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Design And Implement Attractive Tools For Coloring IRAQ 's Political Map With Minimum Cost

تصميم وتنفيذ أدوات فعالة لتلوين خارطة العراق السياسية وبأقل كلفة

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Abstract:

Many optimization problems such as certain events cannot occur at the same time ,or certain members of a set of objects cannot be adjacent . In map coloring problem, in which colors must be chosen for countries on a map in a way that makes bordering countries with different colors.

The aim of this paper is to design and implement attractive tools for coloring IRAQ's political map with minimum number of colors called (chromatic number) that reduce the cost of coloring using graph vertex coloring algorithm. The IRAQ's political map was drawn manual by designing software , then , represented by equivalence graph data structure , where the nodes denoted to the cities in the map and the edges denoted to the political boundaries between these cities . Also these tools can be implemented to coloring any graph data structure which represent any country's map and finding the chromatic number of it with minimum cost .

الخلاصة //

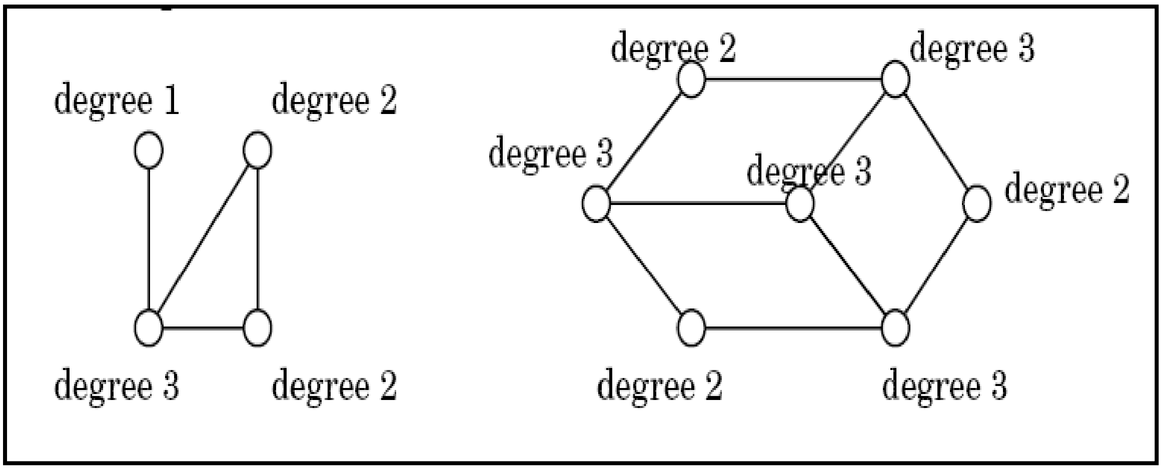
هناك العديد من مسائل الامثلية مثلا عدم وقوع احداث معينة او عدم تجاور شيئين في الوقت نفسه في مسالة تلوين الخرائط يجب عدم تلوين مدينتين متجاورتين في الخارطة باللون نفسه .

يهدف البحث الى تصميم وتنفيذ ادوات لتلوين خارطة العراق السياسية بأقل عدد من الالوان وهذا مايدعى بـ (العدد اللوني) بحيث يقلل من كلفة التلوين باستخدام خوارزمية تلوبن عقدد المخطط ، حيث رسمت خارطة العراق السياسية باستخدام ايعازات الرسم ، ثم تم تمثيل الهيكل البياني المخطط المكافئ لها ، اذ ان العقد تشير الى المدن في الخارطة والحواف تمثل الحدود السياسية بين تلك المدن، كما ان هذه الادوات يمكنها تلوين اي مخطط يمثل خارطة اي بلد وايجاد العدد اللوني له وبأقل كلفة .

1.Introduction to Graph Data Structure

A graph is a collection of vertices or nodes, pairs of which are joined by lines or edges. A graph *G= (V, E)* is an ordered pair of finite sets *V* and *E*. The elements of *V* are called vertices (vertices are also called nodes and points). The elements of *E* are called edges (edges are also called arcs and lines). Each edge in *E* joins two different vertices of *V* and is denoted by *(i, j)*, where *i* and *j* are the two vertices joined by *E* . Graphs can be used not only to represent physical relationships ,but also to represent logical relationships , biological relationships ,arithmetic relationships [1,2] .

The degree of a vertex in a graph is the number of edges which have that vertex as an endpoint. See figure (1) .



Figure(1) Degree of graph's Vertex

In order to process graph with computer program ,we first need to decide how to represent them within the computer .We have two ways of representing a graph or digraphs [3]:

1. Adjacency matrix : The most straightforward representation for graphs is adjacency matrix of an n-vertex graph $G= (V, E)$ is an $n\times n$ matrix A. Each element of A is either zero or one . We shall assume that $V= (1, 2,\dots n)$.

If G is an undirected graph, Then the elements of A are defined as follow:

$$A(i,j) = \begin{cases} 1 & E \in \text{ If } (i,j) \text{ or } (j,i) \\ 0 & \text{Otherwise} \end{cases} \quad \dots (1)$$

If G is a digraph, then the elements of A are defined as follow:

$$A(i,j) = \begin{cases} 1 & E \in \text{ If } (i,j) \\ 0 & \text{Otherwise} \end{cases} \quad \dots (2)$$

2. Linked Adjacency lists : An alternative to the adjacency matrix representation is a data structure containing , for each vertex V , a linked list indicating which vertices are adjacent to V . The data in the adjacency lists will vary with the problem , but there is fairly standard basic structure that is useful form many algorithms [1] .First representation was used in this paper . Most algorithms for solving problems on a graph or digraph examine or process each vertex or edge. The order in which vertices and edges were considered was a fundamental part of method used to solve the problem. There are two traversal strategies (Depth First Search and Breadth first search).

2. Graph Coloring Problem as State Space Tree

The graph (or vertex) coloring problem, which involves assigning colors to vertices in a graph such that adjacent vertices have distinct colors, arises in a number of scientific and engineering applications such as scheduling, register allocation, optimization and parallel numerical computation [2,4] .

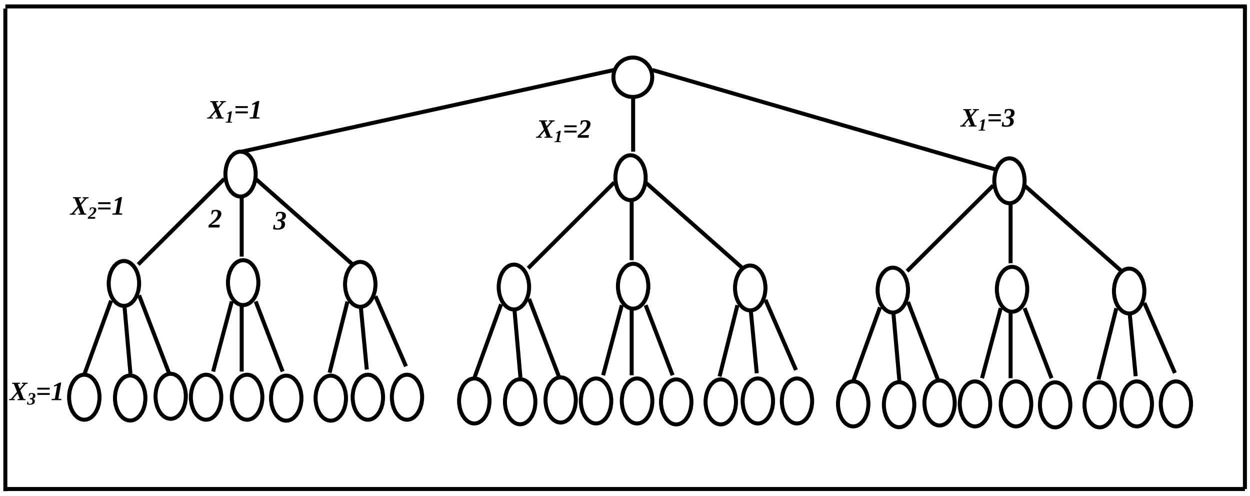
Mathematically, Let G be a graph and m be a given positive integer .We want to discover whether the nodes of G can be colored in such a way that no two adjacent nodes have the same color yet only m color are used .This is termed the *m-colorability decision problem* [5].

If d is the degree of given graph , then it can be colored with *d+1* colors .The *m-colorability optimization problem* asks for smallest integer *m* for which the graph *G* can be colored .This integer is referred to as the *chromatic number* of the graph . The problem is often to determine the minimum cardinality (the number of colors) for a given graph *G* or to ask whether it is able to color graph *G* with a certain number of colors.

Definition : The class of color contain vertex with same color [4] .

A map can easily be transformed into graph , each region of the map becomes a node, and if two regions are adjacent ,then the corresponding nodes are joined by an edge .

Suppose a graph represented by its adjacency matrix *A[1..n,1..n]*,where *A[i,j]=1* if (i,j)is an edge of G ,and *A[i,j]=0* otherwise . The colors are represented by the integers *1,2,…,m* and the solutions are given by $(x_1,x_2,...x_n)$, where x_i is the color of node *i* . he state space tree used is a tree of degree m and height *n+1*. Each node at level *i* has *m* children corresponding to the *m* possible assignments to x_i $,1<=i<=n$. Nodes at level *n+1* are leaf nodes .Figure (2) shows the state space tree when *n=3* and *m=3*[5] .



Figure(2) State Space Tree When *n*=3 *m*=3

Least number of colors needed to color the graph is called it's Chromatic number , χ (G). For example the chromatic number of a complete graph of *n* vertices(a graph with an edge between every two vertices) is $\chi(G) = n$.

Cost of coloring depending on two parameters ,first, the chromatic number that result from coloring algorithm (number of colors required for coloring a map) ,second, the cost of each color , as we show from figure(3) ,the basic colors (Red,Blue,Green)have low cost .

Equation (3) used to compute the coloring cost :

$$COST_{coloring} = \sum_{i=1}^{\chi(G)} Cost(i).Count \qquad \dots (3)$$

Where, i is the index of color , $\chi(G)$ is the chromatic number , Cost(i) is the cost of color i , and Count is the number of nodes colored in ith color .

The solution is given by the $(x_1,x_2,...,x_n)$,where x_i is the color of node *i*. All assignments of *1,2,…,m* to the vertices of the graph such that adjacent vertices are assigned distinct integer are evaluated to obtain minimum cost , (the colors sorted according to their cost i.e. the RED color is the first ,and the BLUE color is the second ,and so on)[8] ,see figure (3).

Graph Coloring Algorithm

Inputs: G(V,E)

Outputs: Colored G(V,E) with minimum cost

Begin of Graph Coloring Algorithm

*Step*1(Initialization) :

1.1 Assigning the graph to its adjacency matrix .

The graph is represented by its boolean adjacency matrix *A[1:n,1:n]*.

1.2 Represent the colors by the integers *1,2,…,m*

1.3 Assigning index of the vertex to color .

*Step*2(Process each vertex and its edge) :

2.1 Process first vertex and coloring it with low cost color (see colors table in figure(3)).

2.2- Process each edge (look at the vertex on the other end) .

2.2.1 If it is not colored we color it with new color and continue.

2.2.2 If the vertex is already colored with same color we must change the current color by adding new color .

2.2.3 Try another edges which connected to our vertex if there exist .

*Step*3(Limitations) :

3.1continue until no two connected vertices *(v,w)* have same color

Step4(Find Coloring Cost) :

4.1 Compute the total coloring cost using equation (3) .

End of Graph Coloring Algorithm

4. Implementation and Results

We use above algorithm to coloring Iraq's map and also any graph we want ,then compute the cost of coloring depending on the cost color table .

The attractive tools are display the graph equivalence to Iraq's map and the cost color table ,also draw any graph then implement coloring algorithm on it ,see figure(3) :

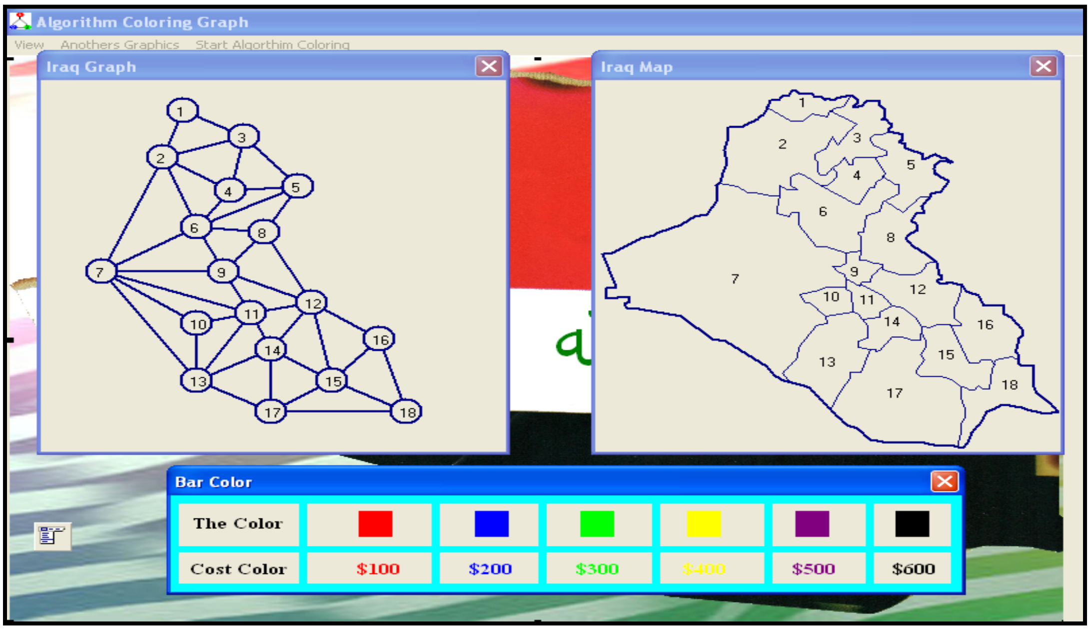


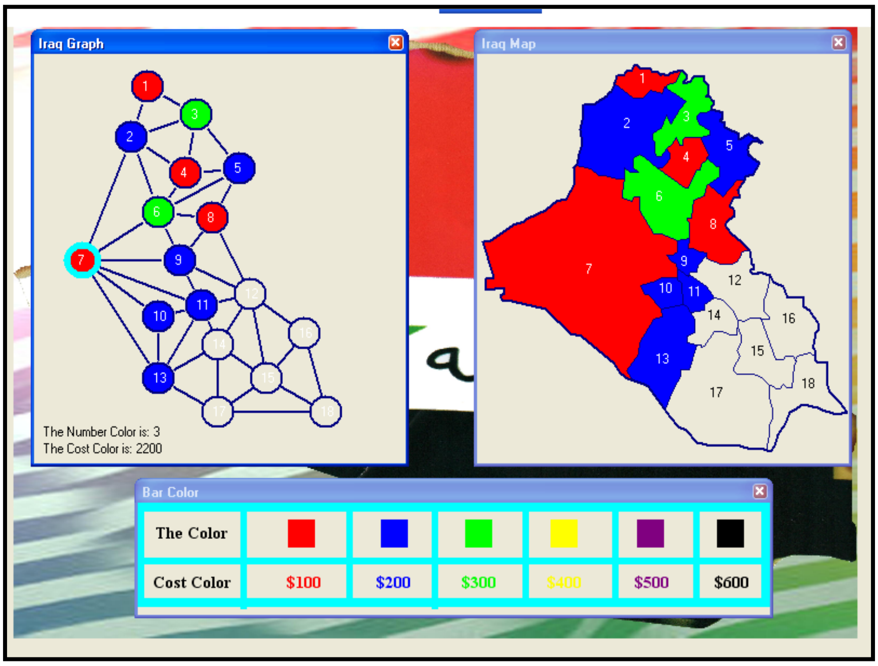
Figure (3) Graph equivalence to Iraq's map and the cost color table

We will see graph contain 18 vertices represent the number of IRAQ's PROVINCES, the map drawn in by instructions of graphics in Delphi language, figure (4) shown numbers , Provinces name and Provences center name of IRAQ's MAP :

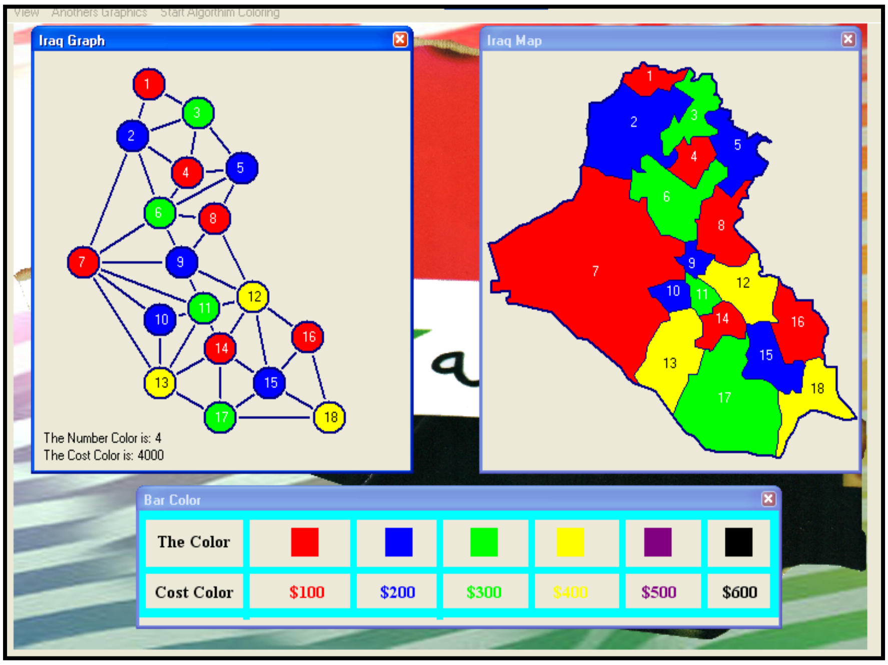
Number City	Province City	Center City
1	Dihouk	Dihouk
2	Nineveh	Mosul
3	Erbil	Erbil
4	Ta'mim	Kirkuk
5	Suleimaniya	Suleimaniya
6	Saladdin	Tikrit
7	Anbar	Ramadi
8	Diyala	Baquba
9	Baghdad	Baghdad
10	Karbala	Karbala
11	Babylon	Hilla
12	Wassit	Kut
13	Najaf	Najaf
14	Qadisiya	Diwaniya
15	Thi Qar	Nasiriya
16	Maysan	Amara
17	Muthanna	Samawa
18	Basrah	Basrah

Figure (4) Numbers , Provinces name and Provinces center name of IRAQ's MAP

The START ALGORTHIM COLORS choice always used to coloring the IRAQ's MAP take red color at first because the red color is cheap than another colors , see coloring process in figure (5) and figure (6) .



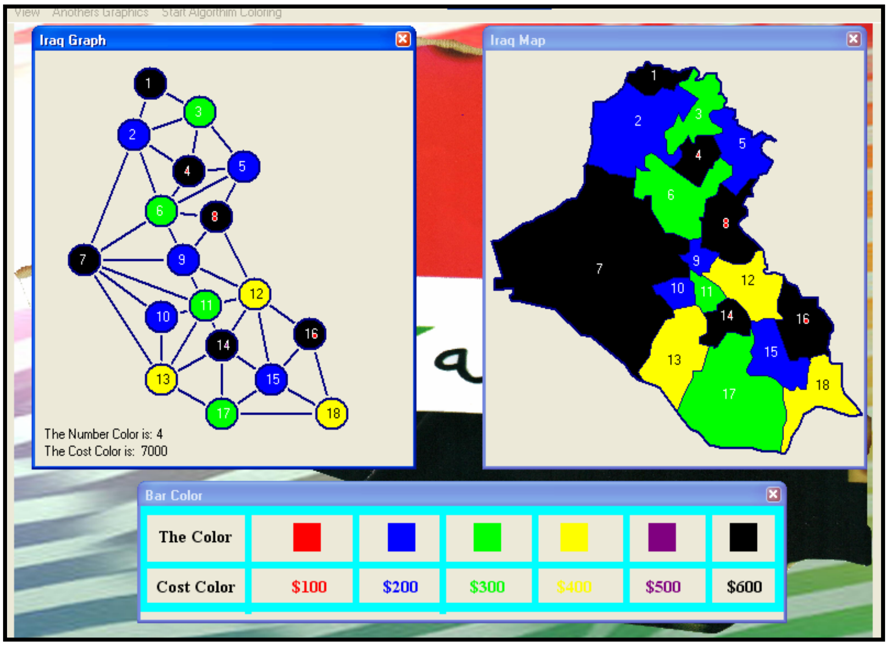
Figure(5) Partial Coloring Process of IRAQ's Map



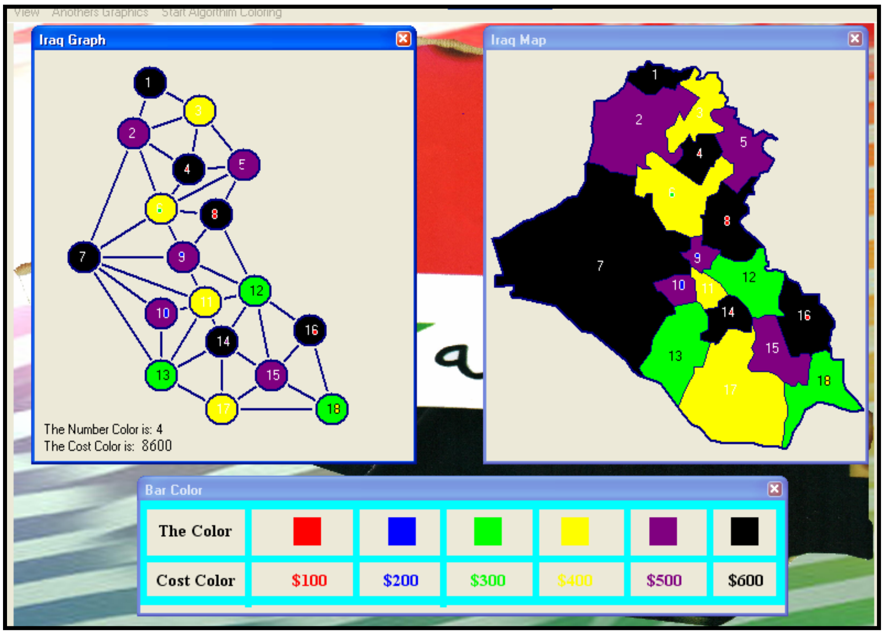
Figure(6) Complete Coloring Process of IRAQ's Map

As shown in above , the IRAQ's Map needed four colors . The cost of coloring equal to (4300)$ depending on cost color table shown in figure (3) . The limitation of coloring algorithm that no two connected vertices *(v,w)* have same color ,therefore the nodes with number 10,11 and 13 (figure 5) in both map and graph are need another color from color table ,see above algorithm .

To show the effect of selected color on coloring cost , we take BLACK color at (note the cost of it is 600$) , the total coloring cost is ,see figure(7).



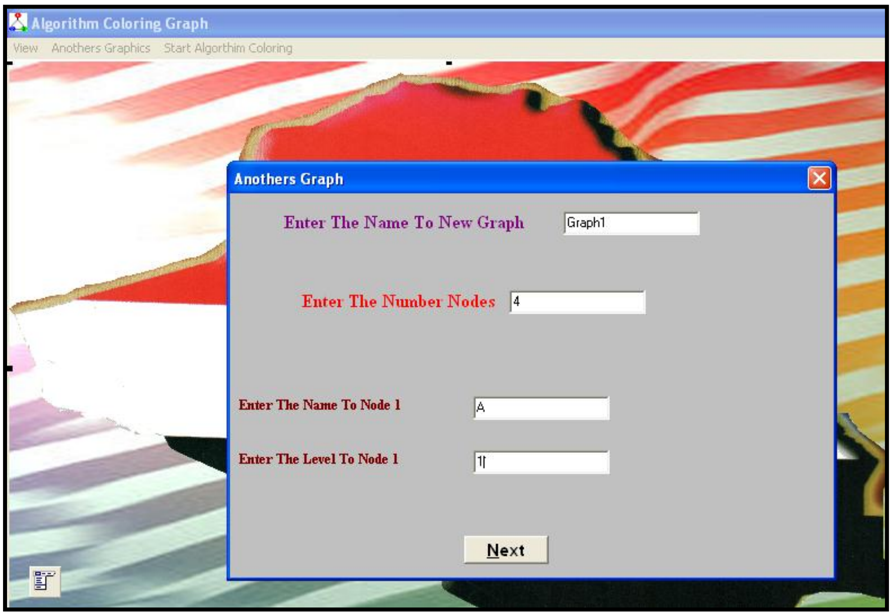
Figure(7) Coloring Start with High Color Cost(BLACK)



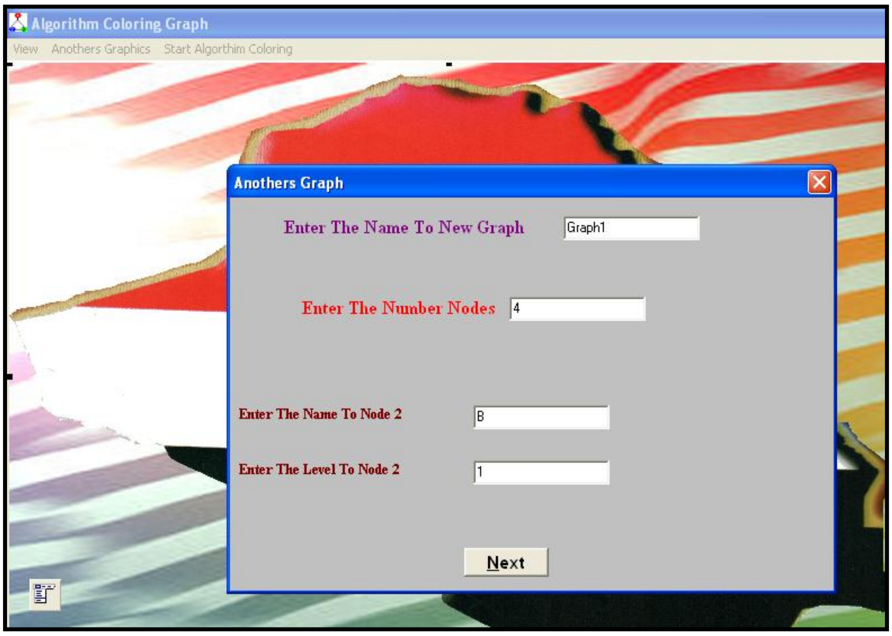
Figure(8) Coloring with High Color Cost

Note that the cost different when choosing high color cost

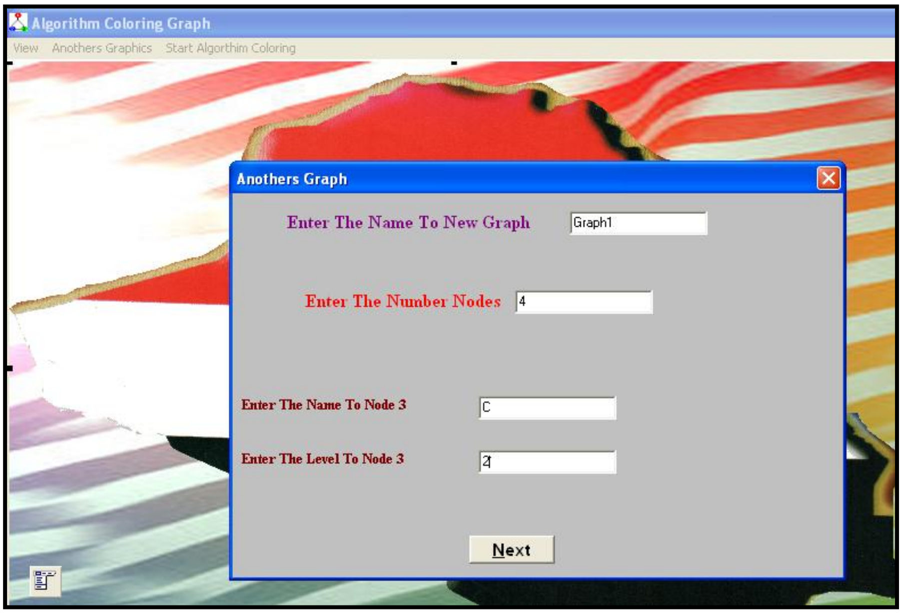
The ANOTHERES GRAPHICS menu item used to entering any graph you want to coloring it by this algorithm , we must entering the name of graph ,number of node ,name of these nodes , the level for each node and the connect for each node to others by (0 or 1)see figures(9,10,11,12,13, 14 ,15 and 16) .



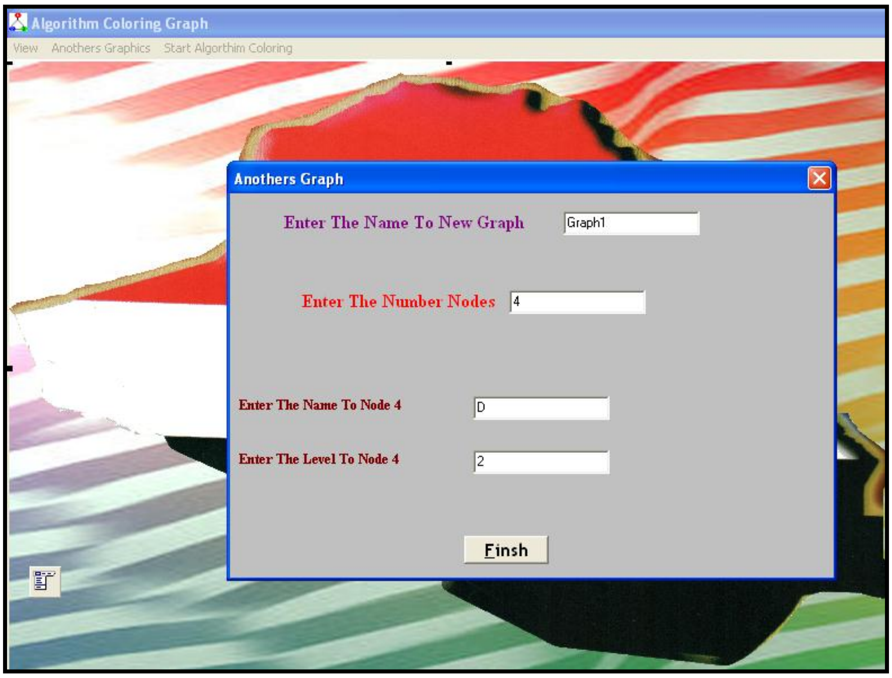
Figure(9) Construct any Graph



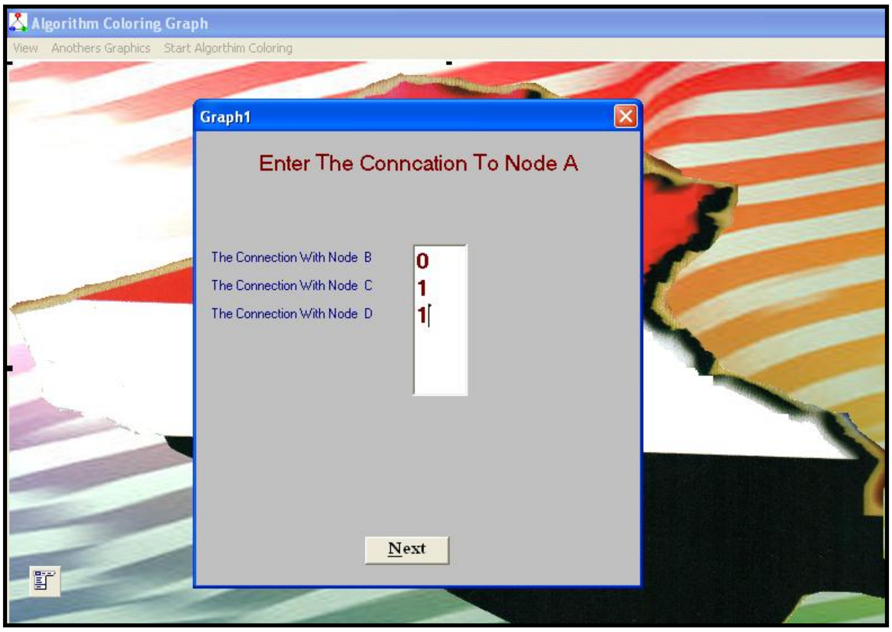
Figure(10) Construct any Graph(Continued)



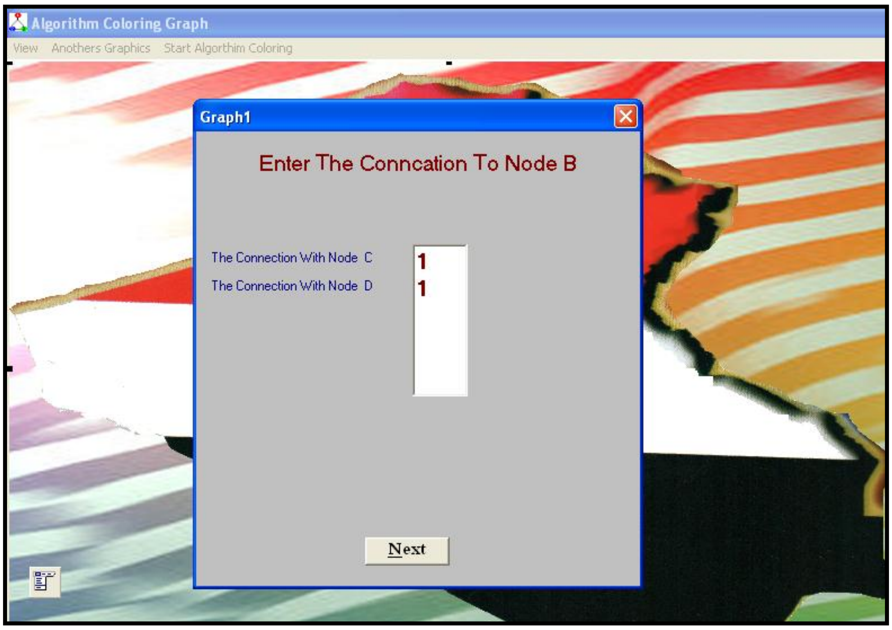
Figure(11) Construct any Graph(Continued)



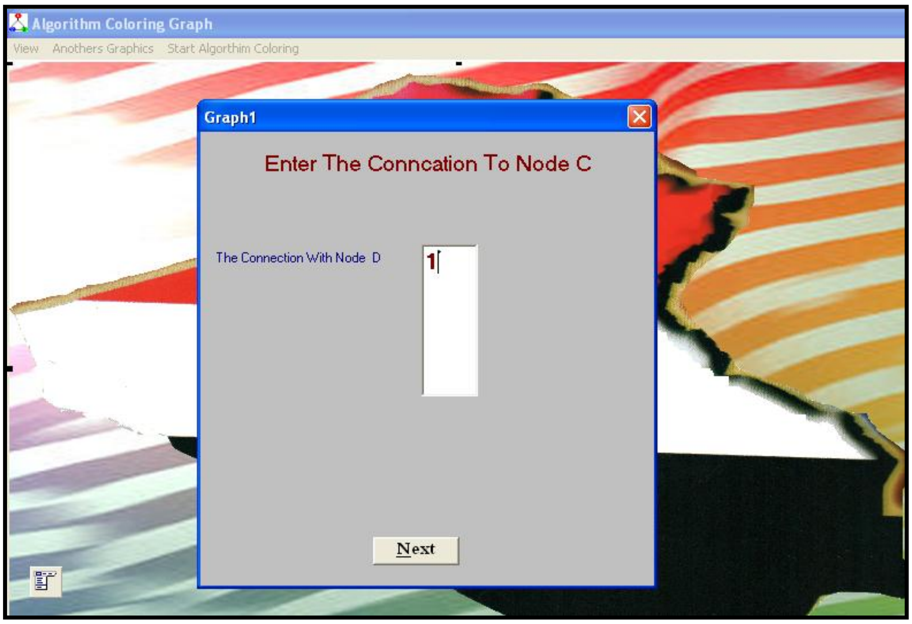
Figure(12) Construct any Graph(Continued)



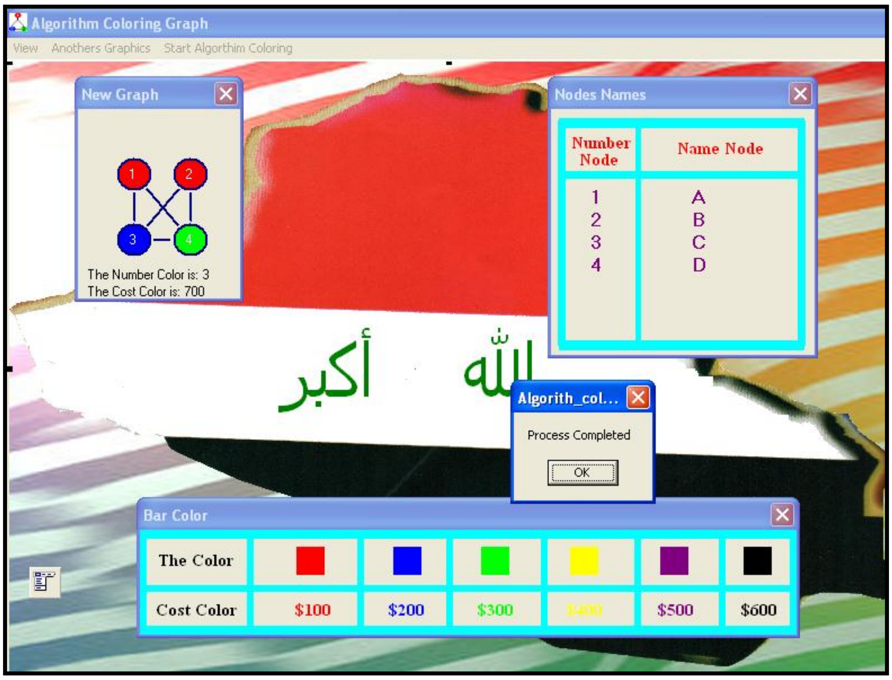
Figure(13) Construct any Graph(Continued)



Figure(14) Construct any Graph(Continued)

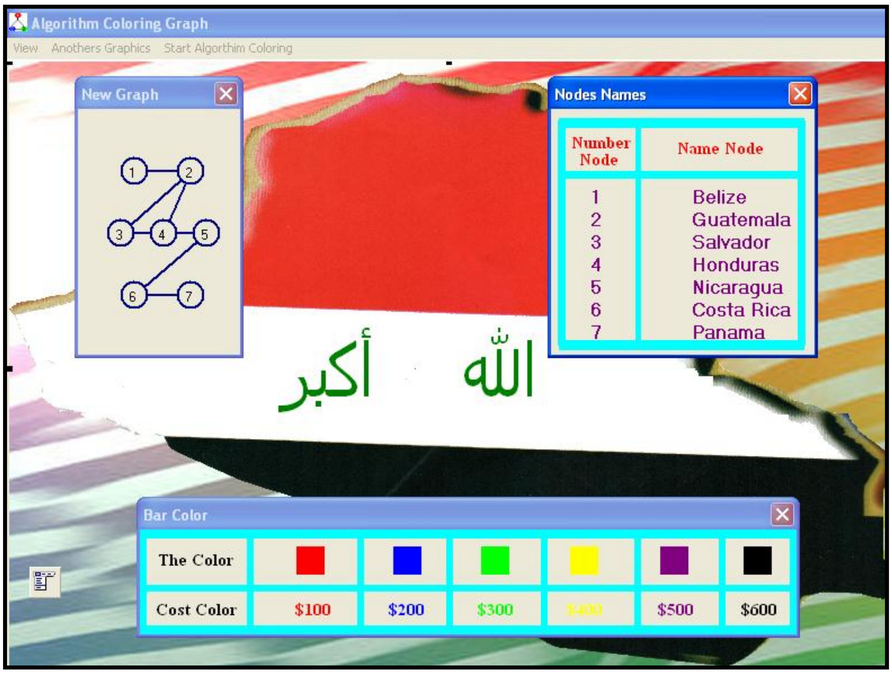


Figure(15) Construct any Graph(Continued)

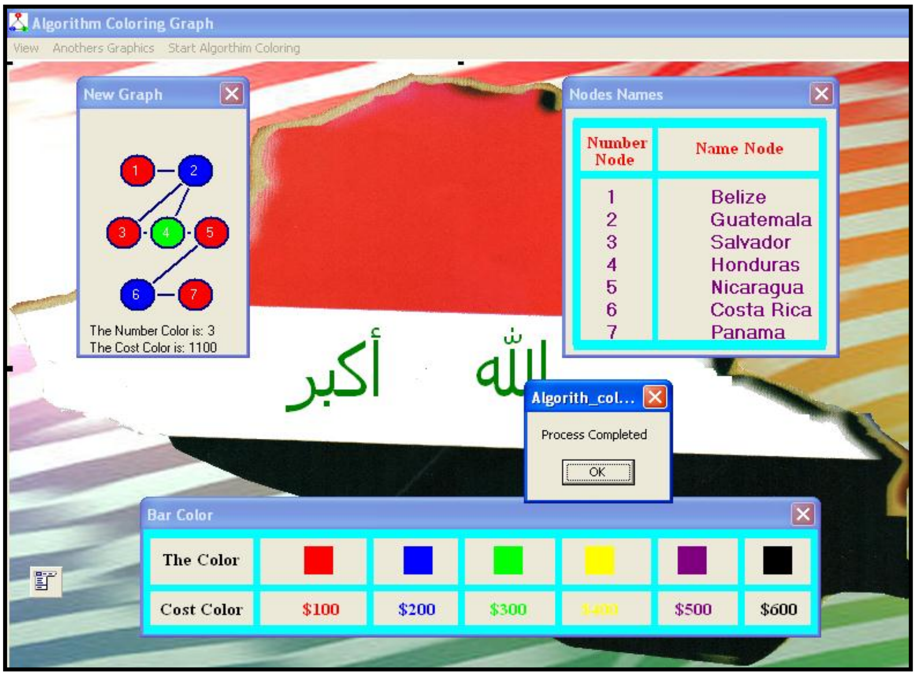


Figure(16) The resulting of construct Graph and coloring

We can use above menu item to entering the graph represent any map , if we take the map of Central America (contain 7 countries) the graph represent this map and the results of coloring process shown in figure (17) and figure (18) :



Figure(17) Partial Coloring Process of Equivalence Graph to Central America's Map



Figure(18) Complete Coloring Process of Equivalence Graph to Central America's Map

As shown in above , the Central America's Map needed three colors . The cost of coloring equal to (1100)$ depending on cost color table shown in figure (3) .

8. Conclusion

Graphs are easy to represent in a computer , and useful in describing a huge variety of problems . The Map of our country (IRAQ) need only chromatic number = 4 , although the degree of equivalence graph of IRAQ's map is (6) then, the chromatic number is not necessary equal to degree of graph .

The class of color contain large number of vertex , is a good , the class of red color in IRAQ's map is 6 that mean there are six city colored with red color (the red color is chipset) which reduce the cost of coloring , five city colored with blue color and so on .

9. Future works

1. Check another method of coloring using Genetic Algorithm .
2. Solving (Framer , Goat , Wolf , Cabbage) problem in graph coloring algorithm .
3. Modify this software to represent any map of country entering by scanner (with special properties and use segmentation of the image of these map to implement coloring algorithm .

10.References

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