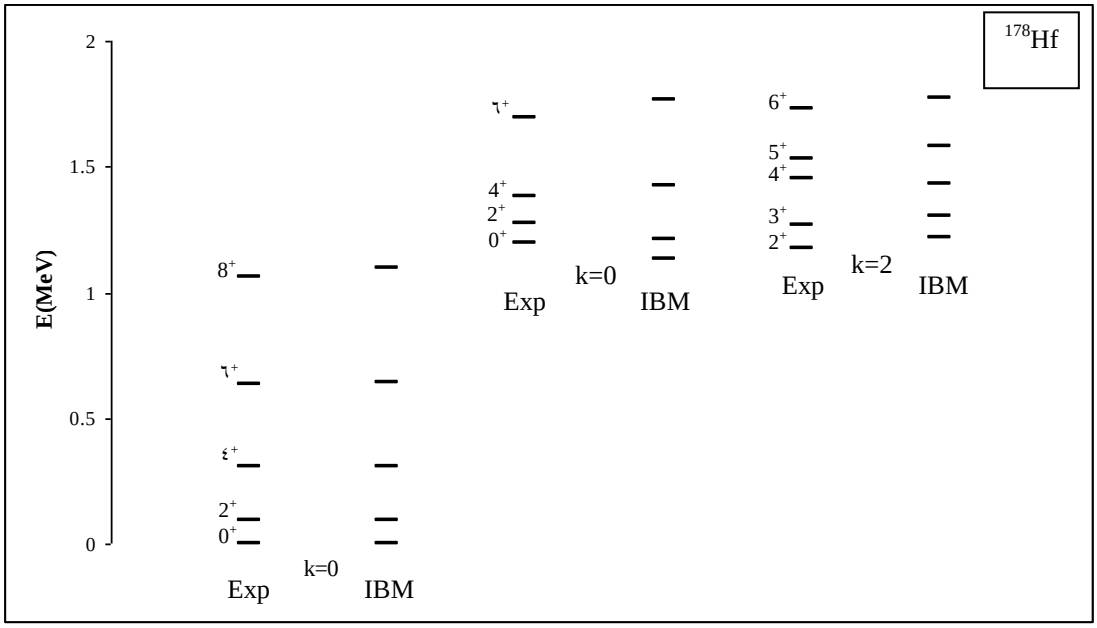

Fig. (7) Levels in ^{178}Hf compared with the results of IBM-1 calculations.

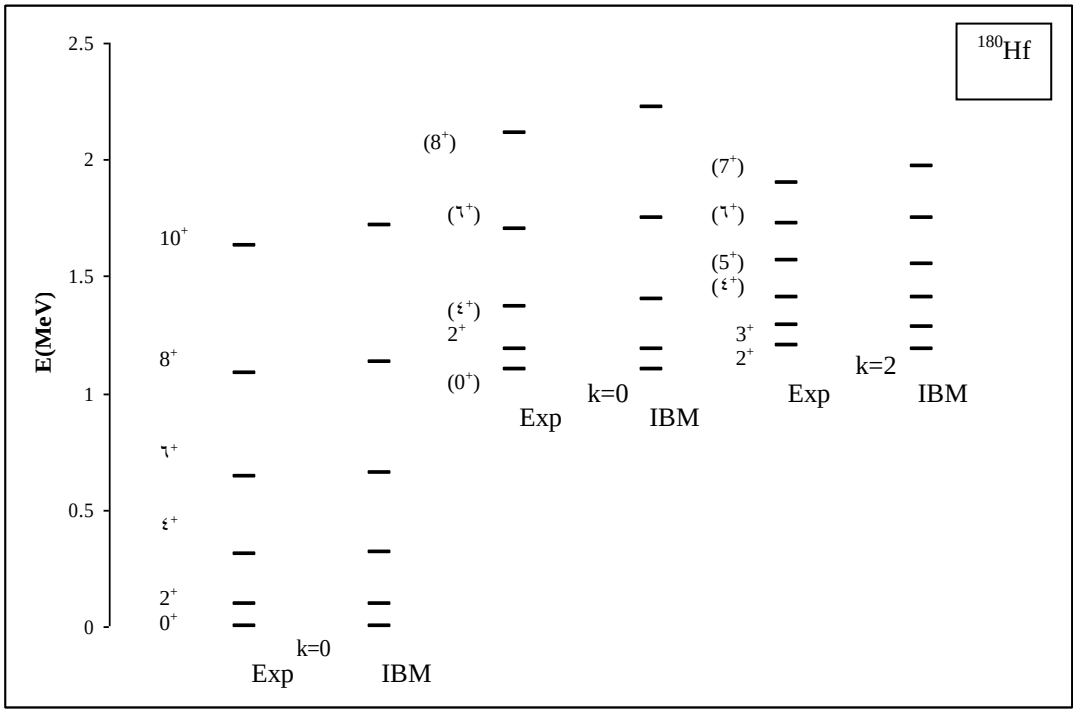

Fig. (8) Levels in ^{180}Hf compared with the results of IBM-1 calculations. Numbers in bracket refer to states whose spin and parity have been indefinitely assigned.

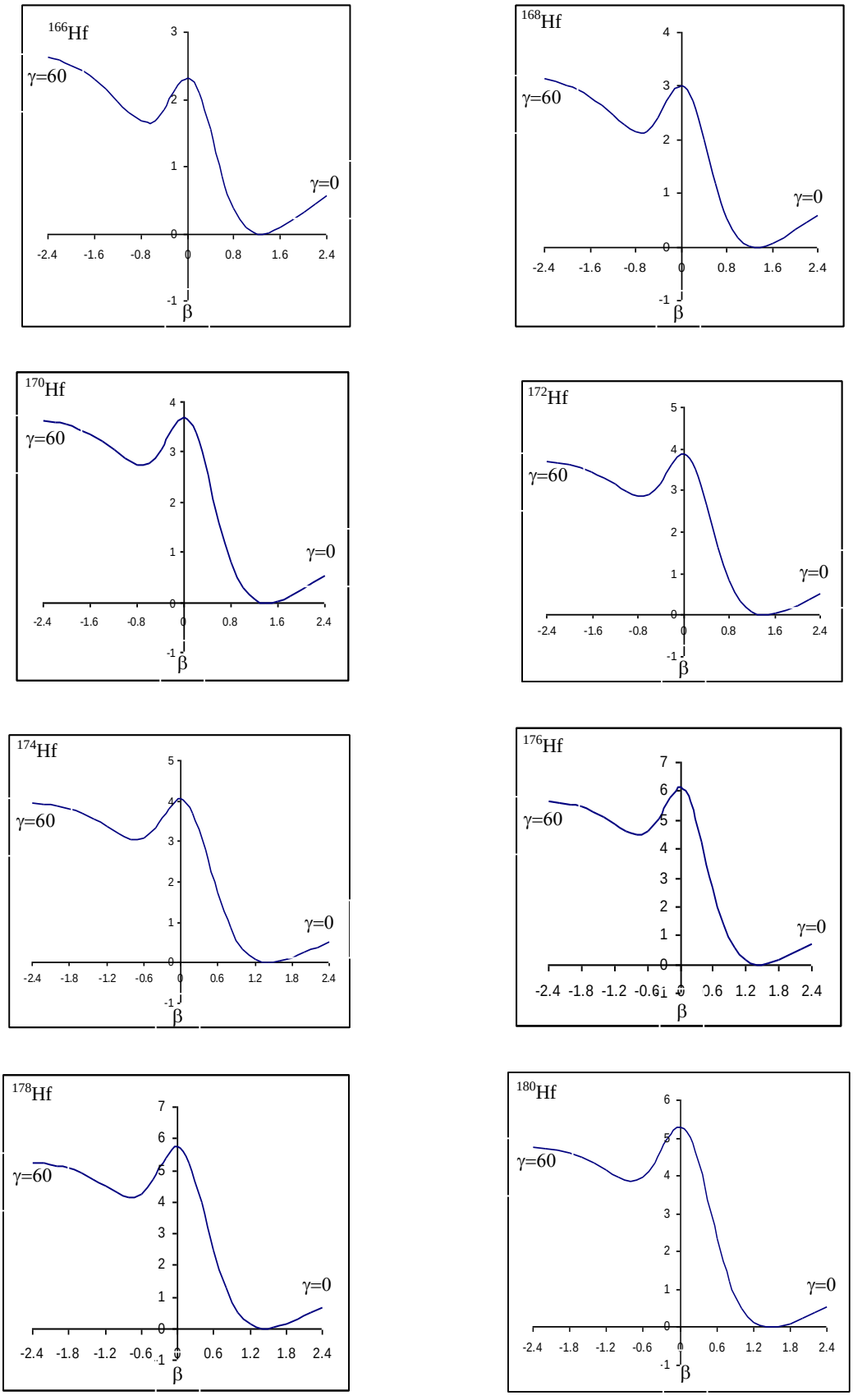

Fig. (9) Potential Energy surfaces for $^{166\text{-}180}$Hf nuclei.