Optimal Perturbed H Control Problems with Unbounded Control Operator

Radhi Ali Zboon — Department of Mathematics, College of science., AL-Mustansirya University

Jehad Ramadhan Kider — Department of Applied science, (University of Technology)

Samir Kasim Hassan — Department of Mathematics, College of Education, Al-Mustansirya University

Abstract

The H_∞-optimal control problem in infinite dimensional spaces is presented. Some optimal control problems and solvability, controllability, null controllability, perturbed Recatti operator equation as well as some of its properties are discussed and developed via evolution perturbed strongly continuous semigroup generated by the linear unbounded perturbed generators.

1 Introduction

Stabilization and H_∞ control for uncertain systems has received considerable interest in the last decades; see [5,10,11,12,14,16,17,18] and references therein .In particular , in [16] it has shown that the solution of the H_∞ control problem for affine nonlinear systems can be obtained by solving Hamilton-Jacobi equation, which is nonlinear version of the Riccati equation considered in the corresponding linear H_∞ control theory. Furthermore the H_∞ control problem for more general nonlinear systems is considered in [6,13], where the sufficient conditions are obtained on the solution of Riccati-like equations or inequalities. It is well known that the solution of the H_∞ control problem for finite-dimensional systems can be obtained by solving either linear matrix inequalities or algebraic/differential Riccati-like equation. For infinite-dimensional control systems, the investigation of this problem is more complicated and requires sophisticated techniques from semigroup theory. The linear time varying case is considered in [6], and necessary and sufficient conditions for H_∞ control problem are given in terms of two Riccati-like operator equation. The nonlinear H_∞ control problem for infinite-dimensional system is still under active investigation.

In this paper, we suggested the optimal perturbed unbounded H_∞ control problem for uncertain nonlinear systems in real Hilbert space. Some controllability concepts ,like globally null-controllability and solvability semilinear system have been discussed and developed or some results depending on solvability of differential Perturbed Ricati Operator Equation corresponding the problem.

Consider a nonlinear uncertain system of the form,[4]:

$\dot{x}(t)$ F(t, x(t), u(t), w(t)), x(0) x_0,

z(t) G(t, x(t), u(t)),

(1)

where x(t) H is the state, u(t) U is the control function, w W is the uncertain input function, z Z is the observation output function, H, U, Z, W are real Hilbert spaces,

F : $\mathbb{R}^+$HUW H, G : $\mathbb{R}^+$HU Z (where $\mathbb{R}^+$ is a positive reals) are given nonlinear functions .In the sequel, we say that the uncertainty w(t) is an admissible if w L^2 ([0,):W).

We assume that for every $x_0 \in H$, the admissible control function u L^2 ([0, t]: U) is a Banach space of 2-integrable functions with its domain [0, t) into U and an admissible uncertainty function w(t), the system (1) in two equations has a unique solution x(t, x_0) given in the integral form:

$$x(t, x_0)\ x_0 + \int_0^t F(s, x(s), u(s), w(s))\, ds,\ t\ [0,).$$

Definition (1.1), [4]:

The system (1) is robustly stabilizable if there exists a feedback control operator u(t) K(x(t)) such that the solution $x(t, x_0)$ of the closed loop system(from output z(t) depend on u(t) and u(t) K(x(t)))

$\dot{x}$ F(t, x(t), k(x(t)), w(t)),

z(t) G(t, x(t), u(t)),

belong to L^2([0,):H) for all uncertainties w L^2([0,): W), where K is a linear bounded operator.

Remark (1.1), [13]

- The H control problem for the system (1) is considered as follows:

  Given the scalar > 0, find a feedback control u(t) k(x(t)), such that:

  i. The control system (1) is robustly stabilizable.

  ii. There is a number $c_0 > 0$, such that:

$$\sup_{x_0 \in H} \frac{\int_0^{\infty} \| z(t) \|_Z^2 \, dt}{c_0 \| x_0 \|_H^2 + \int_0^{\infty} \| w(t) \|_W^2 \, dt}$$

, for all non-zero admissible uncertainties w(t).

- From definition (1.1), one can say that the H_∞ optimal control problem for system (1) has a solution and the feedback control u(t) k(x(t)) is robustly stabilizable.

- If the conditions (i-ii) in (1) hold for all x_0 H, u(t) U, w(t)W, for some neighborhood of the origin, then we say the H optimal control problem has a local solution.

- The robust stabilizability implies that the closed-loop system can be made L^2-stable for all admissible uncertainties wW.

- The H optimal control condition (ii) in (1), can be guaranteed under non-zero initial conditions.

Definition (1.2) , [7]:

A set T $\{T(t, s)\}_{ts0}$ L(H) is an evolution semigroup(where L(H) space of linear bounded operators) if:

1- T(t, s) T(t, r)T(r, s), T(s, s) I, $_{t\ r\ s\ 0}$, where I stands for the identity operator.

2- (t, s) T(t, s) is strongly continuous,

3- $\frac{\partial T(t,s)}{\partial t}$ A(t)T(t, s), $\frac{\partial T(t,s)}{\partial s}$ A(t)T(t, s), (where A(t) is a generator of T).

4- ||T(t, s)|| M $e^{\omega(t-s)}$, for all t r s 0 and constants M 1 and ω | |, where | | stands for the set real numbers.

2. Problem Formulation

We consider the following semilinear system (induced by linear and nonlinear parts) :

$\dot{x}$(t) [A(t) + A(t)]x(t) + [B(t) + B(t)]u(t) + $[B_1(t) + B_1(t)]$w(t) + f(t, x(t), u(t), w(t)),
z(t) C(t)x(t) + D(t)u(t) + g(t, x(t), u(t)), (2)

satisfies the following assumptions:

- The operator A(t) : D(A(t)) H is unbounded generating C_o-evolution semigroup {T(t, s)}$_{ts0}$. The operator A(t):D(A(t)) H is a bounded linear operator, such that D(A(t)) D((A(t)), implies that A(t) + A(t) : D(A(t)) H is also unbounded linear operator generating C_o-perturbation evolution semigroup $\{T_A(t, s)\}_{ts0}$ of system (2).
- Let $\overline{H}_t$, t 0, be a Hilbert space in which H is densely and continuously embedded in $\overline{H}_t$ and let $\overline{T}_{\Delta A}(t,s)$: $\overline{H_s}$ $\overline{H_t}$ is a continuously extension C_o-evolution perturbation semigroup of T_A(t, s) generated by a linear unbounded operator $\overline{[A(t)+\Delta A(t)]}$ is the extension generator of the generator $A(t)+\Delta A(t)$ on $\overline{H_t}$.
- B(t):UH is unbounded linear control operator and B(t) : U H, such that

$B(t) + \Delta B(t) : U \to H$, is unbounded perturbed linear control operator, where U is a control space.

$B_1(t) + \Delta B_1(t) \in \mathcal{L}(W, H)$, (The space of all continuous linear operators from W into H).

- such that $D(B_1(t)) \subseteq D(\Delta B_1(t))$,where W is the uncertain Hilbert space and $[B_1(.)+\Delta B_1(.)]w$, is continuous function in $t \in \mathbb{R}^+$ for all $w \in W$, $C(t) : H \to Z$ and $D(t): U \to Z$ are linear bounded operators, and $C(.)x$, $D(.)u$, are continuous functions for $t \in \mathbb{R}^+$ and all $x \in H$, $u \in U$, $w \in W$.

- The functions, $F:\mathbb{R}^+ \times H \times U \times W \to H$, $G:\mathbb{R}^+ \times H \times U \to Z$ are given nonlinear perturbation functions and continuous functions for all (x, u, w) they are also satisfying the following growth conditions:

  $\exists\, c_1, c_2, c_3 > 0$, such that:

  $$\|F(t, x(t), u(t), w(t))\| \le c_1\|x(t)\| + c_2\|u(t)\| + c_3\|w(t)\|, \tag{3}$$

  $\exists\, c_4, c_5 > 0$, such that:

  $$\|G(t, x(t), u(t))\| \le c_4\|x(t)\| + c_5\|u(t)\|. \tag{4}$$

  We shall make the conditions such that the nonlinear system (2) has a unique solution.

  We assume that in addition, we need the following assumptions:

- The functions $[B(.) + \Delta B(.)]u$, $[B_1(.) + \Delta B_1(t)]w$, $C(.)x$, $D(.)u$, $A(.)x$ are bounded such that:

  $\exists\, b_1, b_2, \Delta_a, \Delta_b, \Delta_{b_1}$, c, d, p > 0, such that:

$$\Delta_a \ge \sup_{t\in\mathbb{R}^+}\|A(t)\|_H, b \ge \sup_{t\in\mathbb{R}^+}\|B(t)\|_{\mathcal{L}(U,H_1)}, b_1 \ge \sup_{t\in\mathbb{R}^+}\|B_1(t)\|_{\mathcal{L}(W,H)},$$

$$\Delta_b \ge \sup_{t\in\mathbb{R}^+}\|\Delta B(t)\|_{\mathcal{L}(U,H)}, \Delta_{b_1} \ge \sup_{t\in\mathbb{R}^+}\|\Delta B_1(t)\|_{\mathcal{L}(W,H)}, c \ge \sup_{t\in\mathbb{R}^+}\|C(t)\|_{\mathcal{L}(H,Z)},$$

$$d \ge \sup_{t\in\mathbb{R}^+}\|D(t)\|_{\mathcal{L}(U,Z)}, Q(t) \equiv C^*(t)C(t) \ge \rho I \tag{5}$$

- We assume that:

  $C^*(t)D(t) \equiv 0$, $D^*(t)D(t) \equiv I$, $\forall\, t \ge 0$

Note that this assumption is a standard assumption in H control theory.

We are interested in the relation between the evolution semigroup $\{T(t, s)\}_{ts0}$ generated by A(t) and the perturbation evolution semigroup $\{T_A(t, s)\}_{ts0}$ [see assumption 1] generated by $A(t) + \Delta A(t)$,let

$$G(s,)x \;\; T(t, s)T_A(s,)x. \tag{6}$$

Now

$$\frac{\partial G(s,\tau)x}{\partial s} \;\; A(t)T(t, s)T_A(s,)x + T(t, s)(A(t)+ A(t))T_A(s,)x \;\; A(t)T(t, s)T_A(s,)x + [T(t, s)A(t) + T(t, s)A(t)T_A(s,)x \;\; T(t, s)A(t)T_A(s,)x. \tag{7}$$

By integrating both sides of (7) from to t, we obtain:

$$G(t,)x \;\; G(,)x \;\; \int_{\tau}^{t} T(t, s)A(s)T_A(s,)x \, d.$$

From (4.6), we have:

$$T_A(t,)x \;\; T(t,)x + \int_{\tau}^{t} T(t, s)A(s)T_A(s,)x \, ds \tag{8}$$

The aim of this section is to find sufficient conditions and construct the feedback control for solving the H optimal control for nonlinear perturbed system (2).

<u>Definition(2.3)</u>:

Let T_A be an evolution family on H and let $\overline{H}_t$, t 0, be a real Hilbert space in which H is densely and continuously embedded in $\overline{H}_t$.Assume that $T_A(t, s)$ has a locally uniform bounded extension $\overline{T}_{\Delta A}(t,s)$: $\overline{H_s}$ $\overline{H_t}$ **(which then satisfies (1) in definition (1.2) and is strongly continuous with respect to s**). We call B(t) $\mathcal{L}(U, \overline{H}_t)$, t 0, $T_A$admissible

perturbed control operators if the function

$\overline{T}_{\Delta A}(t,s)$[B(.) + B(.)]u(.) is integrable in $\overline{H}_t$ and

$$(\overline{K_s}\,B(.)+B(.)u)(t)\ \int_{\tau}^{t}\ \overline{T}_{\Delta A}(t,s)[B(s)+B(s)]u(s)\,ds\ \ H, \tag{9}$$

and there are constants t_0, > 0, such that:

$$||\overline{K_s}\,B(.)u)(t)||_H\ \ ||u||_{L^2([s,t],U)}, \text{ for all } 0\ \ s\ \ t\ \ s + t_0 \text{ and } u\ \ L^2([s, t], U). \tag{10}$$

Remark(2.1)

Consider the linear varying perturbation control system:

\ $\dot{x}$ (t) [A(t) + A(t)]x(t) + [B(t) + B(t)]u(t), t 0

x(0) x_0 . (11)

The operators A(t): D(A(t)) H and A : D(A(t)) H defined as in assumption (1) of problem formulation .It should be noted that to find the mild solution of problem (11), let T_A(t, s) has a locally uniformly bounded extension, and by definition(1.2), the perturbation evolution linear operator $\overline{T}_{\Delta A}(t,s)$: $\overline{H_s}$ $\overline{H_t}$ is strongly continuous with respect to s generated

by a linear operator $\overline{[A(t)+\Delta A(t)]}$ which is the extension generator of the generator $A(t)+\Delta A(t)$ on $\overline{H_t}$.Let B(t) $\mathcal{L}$(U: $\overline{H}_t$), t 0, is T_A-admissible perturbed control linear operator defined from U into $\overline{H_t}$,such that [B(.)+B(.)] $\mathcal{L}$(U: $\overline{H_t}$) satisfies the conditions in (9) and (10) where $\overline{H_t}$, t 0 is an extension Hilbert spaces in which H is densely and continuously embedded in $\overline{H_t}$.Let x(.) H be the solution of (11). Then

$$\overline{T}_{\Delta A}(t,s)x(s)\ \ \overline{T}_{\Delta A}(t,0)x(0) + \int_{0}^{t}\ \overline{T}_{\Delta A}(t,s)[B(s) + B(s)]u(s)\,ds.$$

Hence the solution is :

$$x(t) \quad \overline{T}_{\Delta A}(t,0)x(0) + \int_0^t \overline{T}_{\Delta A}(t,s)[B(s) + B(s)]u(s)\,ds.$$

So according to the results above, the following definition will be presented:

Definition (2.4):

A continuous function x C[[0,t):H), given by:

$$x(t) \quad \overline{T}_{\Delta A}(t,0)x_0 + \int_0^t \overline{T}_{\Delta A}(t,s)[B(s) + B(s)]u(s)\,ds , \qquad (13)$$

will be called a mild solution to the linear varying perturbation initial-value control problem .

The concept of controllability is concerned with the question of existence of an admissible control, which stress any state to another state of the system in finite time. Depending on the properties involved on defining different concepts of the controllability various aspects of controllability results can be found in [2], [3], [8], [9] and the reference there in.

Definition (2.5):

The system (11) is called globally null-controllable in time t > 0 if for every initial state x_0 there is a admissible control u(t) L^2([0, t]: U), such that:

$$\overline{T}_{\Delta A}(t,0)x_0 + \int_0^t \overline{T}_{\Delta A}(t,s)[B(s) + B(s)]u(s)\,ds \quad 0.$$

In order to prove our problem , we need the following remark.

Remark (2.3):

Associated with the system (11), we consider the following Perturbed Ricatti Operator Equation (PROE):

$\dot{p}$(t)+[A(t)+A(t)]*p(t)+p(t)[A(t)+A(t)]p(t)[B(t)+B(t)][B(t)+B(t)]*p(t)+Q(t)0,t $\geq$ 0.

(14)

Since A(t) + A(t) is unbounded, it is not clear a priori what a solution of PROE(14) is. We will generalize and define as in [1] that an operator p(t) $\mathcal{L}(\overline{H_t})$ is a mild solution of PROE (14) if the scalar function <p(t)x, x>$^{\overline{H}_t,H}$ is differentiable for every x H; and:

<$\dot{p}$(t)x,x>$^{\overline{H}_t,H}$+<p(t)x,[A(t)+A(t)]x>$^{\overline{H}_t,H}$<p(t)[B(t)+B(t)].[B(t)+B(t)]*p(t)x,x>$^{\overline{H}_t,H}$ + <Q(t)x, x>$^{\overline{H}_t,H}$=0, for all x D(A(t) + A(t)) and t $\geq$ 0.

Definition (2.6) [42]:

Consider the initial value control system is:

$\dot{x}$ (t) A(t)x(t) +B(t) u(t), t 0

x(0) x_0 (15)

where A(t) : H H is unbounded linear operator and B(t) : U H is bounded linear control operator satisfying the conditions in [2], [7] such that the system (15) has a unique solution for every u(t) $\in L^2([0,\infty):U)$.The system (15) is called Q(t)-stabilizable,[see 13], if for every initial state, there is a control u(t)$\in L^2([0,\infty):U)$, such that the cost function will be

$$J(u) \int_0^\infty [<Q(s)\,x(s), x(s)>_H + \|u(s)\|_U^2]\,ds < \infty \qquad (16)$$

Assume that the system (15) is Q(t)-stabilizable, where Q(t)$\in LCO([0,\infty):H^+)$ is bounded on $[0,\infty)$,where (LOC is standard for the set of all linear bounded self-adjoint non-negative definite operator-valued function in H^+ **is a positive real Hilbert space**). Then the PROE of the system (15) has a solution P(t) $\in LCO([0,\infty):H^+)$bounded on $[0,\infty)$.

From the previous remark the following result of globally null-controllable for the suggested linear perturbed dynamical control system in infinite-dimensional spaces with unbounded perturbed control operator .

<u>Proposition (2.1):</u>

If the linear varying perturbed unbounded linear control system (11) is globally null-controllable in some finite time on H_t^+, then for any Q(t) LCO([0,]: H_t^+) bounded on [0,), the PROE (14) has a bounded solution p(t) LCO([0,): $\overline{H}_t^+$).

Proof:

Assume that the system (11) is globally null-controllable in some time $t_1 > 0$. Let us take any operator Q(t) LCO([0,): $H_t^{!}$) and consider the cost function (16).Due to the definition of global null-controllability, for every initial state x_0 H, there is a control u(t) $L_2([0, t_1]: U)$ such that the solution $x(t, t_0)$ of the system, according to the control u(t), satisfies:

$x(t_0)$ x_0, $x(t_1)$ 0.

Let us denote by $u_x(t)$ an admissible control according the solution x(t) of the system. Define:

$\tilde{u}(t)$ $\begin{cases} u_x(t), & t \in [0, t_1) \\ 0, & t \in [t_1, \infty). \end{cases}$ If $\tilde{x}$(.) is the solution corresponding to $\tilde{u}$(.),

$\tilde{x}(t)$ $\overline{T}_{\Delta A}(t,0)x_0 + \int_0^t \overline{T}_{\Delta A}(t,\tau)$ [B() + B()] $\tilde{u}$() d .for t [0,), then:

$\tilde{x}(t)$ $\overline{T}_{\Delta A}(t,0)\, x_0 + \int_0^t \overline{T}_{\Delta A}(t,\tau)$ [B() + B()] $\tilde{u}$() d 0.

For $t > t_1$.Therefore, every initial state x_0, there is a control $\tilde{u}(t)$ L^2([0,): U), such that:

$J(\tilde{u})\int_0^{\infty}$ [<Q(s) $\tilde{x}$(s), $\tilde{x}$(s)>$_H$+|| $\tilde{u}$(s)||$_U^2$]ds $\int_0^{t_1}$ [<Q(s) $\tilde{x}$(s), $\tilde{x}$(s)>$_H$+|| $\tilde{u}$(s)||$_U^2$] ds <

which implies that (11) is Q(t)-stabilizable and hence Remark (2.4), implies that the PROE (14) has a solution.

Theorem (2.1):

Suppose that assumptions (1)-(7) of problem formulation (2) are held. The H_∞ optimal control problem for semi-linear system (2) has a solution if:

- B(t) $\mathcal{L}$(U, $\overline{H}_t$), t 0, is $T_A$admissible perturbed control linear defined from U into $\overline{H_t}$, such that [B(.) + B(.)] $\mathcal{L}$(U, $\overline{H}_t$) satisfies the conditions in (9) and (10) where $\overline{H}_t$, t 0 is a extension Hilbert spaces in which H is densely and continuously embedded in $\overline{H_t}$.
- System (4.11) is globally null-controllable in finite time and

  $$p \quad \sup_{t\in\ +} \|p(t)\|_{\overline{H}_t},$$

  where p(t) is the solution of PROE (14)
- 1 $2c_1p + c_2(b + {}_b)p^2 + p_a + p^2(b_1 + c_3 + \Delta_{b_1})^2 + (c_4 + c_5(b + b)p)(2c_4 + 2dp(b + {}_b) + c_4 + c_5(b + {}_b)p) > 0.$ (17)
- The feedback control :

  u(t) [B(t) + B(t)]*p(t)x(t), (18)

  robustly stabilizing the system (11).

Proof:

Since the system (11) is a globally null-controllable infinite time.Thus from proposition (2.1),the PROE (14) has a bounded linear solution p(t) LCO([0,): $\overline{H}_t^+$), t 0, where Q(t) C*(t)C(t) + I 0 is a bounded linear operator on [0,).

Let us consider the scalar function:

$$V(t, x(t)) \quad <p(t)x(t), x(t)>_{\overline{H}_t,H}$$

Using the feedback control (18) the derivative of V(.) along the solution x(t) of the closed loop system is:

$$\dot{V}(t, x(t)) \quad <\dot{p}(t)x(t), x(t)>_{\overline{H}_t,H} + <p(t)\dot{x}(t), x(t)>_{\overline{H}_t,H} + <p(t)x(t), \dot{x}(t)>_{\overline{H}_t,H}$$

$$<p(t)x(t), x(t)>^{\bar{H}_t,H} + <p(t)\dot{x}(t), x(t)>^{\bar{H}_t,H} + <x(t),p^*(t)\dot{x}(t)>^{\bar{H}_t,H} \quad . \qquad (19)$$

Now, since p(t) LCO([0,), $\bar{H}_t^+$), implies that p(t) p*(t), t 0, and we have that:

$$\dot{V}(t, x(t)) \;\; <\dot{P}(t)x(t), x(t)>^{\bar{H}_t,H} + 2<p(t)\dot{x}(t), x(t)>^{\bar{H}_t,H}$$

By using (14), (12) and (5), we have that:

$\dot{V}(t, x(t))$ <([A(t) + A(t)]*p(t) p(t)[A(t) + A(t)] + p(t)[B(t) + B(t)][B(t)

+B(t)]*p(t) C*(t)C(t)I)x(t),x(t)>$^{\bar{H}_t,H}$ + 2p(t)(A(t)x(t) + B(t)u(t) + B_1(t)w(t)

+ [A(t)x(t) +B(t)u(t) +B_1(t)w(t)] + F(t, x, u, w)), x(t)>$^{\bar{H}_t,H}$

<p(t)x(t), [A(t) + A(t)]x(t)>$^{\bar{H}_t,H}$ <p(t)[A(t) +A(t)]x(t), x(t)>$^{\bar{H}_t,H}$ +<p(t)[B(t) + B(t)][B(t) + B(t)]*p(t)x(t), x(t)>$^{\bar{H}_t,H}$ <C*(t)x(t), x(t)>$^{\bar{H}_t,H}$ <x(t), x(t)>$_H$ + 2<p(t)A(t)x(t), x(t)>$^{\bar{H}_t,H}$ +2<p(t)B(t)u(t), x(t)>$^{\bar{H}_t,H}$+2<p(t)B_1(t)w(t), x(t)>$^{\bar{H}_t,H}$+2<p(t)A(t)x(t), x(t)>$^{\bar{H}_t,H}$ + 2<p(t)B(t)u(t), x(t)>$^{\bar{H}_t,H}$ + 2<p(t) B_1(t)w(t), x(t)>^{H_1,H}+<p(t)f(t, x, u, w), x(t)>$^{\bar{H}_t,H}$.

From proposition (2.1), we have that p(t) LOC([0,), $\bar{H}_t^+$), therefore:

<p(t)x(t), [A(t) + A(t)]x(t)>$^{\bar{H}_t,H}$ <x(t), p*(t)[A(t) + A(t)]x(t)>$^{\bar{H}_t,H}$

<x(t), p(t)[A(t) + A(t)]x(t)>$^{H,\bar{H}_t}$ <p(t)[A(t) + A(t)]x(t), x(t)>$^{\bar{H}_t,H}$.

Also from (18), we have that:

$$<p(t)B(t)u(t), x(t)>^{\bar{H}_t,H} \;\; <p(t)B(t)[B(t) + B(t)]^*p(t)x(t), x(t)>^{\bar{H}_t,H}.$$

Hence:

V(t, x(t)) <x(t),x(t)>$_H$

C*(t)C(t)x(t),x(t)>$_H$<p(t)B(t)[B(t)+B(t)]*p(t)x(t),x(t)>$^{\bar{H}_t,H}$+2<p(t) B_1(t)w(t),x(t)>$^{H_{-1},H}$ + 2<p(t)F(t,x,u,w),x(t)>$^{\bar{H}_t,H}$+2<p(t)A(t)x(t),x(t)>$^{\bar{H}_t,H}$

2<p(t)B(t)[B(t) +B(t)]*p(t)x(t), x(t)>$^{\bar{H}_t,H}$ + 2<p(t)B_1(t)w(t), x(t)>$^{\bar{H}_t,H}$ (20)

Now, since:

$<C(t)x(t),C(t)x(t)>_H \ 0$, $<[B(t)+B(t)]^*p(t)x(t), B^*(t)p(t)x(t)>_U \ 0$,

$<p(t)B(t)[B(t) + B(t)]^*p(t)x(t), x(t)>^{\Pi_t, H} \ 0$,

and using condition (3), the relation (19) becomes:

$\dot{V}(t,x(t)) \|x(t)\|^2_H + 2<p(t)B_1(t)w(t),x(t)>^{\Pi_t,H} + 2<p(t)F(t,x,u,w),x(t)>^{\Pi_t,H}$

$+2<p(t)A(t)x(t), x(t)>^{\Pi_t,H} + 2<p(t)B_1(t)w(t), x(t)>^{\Pi_t,H} \|x(t)\|^2_H$

$+2<p(t)B_1(t)w(t),x(t)>^{\Pi_t,H} + 2<p(t)F(t,x,u,w),x(t)>^{\Pi_t,H} + 2<p(t)A(t)x(t),x(t)>^{\Pi_t,H} + <p(t)B_1(t)w(t),x(t)>^{\Pi_t,H} \|x(t)\|^2_H + 2\|p(t)\|^{\Pi_t} + \|B_1(t)\|_H \|w(t)\|_w \|x(t)\|^H + 2\|p(t)\|^{\Pi_t} \|F(t, x, u, w)\|^H \|x(t)\|_H + 2\|p(t)\|^{\Pi_t} \|A(t)\|_H \|x(t)\|^2_H + 2\|p(t)\|^{\Pi_t} \|B_1(t)\|_{\mathcal{L}(W,H)} \|w(t)\|_W \|x(t)\|_H$ $\|x(t)\|^2_H + 2\|p(t)\|^{\Pi_t} \|B_1(t)\|_{\mathcal{L}(W,H)} \|w(t)\|_W \|x(t)\|_H + 2\|p(t)\|^{H_t} \|F(t, x, u, w)\|^H \|x(t)\|_H +$ $2\|p(t)\|^{\Pi_t} \|A(t)\|_H \|x(t)\|^2_H + 2\|p(t)\|^{\Pi_t} \|B_1(t)\|_{\mathcal{L}(W,H)} \|w(t)\|_W \|x(t)\|_H$

$\|x(t)\|^2_H + 2pb_1\|w(t)\|_w\|x(t)\|_H + 2p(c_1\|x(t)\|_H + c_2\|u(t)\|^U + c_3\|w(t)\|_W)\|x(t)\|_H +$
$2p_a\|x(t)\|^2_H + 2p^{b_1} \|w(t)\|^W \|x(t)\|_H.$ (21)

From equation (4.18), we have that:

$\|u(t)\|_U \ \|[B(t) + B(t)]^*p(t)x(t)\|_U \ (b + {}_b)p\|x(t)\|_H.$

Thus, relation (4.21) becomes:

$\dot{V}(t,x(t)) \|x(t)\|^2_H + 2pb_1\|w(t)\|_w . \|x(t)\|_H + 2p(c_1\|x(t)\|_H + c_2(b+{}_b) \|x(t)\|_H) + c_3\|w(t)\|_w)\|x(t)\|_H$

$+2p_a\|x(t)\|^2_H + 2p^{b_1} \|w(t)\|_w \|x(t)\|_H.$ (22)

From condition (6)of problem formulation (4) and assumption (2) of the theorem, we obtain:

$\dot{V}(t, x(t)) \ (2pc_1 + c_2(b + {}_b)p + 2p_a \ 1)\|x(t)\|^2_H + (2pb_1 + 2pc_3 + 2p^{a_1})\|w(t)\|_w \|x(t)\|_H,$ (23)

Integrating both sides of (23) from 0 to t, yields:

$<p(t)x(t),x(t)><p(0)x(0),x(0)>^{\Pi_t} {}_1 \int_0^{t_1} \|x(s)\|^2_H ds + 2_2 \int_0^{t} \|w(s)\|_W \|x(s)\|_H ds,$
where

${}_1 \ 2pc_1 + c_2(b + {}_b)p + 2p_a \ 1$, ${}_2 \ 2pb_1 + 2pc_3 + 2p^{b_1}$.

Thus:

$<p(t)x(t),\ x(t)>_{H_t,H} \quad <p(0)x(0),\ x(0)>_{H_t,H} \quad {}_1 \int_0^{t_1} \|x(s)\|_H^2 \, ds \ + \ 2_2 \int_0^t \|x(s)\|_H^2 \, ds \ \int_0^t \|w(s)\|_w^2 \, ds.$

Therefore:

$$\frac{< p(t)x(t), x(t) >_{H_t,H}}{\delta_1} \quad {}_3 + 2_4 \left\{ \int_0^t \| x(s) \|_H^2 \ \ ds \right\}^{1/2},$$

where ${}_3 \quad \dfrac{< p(0)x(0), x(0) >_{H_t,H}}{\delta_1}$, ${}_4 \quad \dfrac{\delta_2 \left\{ \int_0^\infty \| w(s) \|_w^2 \ \ ds \right\}^{1/2}}{\delta_1}$.

By completing the square we obtain:

$$\int_0^t \|x(t)\|_H^2 \ ds \quad {}_4 + \sqrt{\delta_4^2 + \delta_3}, \ t \quad] ^+,$$

letting t , the last inequality gives:

$$\lim_{t \to \infty} \int_0^t \| x(s) \|_H^2 ds \quad \int_0^\infty \| x(s) \|_H^2 \ ds \quad {}_4 + \sqrt{\delta_4^2 + \delta_3}.$$

Thus x L^2([0,):H^+), which implies that the closed loop control is robustly stabilizable

We need to show that the condition (ii) in remark (1.1) is satisfied

Now, consider the following relation:

$$\int_0^\infty [\|z(t)\|^2 \quad \|w(t)\|^2] \ dt \quad \int_0^\infty [\|z(s)\|_Z^2 \quad \|w(s)\|_W^2 \ \|w(t)\|^2 + \dot{V}(t, x(t))] \ dt$$

$$\int_0^\infty \dot{V}(t, x(t)) \ dt,$$

for any x(t) H and nonzero w(t) L_2([0,), W), we have:

$\dot{V}(t, x(t))\ \ \|x(t)\|^2_H\ \ \langle C^*(t)C(t)x(t), x(t)\rangle_H\ \ \langle p(t)B(t)[B(t) + B(t)]^*p(t)x(t), x(t)\rangle_{\bar{\Pi}_t,H}$ + $2\langle p(t)B_1(t)w(t),x(t)\rangle_{\bar{\Pi}_t,H} + 2\langle p(t)f(t, x, u, w), x(t)\rangle_{\bar{\Pi}_t,H} + 2\langle p(t)A(t)x(t), x(t)\rangle_{\bar{\Pi}_t,H}$

$2\langle p(t)B(t)B^*(t)p(t)x(t), x(t)\rangle_{\bar{\Pi}_t,H} + 2\langle p(t)B_1(t)w(t), x(t)\rangle_{\bar{\Pi}_t,H}$

Moreover, from the proof of proposition (2.1), the solution x(t) 0 when t , we have:

$$\int_0^\infty \dot{V}(t, x(t))\, dt\ \ V(, x())\ \ V(0, x_0)\ \ V(0, x_0)\ \ \langle p(0)x_0, x_0\rangle_{\bar{\Pi}_t,H}$$

where the initial condition p(0) 0. Therefore:

$$\int_0^\infty [\|z(s)\|^2_Z\ \|w(s)\|^2_w]dt\ \ \int_0^\infty [\|z(s)\|^2_Z\ \|w(s)\|^2_w + \dot{V}(t, x(t))]dt$$

By (4.2), we have that:

$\langle z(t),z(t)\rangle_Z\ \ \langle C(t)x(t),z(t)\rangle_Z + \langle D(t)u(t),z(t)\rangle_Z + \langle G(t,x,u),z(t)\rangle_Z\ \langle C(t)x(t), C(t)x(t)\rangle_Z + \langle C(t)x(t), D(t)x(t)\rangle_Z + \langle C(t)x(t), G(t, x, u)\rangle_Z + \langle D(t)u(t), C(t)x(t)\rangle_Z + \langle D(t)u(t), G(t, x, u)\rangle_Z + \langle G(t, x, u), C(t)x(t)\rangle_Z + \langle G(t, x, u), D(t)u(t)\rangle_Z + \langle G(t, x, u), g(t, x, u)\rangle_Z$. (24)

By using condition (6) of problem formulation (4), we have that:

$$\langle C(t)x(t), D(t)x(t)\rangle_Z\ \ \langle x(t), C^*(t)D(t)x(t)\rangle_H\ \ 0, \tag{25}$$

$$\langle D(t)u(t), C(t)x(t)\rangle_Z\ \ \langle C^*(t)D(t)u(t), x(t)\rangle_H\ \ 0, \tag{26}$$

and

$$\langle D(t)u(t), D(t)u(t)\rangle_Z\ \ \langle D^*(t)D(t)u(t), u(t)\rangle_U\ \ \langle u(t), u(t)\rangle_U$$
$$\langle [B(t)+B(t)]^*p(t)x(t), [B(t) + B(t)]^*p(t)x(t)\rangle_U \tag{27}$$

From (25), (26) and (27), equation (24) becomes:

$\|z(t)\|^2_Z\ \ \langle C^*(t)C(t)x(t), x(t)\rangle_H + \langle G(t, x, u), C(t)x(t)\rangle_Z + \langle [B(t) + B(t)]^*p(t)x(t), [B(t) + B(t)]^*p(t)x(t)\rangle_U + \langle G(t, x, u), D(t)u(t)\rangle_Z + \langle G(t, x, u), C(t)x(t)\rangle_Z + \langle G(t, x, u), D(t)u(t)\rangle_Z + \|G(t, x(t), u(t))\|^2_Z$.

Hence:

$\|z(t)\|^2_Z\ \ \langle C^*(t)C(t)x(t), x(t)\rangle_H + 2\langle G(t, x(t), u(t)), C(t)x(t) + D(t)u(t)\rangle_Z + \langle p(t)[B(t) + B(t)][B(t) + B(t)]^*p(t)x(t), x(t)\rangle_{\bar{\Pi}_t,H} + \|G(t, x(t), u(t))\|^2_Z$. (26)

From (4.26), we have that:

$$\int_0^\infty [\|z(t)\|_Z^2 \ \|w(t)\|_W^2]\, dt \int_0^\infty [\|z(t)\|_Z^2 \|w(t)\|_W^2]dt + \dot{V}(t, x(t)) + \langle p(0)x_0, x_0\rangle^{\bar{\Pi}_t,H}\, dt$$

$$\int_0^\infty [\langle C^*(t)C(t)x(t), x(t)\rangle_H + \langle p(t)[B(t) + B(t)][B(t)+ B(t)]^*p(t)x(t), x(t)\rangle^{\bar{\Pi}_t,H} + 2\langle G(t,$$

$$x, u),C(t)x(t)\ D(t)u(t)\rangle_Z+\|G(t, x(t), u(t))\|_Z^2 \langle C^*(t)C(t)x(t), x(t)\rangle_H + 2\langle p(t).$$

$$F(t, x(t),u(t),w(t)),x(t)\rangle^{\bar{\Pi}_t,H} + 2\langle p(t)A(t)x(t), x(t)\rangle^{\bar{\Pi}_t,H} \ 2\langle p(t)B(t)[B(t) +$$

$$B(t)]^*p(t)x(t),x(t)\rangle^{\bar{\Pi}_t,H} + 2\langle p(t)B_1(t)w(t), x(t)\rangle^{\bar{\Pi}_t,H}\langle w(t), w(t)\rangle_W+\langle p(0)x_0, x_0\rangle^{\bar{\Pi}_t,H}$$

$$\langle x(t), x(t)\rangle_H .$$

$$\int_0^\infty [\|z(t)\|_Z^2 \ \|w(t)\|_W^2]\, dt \int_0^\infty \langle x(t), x(t)\rangle_H + 2\langle G(t, x, u), C(t)x(t) + D(t)u(t)\rangle_Z$$

$$+\|g(t, x(t), u(t))\|_Z^2 + 2\langle p(t)B_1(t)w(t), x(t)\rangle^{\bar{\Pi}_t,H} \ 2\langle p(t)B(t)[B(t) + B(t)]^*p(t)x(t),$$

$$x(t)\rangle^{\bar{\Pi}_t,H} + 2\langle p(t)B_1(t)w(t), x(t)\rangle^{\bar{\Pi}_t,H} \ \langle w(t), w(t)\rangle^{W} + \langle p(0)x_0, x_0\rangle^{\bar{\Pi}_t,H}$$

$$+ 2\langle p(t)f(t, x(t), u(t), w(t)), x(t)\rangle^{\bar{\Pi}_t,H}.$$

$$\int_0^\infty [\|z(t)\|_Z^2 \ \|w(t)\|_W^2]\, dt \int_0^\infty \|x(t)\|_H^2 + 2\|G(t, x(t),$$

$$u(t))\|_Z(\|C(t)\|^Z\|x(t)\|_H+\|D(t)\|^Z\|u(t)\|_U)+\|G(t, x(t), u(t))\|_Z^2 + 2\langle p(t)B_1(t)w(t),$$

$$x(t)\rangle^{\bar{\Pi}_t,H}+2\langle p(t)A(t)x(t),x(t)\rangle_H \ 2\langle p(t)B(t)\ [B(t) + B(t)]^*p(t)x(t), x(t)\rangle^{\bar{\Pi}_t,H} +$$

$$2\langle p(t)B_1(t)w(t),x(t)\rangle^{\bar{\Pi}_t,H}\langle w(t),w(t)\rangle_W\langle p(0)x_0, x_0\rangle^{\bar{\Pi}_t,H}+ 2\langle p(t)f(t, x(t), u(t),w(t)),$$

$$w(t)\rangle^{\bar{\Pi}_t,W}$$

$$\int_0^\infty [\|z(t)\|_Z^2 \ \|w(t)\|_W^2]\, dt \int_0^\infty \|x(t)\|_H^2 + 2\|G(t, x(t), u(t))\|_Z(\|C(t)\|^Z\|x(t)\|^H$$

$$+ \|D(t)\|^Z \ \|u(t)\|^U)+ \|g(t, x(t),u(t))\|_Z^2 + 2\|p(t)\|^{H_t} \|B_1(t)\|^H \ \|w(t)\| \ \|x(t)\|^H +$$

$$2\|p(t)\|^{\bar{\Pi}_t} \|A(t)\|^H \|x(t)\|_H^2 \ 2\|p(t)\|^{\bar{\Pi}_t} \|B(t)\|^H \ \|[B(t) + B(t)]^*\|^U \ \|p(t)\| \ \|x(t)\|_H^2 +$$

$2\|p(t)\|_{\overline{\Pi}_t} \|B_1\|_H \|w(t)\|_W\|x(t)\|_H \quad \|w(t)\|_W^2 \quad \|p(0)\|_{\overline{\Pi}_t} \|x_0\|_H^2 + 2\|p(t)\|_{\overline{\Pi}_t} \|F(t, x(t), u(t), w(t))\|_H$

$\|x(t)\|_H \int_0^\infty \|x(t)\|_H^2 + 2c_4C\|x(t)\|_H^2 + 2c_4d\|x(t)\|_H^2 + 2c_5p(b + {}_b)c\|x(t)\|_H^2 + 2c_5p(b + {}_b)d\|x(t)\|_H^2 + c_4^2\|x(t)\|_H^2 + 2c_4c_5p(b + {}_b)\|x(t)\|_H^2 + c_5^2 p^2(b + {}_b)^2 \|x(t)\|_H^2$

$+2pb_1\|w(t)\|_W\|x(t)\|_H + 2p_a\|x(t)\|_H^2 + 2p^{b_1} \|w(t)\|_W \|x(t)\|_H^2$

$\|w(t)\|^2 \quad \|p(0)\|_{\overline{\Pi}_t} \|x_0\|_H^2 + 2p(c_1\|x(t)\|_H + c_2p(b + {}_b)\|x(t)\|_H + c_3\|w(t)\|_W)\|x(t)\|$

$\int_0^\infty [1 + c_4c + 2c_4d + 2c_5p(b + {}_b)c + 2p_a + 2c_5p(b + {}_b)d + c_4^2 + 2c_4c_5p(b + {}_b) + c_5^2 p^2(b + {}_b)^2 + 2c_1p + c_2p^2(b + {}_b)]\|x(t)\|_H^2 + [2p^{b_1} + 2pb_1 + 2pc_3]\|w(t)\|_W \quad \|x(t)\|_H$

$\|w(t)\|_W^2 + \|p(0)\|_{\overline{\Pi}_t} \|x_0\|_H \int_0^\infty [2c_1p + c_2(b + {}_b)p^2 + p_a + p^2(b_1 + c_3 + {}^{b_1})^2 + (c_4 + c_5(b + {}_b)p)(2c + 2dp(b + {}_b) + c_4 + c_5(b + {}_b)p) \quad 1] \|x(t)\|_H \, dt + \|p(0)\|_{\overline{\Pi}_t} \|x_0\|_H^2$

From (4.17), we obtain:

$1 + 2c_1p + c_2(b + {}_b)p^2 + p_a + p^2(b_1 + {}^{b_1})^2 + (c_4 + c_5(b + {}_b)p)(2c + 2dp(b + {}_b) + c_4 + c_3(b + {}_b)p) < 0$ .Hence:

$$\int_0^\infty [\|z(t)\|_z^2 \quad \|w(t)\|_W^2] \, dt \quad \| p(0) \|_{H_t} \| x_0 \|_H^2.$$

Setting $c_0 \quad \frac{\| p(0) \|}{\gamma}$, therefore: $\int_0^\infty [\|z(t)\|_z^2 \quad \|w(t)\|_W^2] \, dt < c_0\|x_0\|_H^2$

$$\int_0^\infty \|z(t)\|_z^2 \, dt < c_0\|w(t)\|_W^2 + \int_0^\infty \|w(t)\|_W^2 \, dt \qquad \frac{\int_0^\infty \| z(t) \|_z^2 \, dt}{\int_0^\infty \| w(t) \|_w^2 \, dt + c_0 \| x_0 \|_H^2} <$$

for all w(t) L^2([0,), W) and x_0 X provided condition(ii) in remark(1.1). This completes the proof of the theorem.

د.راضي علي زبون
قسم الرياضيات
كلية العلوم
الجامعة المستنصرية

د. جهاد رمضان خضر
فرع الرياضيات
الجامعة التكلونوجية

د.سمير قاسم حسن
قسم الرياضيات
كلية التربية
الجامعة
لمستنصرية

المستخلص

لقد تم عرض مسالة سيطرة مثلى من نوع H_∞ والمعرفة على فضاء ذات بعد غير منتهي. ولقد ناقشنا وطورنا قابلية الحل وقابلية السيطرة و قابلية السيطرة الملغية و كذلك معادلة ريكاتي ذات مؤثر قلقلة وبعض الخواص الاخرى لمسائل سيطرة مثلى بمنهجية شبه الزمرة المقلقلة ذو المعلمتين و مستمر بقوة والذي يتولد بمولد قلقلة غير مقيد.

References

[1] Balachandran,K., "Existence of solutions and controllability of nonlinear integrodifferential systems in Banach spaces ", 2002.
[2] Bensoussan,A. and Da Parto G., Delfour, M. C. and Mitter, S.K., "Representation and control of infinite-dimensional system", Vol., II, Birkhauser, 1992.
[3] Conti, R., "Infinite dimensional linear controllability". Math. Reports, University of Minnesota, 1982, 82-127.
[4] Fahmy S.F.F and Banks S.P .," Robust $H\infty$ control of uncertain nonlinear dynamical systems via linear time-varying", Nonlinear Functional and its Application ,63(2005); 2315-2327.
[5] Francis, B.A.," A Course in $H\infty$ Control Theory". Springer-Verlag, Berlin.
[6] Ichikawa, A., " Product of nonnegative operators and infinite-dimensional H_∞ Riccati equations". Systems and Control Letters", **41**(2000), 183-188.
[7] Pazy, A., "Semigroup of linear operator and applications to partial differential equation", Springer-Verlage, New York, Inc., 1983.
[8] Phat, V.N., "Constrained control problems of discrete processes". World Scientific, Singapor, 1996.
[9] Phat, V.N. and Kiet, T.T., "Globall controllability to a target of discrete-time systems in Banach spaces". Nonl. Funct. Anal. Appl., 5(2000), 23-37.
[10] Petersen I.R., V.A. Ugrinovskii and A.V. Savkin, "Robust Control Design Using H_∞ Methods". Springer-Verlag, London, 2000.
[11] Petersen I.R. and A.V. Savkin, "Robust Kalman Filtering for Signal and Systems

with Large Uncertainties ". Birkhauser, Boston, 1999.
[12] Phat V. N., New stabilization criteria for linear time-varying systems with state delays and norm-bounded uncertainties. IEEE Trans. On Autom *Contr.,* **47**(2002), 2095-2098.
[13] Phat V.N.,"Nonlinar *H*∞ optimal control in Hilbert spaces via Riccati operator equations". Nonlinear Functional. Analysis and its Application 9(2004);79-92
[14] Savkin A.V and I.R. Petersen, Robust *H*∞ control of uncertain systems with structured uncertainty. J. Math. Syst. Estim. Contr., **6**(1996), 339-342.
[15] Shengyuan Xu, James Lam and Chengwu Yang ," Robust *H*∞ Control for Uncertain Singular Systems with State Delay ", 31(2003),1213-1223.
[16] Van der Schaft A.J , On a state space approach to nonlinear *H*∞ control.Syst. Contr. Letters, **16**(1991), 1-8.
[17] Zhijian Ji, Xiaoxia Guo, Long Wang and Guangming Xie," Adaptive Robust *H*∞ State Feedback Control for Linear Uncertain Systems With Time-Varying Delay",22(2008),845-858.
[18] Zhijian Ji, Xiaoxia Guo, Long Wang and Guangming Xie," Robust *H*∞ Control and Quadratic Stabilization of Discrete-time Switched Systems With Norm-Bounded Time-Varying Uncertainties, 9(2007), 352-361.