# The set of the homogeneous linear reciprocal block maps

Alaa H.H. Al-Ka'bi

Department of Mathematics - College of Since the computer and Mathematics,

University of Qadisia

Abstract

In this paper ,we introduce new definition for set of the block maps reciprocal via block maps linear homogeneous ,inhomogeneous ,odd and even . while The classical definition is $\Im(f)=\{g\in F : g$ commute with $f$ under impacts $\circ\}$

## 1-Preliminaries

let $(X,T,\pi)$ be a topological transformation group,

We adopt the set of symbols $\zeta=\{0,1\}$ as the alphabet of our shift space , n-block means the function $\beta_n : I_p^q \to \zeta$ where $I_p^q=\{i\in Z : p\le i\le q : \mathrm{p,q}\in \mathrm{Z}\}$ , $B_n$ means the set all n-blocks, the n-block map $f$ defined by $f:\mathrm{B_n}\to\zeta$[2], $I$ identity block map defined by $I(a_1a_2...a_n)=a_1a_2...a_n\ \forall a_1a_2...a_n\in B_n$ , 0,1constants block map defined by $0(a_1a_2...a_n)=0, 1(a_1a_2...a_n)=1\forall a_1a_2...a_n\in B_n$, $F_n$ a set of all n- block maps and $F$ a set of all block maps[1],[3].

The alphabet we adopt is $\zeta=\{0,1\}$ ,and define translation operator $(\Psi)$ as follows $\Psi f(a_0a_1a_2\dots a_n)= f(a_1a_2\dots a_n)$ such $\mathrm{a_i}\in\zeta$ ,$\theta(f)=\min\{n : f\in F_n\}$ and can written any block map it's say $g$ as form $g=I\cdot\Psi qg+\Psi rg \ni qg, rg\in F_{n-1}$ such that $qg(a_1\dots a_n)=g(0a_1\dots a_n), rg(a_1\dots a_n)=g(0a_1\dots a_n)+g(1a_1\dots a_n)\ \forall a_i\in\zeta$ and have $q(g\circ f)=qg\circ f, q(f\circ g)=qg\ \ \forall f,g\in F$ [5]. We define set of block maps commuting $\Im(f)=\{g\in F : g\circ f = f\circ g\}$. In research our we define the linear block map as follows $f = a_0 + \sum_{i=1}^{n} a_i \Psi^{i-1} I \quad , a_i \in \zeta$ and let

$\gamma = \{ f \in F : f$ linear block map$\}$ , and it is said for $f$ homogeneous if $a_0 = 0$

and inhomogeneous if $a_0 = 1$, and it is said for $f$ even or odd according to value

$card\{i \geq 1 : a_i = 1\}$ even or odd . let $\gamma_H$ be set of all homogeneous linear block map,

and let $\gamma_I$ set all inhomogeneous linear block map. And say for $f$ non-trivial block

map if $card\{i \geq 1 : a_i = 1\} \geq 2$ .We have $(\gamma_H ,+,\circ) \cong (Z_2[x],+,.)$ [4][2].

(2)A New set of the homogeneous linear reciprocal block maps

Preliminaries

In this section , we study the relation between block maps linear homogeneous, inhomogeneous ,odd and even and the composition for block maps .

Theorem (2.1): if $f, g, h$ block maps and $f \in \gamma_H$ then

$$f \circ (g + h) = f \circ g + f \circ h \ \forall \text{g, h}$$

Proof: Since $f \in \gamma_H$ , then there exists $a_1 \ldots a_n = 0$ or $1$ such that

$$f \circ [g + h] = \sum_{i=1}^{n} a_i \Psi^{i-1} I \circ [\sum_{i=1}^{n} a_i \Psi^{i-1} I \circ g + \sum_{i=1}^{n} a_i \Psi^{i-1} I \circ h] = f \circ g + f \circ h$$

Theorem (2.2): if $f$ is block map and $f \in \gamma_H$ then $\gamma_H \subseteq \Im(f)$.

Proof : Let $g$ be homogeneous linear block map, and Since

$$(\gamma_H ,+,\circ) \cong (Z_2[x],+,.)$$

then $(\gamma_H ,+,\circ)$ is commuting ring ,and so then $g \circ f = f \circ g$ for all $\text{g} \in \gamma_\text{H}$

Theorem (2.3) : let $f$ be non-trivial block map and $f \in \gamma$ then $\Im(f) \subseteq \gamma$.

Proof : we will prove by using the induction on value $\theta(f)$

let $g \in \Im(f)$ , $g \circ f = f \circ g$ constant , and so $q(g \circ f) = qg$ .

Now we can written $g$ as form $g = b.I + \Psi rg$ such that

$b$ constant. so

$$g \circ f = b[a_0 + \sum_{i=1}^{n} a_i \Psi^{i-1} I] + \Psi rg \circ f$$

$$f \circ g = b\sum_{i=1}^{n} a_i \Psi^{i-1} I + f \circ \Psi rg$$

$$g \circ f + f \circ g = a_0 b + \Psi(rg \circ f + f \circ rg)$$

we notice that $rg \circ f + f \circ rg$ constant , and by using the induction

then $rg$ constant , this completes the proof .

Theorem (2.4): let $f \in \gamma_H$ and $g \in \gamma_I$ then

$g \circ f = f \circ g$ if and only if $f$ odd.

Proof : we can written $f, g, h$ as form

$$f = \sum_{i=1}^{n} a_i \Psi^{i-1} I, g = 1 + \sum_{j=1}^{m} b_j \Psi^{j-1} I$$

such that

$a_i, b_j = 0 or 1 \ \forall i = 1, \dots, n \ \mathrm{j} = 1, \dots, \mathrm{m}$

and so that $g \circ f = 1 + \sum_{i=1}^{n} \sum_{j=1}^{m} a_i b_j \Psi^{i+j-2} I$

by using theorem (2.1) $f \circ g = \sum_{i=1}^{n} a_i + \sum_{i=1}^{n} \sum_{j=1}^{m} a_i b_j \Psi^{i+j-2} I$

and so that $\sum_{i=1}^{n} a_i = 1$ if and only if $f$ odd .

Theorem (2.5) : let $g, h \in \gamma_I$ then $g \circ h = h \circ g$ if and only if $g, h$

either both odd or both even.

Proof : We can written $h$ as form $h = 1 + \sum_{i=1}^{n} c_i \Psi^{i-1} I$ such that

$c_i = 0 or 1$ for all $i = 1 \dots n$ and $g$ as in the theorem(2.4).

Since $\sum_{j=1}^{m} b_j \Psi^{j-1} I \in \gamma_H$ and by using theorem(2.1) then

$$g \circ h = 1 + \sum_{j=1}^{m} b_j + \sum_{j=1}^{m} \sum_{i=1}^{n} b_j c_i \Psi^{i+j-2}$$

and $$h \circ g = 1 + \sum_{i=1}^{n} c_i + \sum_{j=1}^{m} \sum_{i=1}^{n} b_j c_i \Psi^{i+j-2}$$

and so that $\sum_{i=1}^{n} c_i = \sum_{j=1}^{m} b_j$ if and only if $g, h$ either both odd or both even.

Theorem(2.6): if $f, g$ are block maps and $f \in \gamma_H$ and $f$ is non trivial

1. if $f$ odd map then $\Im(f) = \gamma$.
2. if $f$ even map then $\Im(f) = \gamma_H$.

Proof:(1) from theorem (2.2) then $\Im(f) \subseteq \gamma$.

Let $g \in \gamma$, if either $g \in \gamma_I$ and by using theorem(2.5) then

$g \circ f = f \circ g$, or $g \in \gamma_H$ and by using theorem(2.2) then $g \in \Im(f)$

this completes the proof .

(2) from theorem(2.2) then $\gamma_H \subseteq \Im(f)$.

Let $g \in \Im(f)$ i.e. $g \circ f = f \circ g$ ,and will proof by contradiction

i.e. $g \notin \gamma_H$ and By using theorem (2.3) then $g \in \gamma_I$ and by using

theorem (2.4) then $f$ is odd ,and this contradiction .

Theorem(2.7) : if $f, g$ are block maps and $f \in \gamma_I$ and $f$ is non trivial

1. if $f$ odd map then $\Im(f) = \{g \in \gamma : g \text{ is odd}\}$.
2. if $f$ even map then

$$\Im(f) = \{g \in \gamma_H : g \text{ is odd}\} \bigcup \{g \in \gamma_I : g \text{ is even}\}.$$

proof : (1) let $g \in \Im(f)$ and by using theorem (2.3) then $g \in \gamma$, and since $g \in \gamma$, either $g \in \gamma_H$ and by using theorem(2.4) then $g$ is odd , or $g \in \gamma_I$ and $f$ is odd and by using theorem(2.5) then $g$ is odd , this completes the proof .

proof (2): let $g \in \Im(f)$ and by using theorem (2.3) then $g \in \gamma$, either $g \in \gamma_H$ and we have $g \circ f = f \circ g$ and by using theorem (2.4) then $g$ is odd .or $g \in \gamma_I$ and $f$ is even by using theorem(2.5) then $g$ is even, this completes the proof .

## Reference


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$$\Im(f) = \{g \in F : g \text{ commute with } f \text{ under impacts } \circ \}$$